

Particle Motion Problems

1) A particle moves along the x-axis so that its position at time t is given by $s(t) = t^2 - 6t + 5$.

- For what value of t is the velocity of the particle zero?
- For what intervals does the particle move right? Left?
- What is the acceleration and does it change?

2) A particle moves along the x-axis so that its position at time t is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$.

- For what value of t is the velocity of the particle zero?
- For what intervals does the particle move right? Left?
- What is the acceleration and does it change?

3. The position of a particle moving in a straight line during a 5 second trip is $s(t) = t^2 - t + 10$ cm. Find the time t at which the instantaneous velocity is equal to the average velocity for the entire trip.

4. A particle moving along a line has position $s(t) = t^4 - 18t^2$ meters at time t seconds. At which time does the particle pass through the origin? At which times is the particle instantaneously motionless (zero velocity)?

5. The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is..

Particle Motion Problem Solutions

1) $s(t) = t^2 - 6t + 5$
 $v(t) = 2t - 6$
 $a(t) = 2$

a) $2t - 6 = 0$
 $2t = 6$ $t = 3$

b) $t=0$ $v(t)=\text{neg}$ left | $t=10$ $v(t)=\text{pos}$ right | t
3

left	$(-\infty, 3)$	$-\infty < t < 3$
right	$(3, \infty)$	$3 < t < \infty$

c) $a(t) = \boxed{2}$ for any time t .

2) $x(t) = 2t^3 - 21t^2 + 72t - 53$
 $v(t) = 6t^2 - 42t + 72$

a) $\Rightarrow 6(t^2 - 7t + 12) = 6(t-4)(t-3)$

Velocity zero at $t=4, 3$

b) \rightarrow $t=0$ $v(t)=\text{pos}$ right | $t=3.5$ $v(t)=\text{neg}$ left | $t=4$ $v(t)=\text{pos}$ right | t
3 4

right $(-\infty, 3) \cup (4, \infty)$
left $(3, 4)$

c) $a(t) = 12t - 42$
yes acceleration changes at $12t - 42 = 0$
 $t = \frac{42}{12} = \boxed{3.5}$

3) $s(t) = t^2 - t + 10$
 $v(t) = 2t - 1$

↑ instantaneous velocity

Avg velocity of 5-sec trip

$$\frac{s(5) - s(0)}{5} = \frac{30 - (10)}{5} = 4$$

$$v(t) = 2t - 1 = 4$$

$$2t = 5$$

$t = 5/2 \text{ sec}$

$$4) s(t) = t^4 - 18t^2$$

a) pass through origin (when $s(t) = 0$)

Factor

$$t^4 - 18t^2 = t^2(t^2 - 18) = 0$$

$$\therefore \boxed{t = 0 \text{ sec}} \text{ or } \boxed{t = 3\sqrt{2} \approx 4.24 \text{ sec}}$$

b) when $v(t) = 0$?

$$4t^3 - 36t = 0$$

$$t(4t^2 - 36) = 0$$

$$\downarrow$$

$$t = 0 \text{ sec}$$

$$\downarrow$$

$$4t^2 - 36 = 0$$

$$4t^2 = 36$$

$$t^2 = 9$$

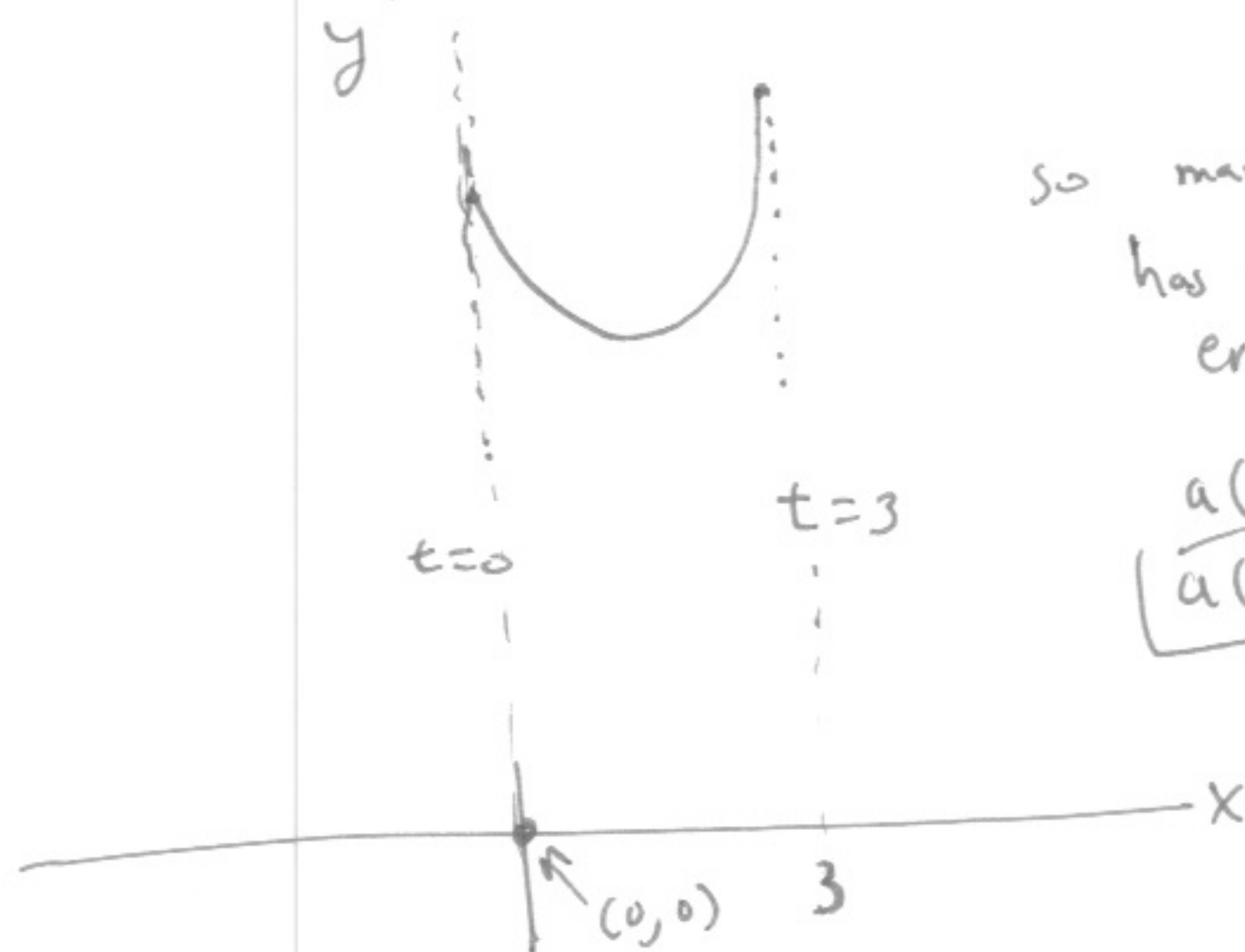
at $t = 0, 3, -3$ seconds

usually AP test
will state $t \geq 0$
for answers.

$$5) v(t) = t^3 - 3t^2 + 12t + 4 \quad \text{for } 0 \leq t \leq 3$$

$$a(t) = 3t^2 - 6t + 12$$

parabola looks like



so max acceleration
has to be at the
end points

$$a(0) = 12$$

$$\boxed{a(3) = 21} \leftarrow \text{max}$$

$$\boxed{t=3}$$