7.1 nth Roots and Rational Exponents

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$\sqrt[n]{a}$ is read as the "*n*th root of *a*"

a is the *radicand* and *n* is the *index (or root)*

$\sqrt[4]{16}$ is read as the 4th root of 16

$\sqrt[x]{y}$ or $y^{1/x}$

$\boxed{\sqrt[x]{y}}$ power Key

Ex. 1 Find the real *n*th roots:

$n = 3$, $a = -343$ $\sqrt[3]{-343} = -7$ $n = 4$, $a = 81$ $\sqrt[4]{81} = 3$

$(-7)^3 = -343$ $(-3)^4 = 81$ $3^4 = 81$

positive principal root

Ex.2 **Evaluate:**

$\sqrt[3]{-512} = -8 \qquad (-8)^3 = -512$

$\sqrt[6]{4096} = 4 \qquad 4^6 = 4096$

$\sqrt{24}$

$2\sqrt{6}$

$\sqrt{x^2} = \sqrt{24}$

$\pm 2\sqrt{6}$

Ex.3 **Solve for** ***x*****.**

$6x^4 = 3700$

$\sqrt[4]{x^4} = \sqrt[4]{\frac{3700}{6}}$

$x = \pm \sqrt[4]{(3700 \div 6)} \boxed{=}$

$\approx \pm 4.98$

$\frac{3700}{6} \boxed{=} \sqrt[4]{}$

$\sqrt[3]{(x+1)^3} = \sqrt[3]{18}$

$x + 1 = \sqrt[3]{18}$

$x = \sqrt[3]{18} - 1 \approx 1.62$

Definition of Rational Exponent

If $b > 0$, $n > 0$, and m and n are integers, then

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

Ex.4 **Write using rational exponents.**

$\sqrt[2]{6^1} = 6^{1/2}$

$\left(\sqrt[8]{80}\right)^2 = 80^{2/8} = 80^{1/4}$

Ex.5 **Evaluate without a calculator.**

$64^{5/6}$ $\left(\sqrt[6]{64}\right)^5$

$2^5 = 32$

$(-32)^{-3/5} = \frac{1}{(-32)^{3/5}} = \frac{1}{\left(\sqrt[5]{-32}\right)^3}$

$= \frac{1}{(-2)^3} = -\frac{1}{8}$

Ex.6 **Simplify to a single power of *x*.**

$$\frac{\sqrt[4]{x} \cdot \sqrt[5]{x^2}}{\sqrt[2]{x^1}} = \frac{x^{1/4} \cdot x^{2/5}}{x^{1/2}} = \frac{x^{\frac{\overset{13}{5+8}}{20}}}{x^{\frac{10}{20}}} = x^{3/20}$$

Ex.7 **Find *x*.**

$$9\sqrt[3]{\sqrt[4]{3}^1} = 3^x$$

$$9\sqrt[3]{(3^{1/4})^1}$$

$$3^2 \cdot (3^{1/4})^{1/3} = 3^x$$

$$3^2 \cdot 3^{1/12} = 3^x$$

$$3^{25/12} = 3^{x}$$