

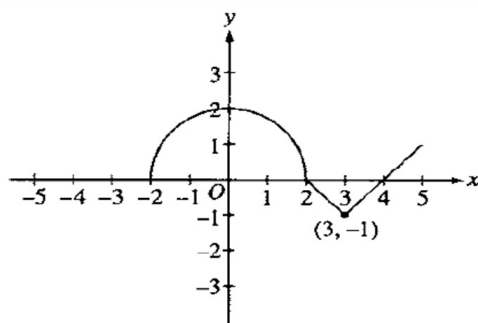
4)

Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

- Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p . Show the analysis that leads to your conclusion.
- For what values of the constant p does f have 3 distinct roots?
- Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.

Page 1

5)



The graph of the function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t) dt$.

- Find $g(3)$.
- Find all the values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.
- Write an equation for the line tangent to the graph of g at $x = 3$.
- Find the x -coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

Page 2

6)

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

- (a) Use separation of variables to find an expression for v in terms of t , where t is measured in seconds.
- (b) Terminal velocity is defined as $\lim_{t \rightarrow \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.
- (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?