9. $F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{\frac{1}{4}}$

$F'(x) = \frac{1}{4}(1+2x+x^3)^{\frac{-3}{4}} \cdot \frac{d}{dx}(1+2x+x^3)$

$= \frac{1}{4}(1+2x+3)^{\frac{-3}{4}}(2+3x^2)$

11. $g(t) = \dfrac{1}{(t^4+1)^3} = (t^4+1)^{-3}$

$g'(t) = -3(t^4+1)^{-4} \cdot \frac{d}{dt}(t^4+1) = -3(t^4+1)^{-4} \cdot 4t^3$

$= -12t^3(t^4+1)^{-4}$

13. $y = \cos(a^3+x^3)$ Note: a is a constant

$y' = -\sin(a^3+x^3) \cdot \frac{d}{dx}(a^3+x^3)$

$= -\sin(a^3+x^3) \cdot (0+3x^2)$

$= -3x^2 \sin(a^3+x^3)$

15. $y = x\sec(kx)$ PR Note: k is a constant

$y' = x \cdot \frac{d}{dx}\sec(kx) + \sec(kx) \cdot \frac{d}{dx}x$

$= x\left[\sec(kx)\tan(kx) \cdot \frac{d}{dx}(kx)\right] + \sec(kx) \cdot 1$

$= x\left[\sec(kx)\tan(kx) \cdot k\right] + \sec(kx)$

$= kx\sec(kx)\tan(kx) + \sec(kx)$

$= \sec(kx)\left[kx\tan(kx) + 1\right]$

17 $g(x) = (1+4x)^5(3+x-x^2)^8$ PR

$g'(x) = (1+4x)^5 \cdot \frac{d}{dx}(3+x-x^2)^8 + (3+x-x^2)^8 \cdot \frac{d}{dx}(1+4x)^5$

$= (1+4x)^5 \cdot 8(3+x-x^2)^7 \cdot \frac{d}{dx}(3+x-x^2)$
$+ (3+x-x^2)^8 \cdot 5(1+4x)^4 \cdot \frac{d}{dx}(1+4x)$

$= 8(1+4x)^5(3+x-x^2)^7(1-2x) + 5(3+x-x^2)^8(1+4x)^4 \cdot 4$

$= 4(1+4x)^4(3+x-x^2)^7[2(1+4x)(1-2x) + 5(3+x-x^2)]$

$= 4(1+4x)^4(3+x-x^2)^7[2(1+2x-8x^2) + 15 + 5x - 5x^2]$

$= 4(1+4x)^4(3+x-x^2)^7(17+9x-21x^2)$

19. $y = (2x-5)^4(8x^2-5)^{-3}$ PR

$y' = (2x-5)^4 \cdot \frac{d}{dx}(8x^2-5)^{-3} + (8x^2-5)^{-3} \cdot \frac{d}{dx}(2x-5)^4$

$= (2x-5)^4 \cdot (-3)(8x^2-5)^{-4} \cdot \frac{d}{dx}(8x^2-5)$
$+ (8x^2-5)^{-3} \cdot 4(2x-5)^3 \cdot \frac{d}{dx}(2x-5)$

$= -3(2x-5)^4(8x^2-5)^{-4} \cdot 16x + 4(8x^2-5)^{-3}(2x-5)^3 \cdot 2$

$= 8(2x-5)^3(8x^2-5)^{-4}[-6x(2x-5) + (8x^2-5)^1]$

$= 8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5)$

21. $y=\left(\dfrac{x^2+1}{x^2-1}\right)^3$

$$y'=3\left(\frac{x^2+1}{x^2-1}\right)^2\cdot\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$
$$=3\left(\frac{x^2+1}{x^2-1}\right)^2\cdot\frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}$$
$$=\frac{3(x^2+1)^2}{(x^2-1)^2}\cdot\frac{-4x}{(x^2-1)^2}$$
$$=\frac{-12x(x^2+1)^2}{(x^2-1)^4}$$

23. $y=\sin(x\cos(x))$

$$y'=\cos(x\cos(x))\cdot\frac{d}{dx}(x\cos(x))$$
$$=\cos(x\cos(x))\cdot[x(-\sin(x))+\cos(x)\cdot 1]$$
$$=\cos(x\cos(x))[-x\sin(x)+\cos(x)]$$

25. $F(z)=\sqrt{\dfrac{z-1}{z+1}}=\left(\dfrac{z-1}{z+1}\right)^{\frac{1}{2}}$

$$F'(z)=\frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-\frac{1}{2}}\cdot\frac{d}{dz}\left(\frac{z-1}{z+1}\right)$$
$$=\frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-\frac{1}{2}}\cdot\frac{(z+1)(1)-(z-1)(1)}{(z+1)^2}$$
$$=\frac{1}{2}\cdot\frac{(z-1)^{-\frac{1}{2}}}{(z+1)^{-\frac{1}{2}}}\cdot\frac{2}{(z+1)^2}$$
$$=\frac{(z-1)^{-\frac{1}{2}}}{(z+1)^{\frac{3}{2}}}$$
$$=\frac{1}{(z-1)^{\frac{1}{2}}(z+1)^{\frac{3}{2}}}$$

27. $y = \dfrac{r}{\sqrt{r^2+1}} = \dfrac{r}{(r^2+1)^{1/2}}$ QR

<u>or</u> $y = r(r^2+1)^{-\frac{1}{2}}$ PR

QR $y' = \dfrac{(r^2+1)^{\frac{1}{2}} \cdot 1 - r \cdot \frac{d}{dr}(r^2+1)^{\frac{1}{2}}}{[(r^2+1)^{\frac{1}{2}}]^2}$

$= \dfrac{(r^2+1)^{\frac{1}{2}} - r \cdot \frac{1}{2}(r^2+1)^{-\frac{1}{2}} \cdot \frac{d}{dr}(r^2+1)}{r^2+1}$

$= \dfrac{(r^2+1)^{\frac{1}{2}} - \frac{1}{2}r(r^2+1)^{-\frac{1}{2}} \cdot (2r)}{r^2+1}$

$= \dfrac{(r^2+1)^{\frac{1}{2}} - r^2(r^2+1)^{-\frac{1}{2}}}{r^2+1}$

$= \dfrac{(r^2+1)^{-\frac{1}{2}}[(r^2+1) - r^2]}{(r^2+1)}$

$= \dfrac{1}{(r^2+1)^{\frac{3}{2}}}$

29. $y = \sin(\tan(2x))$

$y' = \cos(\tan(2x)) \cdot \frac{d}{dx}\tan(2x)$

$= \cos(\tan(2x)) \cdot \sec^2(2x) \cdot \frac{d}{dx}(2x)$

$= 2\cos(\tan(2x))\sec^2(2x)$

31. $y = \sin(\sqrt{1+x^2}) = \sin((1+x^2)^{\frac{1}{2}})$

$y' = \cos((1+x^2)^{\frac{1}{2}}) \cdot \frac{d}{dx}(1+x^2)^{\frac{1}{2}}$

$= \cos((1+x^2)^{\frac{1}{2}}) \cdot \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(1+x^2)$

$= x\cos((1+x^2)^{\frac{1}{2}}) \cdot (1+x^2)^{-\frac{1}{2}}$

33 $y = \sec^2(x) + \tan^2(x) = [\sec(x)]^2 + [\tan(x)]^2$

$y' = 2[\sec(x)]' \cdot \frac{d}{dx}\sec(x) + 2[\tan(x)]' \cdot \frac{d}{dx}\tan(x)$

$= 2\sec(x) \cdot \sec(x)\tan(x) + 2\tan(x) \cdot \sec^2(x)$

$= 4\sec^2(x)\tan(x)$

35. $y = \left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)^4$

$y' = 4\left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)^3 \cdot \frac{d}{dx}\left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)$

$= 4\left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)^3 \cdot \frac{[1+\cos(2x)] \cdot \frac{d}{dx}(1-\cos(2x)) - [1-\cos(2x)] \cdot \frac{d}{dx}(1+\cos(2x))}{[1+\cos(2x)]^2}$

$= \frac{4[1-\cos(2x)]^3}{[1+\cos(2x)]^3} \cdot \frac{[1+\cos(2x)][\sin(2x)\cdot 2] - [1-\cos(2x)][-\sin(2x)\cdot 2]}{[1+\cos(2x)]^2}$

$= \frac{4[1-\cos(2x)]^3}{[1+\cos(2x)]^3} \cdot \frac{2\sin(2x) + 2\sin(2x)\cos(2x) + 2\sin(2x) - 2\sin(2x)\cos(2x)}{[1+\cos(2x)]^2}$

$= \frac{4[1-\cos(2x)]^3}{[1+\cos(2x)]^3} \cdot \frac{4\sin(2x)}{[1+\cos(2x)]^2}$

$= \frac{16\sin(2x)[1-\cos(2x)]^3}{[1+\cos(2x)]^5}$

37. $y = \cot^2(\sin(\theta)) = [\cot(\sin(\theta))]^2$

$y' = 2[\cot(\sin(\theta))]' \cdot \frac{d}{d\theta}\cot(\sin(\theta))$

$= 2\cot(\sin(\theta)) \cdot [-\csc^2(\sin(\theta))] \cdot \frac{d}{d\theta}\sin(\theta)$

$= -2\cot(\sin(\theta))\csc^2(\sin(\theta))\cos(\theta)$

39. $y = [x^2 + (1-3x)^5]^3$

$y' = 3[x^2 + (1-3x)^5]^2 \cdot \frac{d}{dx}[x^2 + (1-3x)^5]$

$= 3[x^2 + (1-3x)^5]^2 \cdot [2x + 5(1-3x)^4 \cdot (-3)]$

$= 3[x^2 + (1-3x)^5]^2 [2x - 15(1-3x)^4]$

41. $y = \sqrt{x + \sqrt{x}} = (x + x^{\frac{1}{2}})^{\frac{1}{2}}$

$y' = \frac{1}{2}(x + x^{\frac{1}{2}})^{\frac{-1}{2}} \cdot \frac{d}{dx}(x + x^{\frac{1}{2}})$

$= \frac{1}{2}(x + x^{\frac{1}{2}})^{\frac{-1}{2}} \cdot (1 + \frac{1}{2}x^{\frac{-1}{2}})$

43. $g(x) = [2r\sin(rx) + n]^p$

$g'(x) = p[2r\sin(rx) + n]^{p-1} \cdot \frac{d}{dx}[2r\sin(rx) + n]$

$= p[2r\sin(rx) + n]^{p-1} \cdot 2r\cos(rx) \cdot \frac{d}{dx}(rx)$

$= p[2r\sin(rx) + n]^{p-1} \cdot 2r\cos(rx) \cdot r$

$= 2pr^2\cos(rx)[2r\sin(rx) + n]^{p-1}$

45. $y = \cos\left(\sqrt{\sin(\tan(\pi x))}\right) = \cos\left((\sin(\tan(\pi x)))^{\frac{1}{2}}\right)$

$= -\sin\left((\sin(\tan(\pi x)))^{\frac{1}{2}}\right) \cdot \frac{d}{dx}\left[\sin(\tan(\pi x))\right]^{\frac{1}{2}}$

$= -\sin\left((\sin(\tan(\pi x)))^{\frac{1}{2}}\right) \cdot \frac{1}{2}\left[\sin(\tan(\pi x))\right]^{-\frac{1}{2}} \cdot \frac{d}{dx}\sin(\tan(\pi x))$

$= -\frac{1}{2}\sin\left(\sin(\tan(\pi x))\right)^{\frac{1}{2}}\right)\left[\sin(\tan(\pi x))\right]^{-\frac{1}{2}}$
$\cdot \cos(\tan(\pi x)) \cdot \frac{d}{dx}\tan(\pi x)$

$= -\frac{1}{2}\sin\left(\sin(\tan(\pi x)))^{\frac{1}{2}}\right)\left[\sin(\tan(\pi x))\right]^{-\frac{1}{2}}$
$\cdot \cos(\tan(\pi x)) \cdot \sec^2(\pi x) \cdot \frac{d}{dx}(\pi x)$

$= -\frac{1}{2}\pi \sin\left(\sin(\tan(\pi x)))^{\frac{1}{2}}\right)\left[\sin(\tan(\pi x))\right]^{-\frac{1}{2}}$
$\cdot \cos(\tan(\pi x)) \cdot \sec^2(\pi x)$

47. $h(x) = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot \frac{d}{dx}(x^2+1) = x(x^2+1)^{-\frac{1}{2}}$

$h''(x) = x \cdot \frac{d}{dx}(x^2+1)^{-\frac{1}{2}} + (x^2+1)^{-\frac{1}{2}} \cdot \frac{d}{dx}(x)$

$= x \cdot \left(-\frac{1}{2}\right)(x^2+1)^{-\frac{3}{2}} \cdot 2x + (x^2+1)^{-\frac{1}{2}} \cdot 1$

$= -x^2(x^2+1)^{-\frac{3}{2}} + (x^2+1)^{-\frac{1}{2}}$

$= (x^2+1)^{-\frac{3}{2}}\left[-x^2 + (x^2+1)\right]$

$= (x^2+1)^{-\frac{3}{2}}$

49 $H(t) = \tan(3t)$

$H'(t) = \sec^2(3t) \cdot \frac{d}{dt}(3t) = 3\sec^2(3t)$

$H'(t) = 3[\sec(3t)]^2$

$H''(t) = 3 \cdot 2[\sec(3t)]^1 \cdot \frac{d}{dt}\sec(3t)$

$= 6\sec(3t) \cdot \sec(3t)\tan(3t) \cdot \frac{d}{dt}3t$

$= 6\sec^2(3t)\tan(3t) \cdot 3$

$= 18\sec^2(3t)\tan(3t)$