Warm-Up

1) A dinner cruise on Newport Harbor offers a choice of 4 appetizers, 3 salads, 5 entrees, and 8 desserts. How many different dinners are possible if you choose 1 appetizer, 1 salad, 1 entrée and one dessert?

$4 \cdot 3 \cdot 5 \cdot 8 = 480$

2) If you need to arrange 12 books on a shelf, how many different orders are possible?

$_{12}P_{12} = 12\ 11\ 10 \cdots = 12!$

3) If you have twelve books, and want to arrange three of them on a shelf, how many possible orders?

$12 \cdot 11 \cdot 10 = {}_{12}P_3 = \frac{12!}{(12-3)!} = \frac{12!}{9!} = \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!} \cdots}{\cancel{9!}}$

7-1 Permutations and Combinations

Going Deeper

Essential question: *What are permutations and combinations and how can you use them to calculate probabilities?*

COMMON CORE **Standards for Mathematical Content**

CC.9-12.S.CP.9(+) Use permutations and combinations to compute probabilities of compound events and solve problems.*

Day 2

Suppose the club members also want to choose a secretary from the group of 7 eligible members. In how many different ways can the four positions (president, vice-president, treasurer, secretary) be filled? Explain.

840; there are 7 × 6 × 5 × 4 = 840 permutations

Suppose 8 members of the club are eligible to fill the original three positions (president, vice-president, treasurer). In how many different ways can the positions be filled? Explain.

336; there are 8 × 7 × 6 = 336 permutations

Every student at your school is assigned a four-digit code, such as 6953, to access the computer system. In each code, no digit is repeated. In how many ways are there to assigned a code with the digits 1, 2, 3, and 4 in any order?

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = {}_4P_4 = 24$

4P4 = 24

How many ways can a student government select a president, vice president, secretary, and treasurer from a group of 6 people?

$_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}}$

$= 360$

How many ways can a stylist arrange 5 of 8 vases from left to right in a store display?

$_8P_5$

$$_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

There are 6720 ways that the vases can be arranged.

In the probability world, the order in which items are arranged is important :-)

Awards are given out at a costume party. How many ways can "most creative," "silliest," and "best" costume be awarded to 8 contestants if no one gets more than one award?

${}_8P_3$

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$$

There are 336 ways to arrange the awards.

How many ways can a 2-digit number be formed by using only the digits 5–9 and by each digit being used only once?

${}_5P_2$

$${}_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}}$$

There are 20 ways for the numbers to be formed.

A **combination** is a grouping of objects in which order does not matter. For example, when you choose 3 letters from the letters A, B, C, and D, there are 4 different combinations.

ABC
ABD
BCD
ACD

Combinations

The number of combinations of n objects taken r at a time is given by ${}_nC_r = \frac{n!}{r!(n-r)!}$.

${}_{10}P_5$ ${}_{10}C_5$

A restaurant offers 8 side dishes. When you order an entree, you can choose 3 of the side dishes. In how many ways can you choose 3 side dishes?

Side Dishes	
Beets	Rice
Potatoes	Broccoli
Carrots	Cole slaw
Salad	Apple sauce

$${}_8C_3 = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5!}\,\cancel{3 \cdot 2} \cdot 1} = 56$$

Suppose the restaurant offers a special on Mondays that allows you to choose 4 side dishes. In how many ways can you choose the side dishes?

${}_8C_4$

In general, are there more ways or fewer ways to select objects when order does not matter? Why?

Fewer; when order does not matter, multiple selections are counted as the same combination.

There are 5 boys and 6 girls in a school play. The director randomly chooses 3 of the students to meet with a costume designer. How many ways are there for the director to choose all boys?

$_5C_3 = 10$

5C3=10

An MP3 player has a playlist with 12 songs. You select the shuffle option for the playlist. In how many different orders can the songs be played?

12P12 = 479,001,600

$\underline{12}\ \underline{11}\ \underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$12!$

$_{12}P_{12}$

There are 10 runners in a race. Medals are awarded for 1st, 2nd, and 3rd place. In how many different ways can the medals be awarded?

720

$\underline{10}\cdot\underline{9}\cdot\underline{8}$

$_{10}P_3$

There are 9 players on a baseball team. In how many different ways can the coach choose players for first base, second base, third base, and shortstop?

$_9P_4 = 3024$

You have 15 photographs of your school. In how many different ways can you arrange 6 of them in a line for the cover of the school yearbook?

$_{15}P_6$

A cat has a litter of 6 kittens. You plan to adopt 2 of the kittens. In how many ways can you choose 2 of the kittens from the litter?

15

IDENTICAL KITTENS :-)

$_6C_2 = \frac{6!}{(6-2)!\,2!} = \frac{6\cdot5\cdot\cancel{4!}}{\cancel{4!}\cdot 2} = 15$

An amusement park has 11 roller coasters. In how many ways can you choose 4 of the roller coasters to ride during your visit to the park?

$_{11}C_4$

Twelve skiers are competing in the final round of the Olympic freestyle skiing aerial competition.

In how many different ways can the skiers finish the competition? (Assume there are no ties)

12 11 10 9

12!

$_{12}P_{12}$

How many ways can there be a gold, silver and bronze medalists?

$_{12}P_3$

There are 12 different-colored cubes in a bag. How many ways can Randall draw a set of 4 cubes from the bag?

$$_{12}C_4 = \frac{12!}{4!(12-4)!} = \frac{12!}{4!(8!)}$$

There are 495 ways to draw 4 cubes from 12.

The swim team has 8 swimmers. Two swimmers will be selected to swim in the first heat. How many ways can the swimmers be selected?

$$_8C_2 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6!)}$$

The swimmers can be selected in 28 ways.

Permutations with Repetition.

The number of disitinguishable permutations of n objects where one object is repeated q_1 times, and another is repeated q_2 times, and so on is

$$\boxed{\frac{n!}{q_1!\, q_2! \, ...q_k!}}$$

3 letters M, O, M.
How many distinguishable words can be formed?

$\frac{3!}{2!}$ 3 letters
M is repeated twice

MOM MMO OMM

Find the number of distinguishable permutations of the letters in MISSISSIPPI.

$$\frac{11!}{4!\,4!\,2!}$$

How many ways are there to arrange 4 identical trumpets and 3 identical violins?

$$\frac{7!}{4!\,3!}$$

Find the number of possible 5-card hands that contains no face cards.

${}_{40}C_5$

Find the number of possible 5-card hands that contains 5 black cards.

${}_{26}C_5$

52

26 Red 26 Black

13 Diamonds 13 Clubs

13 Hearts 13 Spades

$$A\,2\,3\,4\,5\,6\,7\,8\,9\,10\,\overbrace{J\,Q\,K}^{\text{Face Cards}}$$

Next year you are taking math, LA, SS, chemistry and Spanish. In how many different orders can you schedule your classes?

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$5!$

77 **Abelardo wants to create several different 7-character screen names. He wants to use arrangements of the first 3 letters of his first name (abe), *followed by* arrangements of 4 digits in 1984, the year of his birth. How many different screen names can he create in this way?**

A 72

B 144

C 288

D 576

$\underline{3}\ \underline{2}\ \underline{1}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

${}_3P_3 \cdot {}_4P_4$

78 **A train is made up of a locomotive, 7 different cars, and a caboose. If the locomotive must be first, and the caboose must be last, how many different ways can the train be ordered?**

A 5040

B 181,440

C 362,880

D 823,543

1 7 6 5 4 3 2 1 1

7!

Recall that the probability of an event is equal to the number of outcomes that result in the event (want) divided by the number of all possible outcomes.

$\frac{\text{number of outcomes in Event you want}}{\text{total possible outcomes}}$

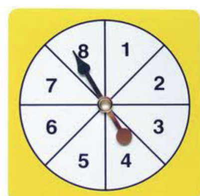
The probability of an event is a number between 0 and 1 that indicates the likelihood the event will occur.

A probability of zero means: NOT / never 0%

A probability of one means: Always

50%

0 $\frac{1}{2}$ 1



A spinner has 8 equal size sectors umbered from 1 to 8. Find the probability:

a. Spinning a 6 $\frac{1}{8}$

b. Spinning a number greater than 5

$\frac{3}{8}$

Teresa and Julia are among 10 students who have applied for a trip to Washington, D.C. ONE students from the group will be selected at random for the trip. What is the probability that Teresa or Julia will be selected? 2/10

Teresa and Julia are among 10 students who have applied for a trip to Washington, D.C. TWO students from the group will be selected at random for the trip. What is the probability that Teresa and Julia will be the 2 students selected?

$\frac{1}{_{10}C_2}$

5 cards are drawn from a standard 52 card deck. What is the probability that they are all red?

26C5/52C5

A card is drawn from a standard 52 card deck. What is the probability that it is a red card? **26/52**

Every student at your school is assigned a four-digit code, such as 6953, to access the computer system. In each code, no digit is repeated. What is the probability that you are assigned a code with the digits 1, 2, 3, and 4 in any order?

4P4 / 10P4

A bag contains 9 tiles, each with a different number from 1 to 9. You choose a tile, put it aside, choose a second tile, put it aside, and then choose a third tile. What is the probability that you choose tiles with the numbers 1, 2, and 3 in that order?

$$\frac{1}{9P3}$$

A bag contains 26 tiles, each with a different letter of the alphabet written on it. You choose 3 tiles from the bag without looking. What is the probability that you choose the tiles containing the letters A, B, and C?

$$\frac{3C3}{26C3} = \frac{1}{2600}$$

There are 5 boys and 6 girls in a school play. The director randomly chooses 3 of the students to meet with a costume designer. What is the probability that the director chooses all boys?

A Let S be the sample space. Find $n(S)$.

The sample space consists of all combinations of 3 students taken from the group of 11 students.

$$n(S) = {}_{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3! \cdot 8!} = \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \underline{165}$$

B Let A be the event that the director chooses all boys. Find $n(A)$.

Suppose the 11 students are $B_1, B_2, B_3, B_4, B_5, G_1, G_2, G_3, G_4, G_5, G_6$, where the Bs represent boys and the Gs represent girls.

The combinations in event A are combinations like $B_2B_4B_5$ and $B_1B_3B_4$. That is, event A consists of all combinations of 3 boys taken from the set of 5 boys.

$$\text{So, } n(A) = {}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot 2 \cdot 1} = \underline{10}.$$

C Find $P(A)$.

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{165} = \underline{\frac{2}{33}}$$

So, the probability that the director chooses all boys is $\underline{\frac{2}{33}}$.

Is the director more likely to choose all boys or all girls? Why?

All girls; ${}_6C_3 = 20$, so $P(\text{all girls}) = \frac{20}{165} = \frac{4}{33}$, which is greater than the probability of choosing all boys, $\frac{2}{33}$

There are 11 students on a committee. To decide which 3 of these students will attend a conference, 3 names are chosen at random by pulling names one at a time from a hat. What is the probability that Sarah, Jamal, and Mai are chosen in any order?

$\underline{\frac{1}{165}}$

A school has 5 Spanish teachers and 4 French teachers. The school's principal randomly chooses 2 of the teachers to attend a conference. What is the probability that the principal chooses 2 Spanish teachers?

$\frac{5}{18}$

There are 6 fiction books and 8 nonfiction books on a reading list. Your teacher randomly assigns you 4 books to read over the summer. What is the probability that you are assigned all nonfiction books?

$\frac{10}{143}$

You are randomly assigned a password consisting of 6 different characters chosen from the digits 0 to 9 and the letters A to Z. As a percent, what is the probability that you are assigned a password consisting of only letters?

$\approx$**11.8%**

There are 9 students on the math team. You draw their names one by one to determine the order in which they answer questions at a math meet. What is the probability that 3 of the 5 seniors on the team will be chosen in any order?

$$\frac{{}_5C3}{{}_9C_3}$$

You put a CD that has 8 songs in your CD player. You set the player to play the songs at random. The player plays all 8 songs without repeating any song.

You have 4 favorite songs on the CD. What is the probability that 2 of your favorite songs are played first, in any order?

$$P=\frac{{}_4C_2}{{}_8C_2}$$

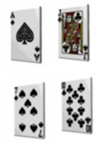
A fruit bowl contains 4 green apples and 7 red apples. What is the probability that a randomly selected apple will be green?

$$\frac{4}{11}$$

TWO apples are randomly selected from a fruit bowl contains 4 green apples and 7 red apples. Without replacement, what is the probability that the 2 apples will be a red and a green?

$$\frac{(4C1)(7C1)}{11C2}$$

A card is drawn from a standard deck of 52 cards. Find the probability the card is either a club or a spade.



$$P(\text{club or spade}) = \frac{13+13}{52}$$
$$= \frac{26}{52} = \frac{1}{2}$$

5 cards are drawn from a standard deck of 52 cards without replacement. Find the probability that they are 3 clubs and 2 spades.

(13C3)(13C2)/(52C5)