1. $\lim_{x\to 2} f(x) = 5$

As x approaches 2, f(x) approaches 5
The graph could have a hole at (2,5)
but still have $f(2) = 3$

2. $\lim_{x\to 1^-} f(x) = 3$

As x approaches 1 from the left,
f(x) approaches 3

$\lim_{x\to 1^+} f(x) = 7$

As x approaches 1 from the right,
f(x) approaches 7

since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ we know

$\lim_{x\to 1} f(x)$ DNE

4. ⓐ $\lim_{x\to 0} f(x) = 3$

ⓑ $\lim_{x\to 3^-} f(x) = 4$

ⓒ $\lim_{x\to 3^+} f(x) = 2$

ⓓ $\lim_{x\to 3} f(x)$ DNE since $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$

[already know $\lim_{x\to 3^-} f(x) = 4$

and $\lim_{x\to 3^+} f(x) = 2$]

ⓔ $f(3) = 3$

5. (a) $\lim_{x \to 1^-} f(x) = 2$

(b) $\lim_{x \to 1^+} f(x) = 3$

(c) we know $\lim_{x \to 1^-} f(x) = 2$ and $\lim_{x \to 1^+} f(x) = 3$

so $\lim_{x \to 1} f(x)$ DNE since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

(d) $\lim_{x \to 5} f(x) = 4$

(e) $f(5)$ is undefined

6. (a) $\lim_{x \to -3^-} h(x) = 4$

(b) $\lim_{x \to -3^+} h(x) = 4$

(c) $\lim_{x \to -3} h(x) = 4$

(d) $h(-3)$ is undefined

(e) $\lim_{x \to 0^-} h(x) = 1$

(f) $\lim_{x \to 0^+} h(x) = -1$

(g) we know $\lim_{x \to 0^-} h(x) = 1$ and $\lim_{x \to 0^+} h(x) = -1$

so $\lim_{x \to 0^-} h(x) \neq \lim_{x \to 0^+} h(x)$

Thus $\lim_{x \to 0} h(x)$ DNE

(h) $h(0) = 1$

(i) $\lim_{x \to 2} h(x) = 2$

(j) $h(2)$ is undefined

(k) $\lim_{x \to 5^+} h(x) = 3$

(l) $\lim_{x \to 5^-} h(x)$ DNE since $h(x)$ does not approach a single number as x approaches 5 from the left

7. (a) $\lim_{t \to 0^-} g(t) = -1$

(b) $\lim_{t \to 0^+} g(t) = -2$

(c) we know $\lim_{t \to 0^-} g(t) = -1$ and $\lim_{t \to 0^+} g(t) = -2$

so $\lim_{t \to 0} g(t)$ DNE since $\lim_{t \to 0^-} g(t) \neq \lim_{t \to 0^+} g(t)$

(d) $\lim_{t \to 2^-} g(t) = 2$

(e) $\lim_{t \to 2^+} g(t) = 0$

(f) $\lim_{t \to 2} g(t)$ DNE since $2 = \lim_{t \to 2^-} g(t) \neq \lim_{t \to 2^+} g(t) = 0$

(g) $g(2) = 1$

(h) $\lim_{t \to 4} g(t) = 3$

10. $\lim_{t \to 12^-} f(t) = 150$ mg

$\lim_{t \to 12^+} f(t) = 300$ mg

These limits show the quick change in the amount of the medicine in the bloodstream at $t = 12$ hours.

11. ⓐ $\lim_{x \to 0^-} f(x) = 1$

ⓑ $\lim_{x \to 0^+} f(x) = 0$

ⓒ we know $\lim_{x \to 0^-} f(x) = 1$ and $\lim_{x \to 0^+} f(x) = 0$

so $\lim_{x \to 0} f(x)$ DNE since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

17. $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2}$

Let $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$

x	$f(x)$
2.5	$f(2.5) \approx 0.714286$
2.1	$f(2.1) \approx 0.677419$
2.05	$f(2.05) \approx 0.672131$
2.01	$f(2.01) \approx 0.667774$
2.005	$f(2.005) \approx 0.667221$
2.001	$f(2.001) \approx 0.666778$
1.9	$f(1.9) \approx 0.655172$
1.95	$f(1.95) \approx 0.661017$
1.99	$f(1.99) \approx 0.665552$
1.995	$f(1.995) \approx 0.666110$
1.999	$f(1.999) \approx 0.666556$

It appears that $\lim_{x \to 2} f(x) = \frac{2}{3}$

19. $\lim_{x \to 0} \dfrac{\sin(x)}{x + \tan(x)}$

Let $f(x) = \dfrac{\sin(x)}{x + \tan(x)}$

x	$f(x)$
1	$f(1) \approx 0.329033$
0.5	$f(0.5) \approx 0.458209$
0.2	$f(0.2) \approx 0.493331$
0.1	$f(0.1) \approx 0.498333$
0.05	$f(0.05) \approx 0.499583$
0.01	$f(0.01) \approx 0.499983$

x	$f(x)$
-1	$f(-1) \approx 0.329033$
-0.5	$f(-0.5) \approx 0.458209$
-0.2	$f(-0.2) \approx 0.493331$
-0.1	$f(-0.1) \approx 0.498333$
-0.05	$f(-0.05) \approx 0.499583$
-0.01	$f(-0.01) \approx 0.499983$

It appears that $\lim_{x \to 0} f(x) = \frac{1}{2}$

20. $\lim_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16}$

Let $f(x) = \dfrac{\sqrt{x} - 4}{x - 16}$

x	$f(x)$
17	$f(17) \approx 0.123106$
16.5	$f(16.5) \approx 0.124038$
16.1	$f(16.1) \approx 0.124805$
16.05	$f(16.05) \approx 0.124902$
16.01	$f(16.01) \approx 0.124980$

x	$f(x)$
15	$f(15) \approx 0.127017$
15.5	$f(15.5) \approx 0.125992$
15.9	$f(15.9) \approx 0.125196$
15.95	$f(15.95) \approx 0.125098$
15.99	$f(15.99) \approx 0.125020$

It appears that $\lim_{x \to 16} f(x) = \frac{1}{8}$

22. $\lim_{x \to 0} \frac{\tan(3x)}{\tan(5x)}$

let $f(x) = \frac{\tan(3x)}{\tan(5x)}$

x	$f(x)$
-0.1	$f(-0.1) \approx 0.5662362068$
-0.01	$f(-0.01) \approx 0.5996798314$
-0.001	$f(-0.001) = 0.5999968$
0.1	$f(0.1) \approx 0.5662362068$
0.01	$f(0.01) \approx 0.5996798314$
0.001	$f(0.001) = 0.5999968$

It appears that $\lim_{x \to 0} \frac{\tan(3x)}{\tan(5x)} = 0.6$

23. $\lim_{x \to 1} \frac{x^6 - 1}{x^{10} - 1}$

let $f(x) = \frac{x^6 - 1}{x^{10} - 1}$

x	$f(x)$
0.9	$f(0.9) \approx 0.7193973436$
0.99	$f(0.99) \approx 0.6120175752$
0.999	$f(0.999) \approx 0.6012001976$
1.1	$f(1.1) \approx 0.4841189962$
1.01	$f(1.01) \approx 0.5880223744$
1.001	$f(1.001) \approx 0.5988002024$

It appears that $\lim_{x \to 1} f(x) = 0.6$