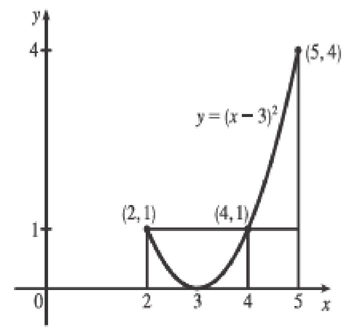
9. (a) $f_{\text{ave}} = \dfrac{1}{5-2}\displaystyle\int_2^5 (x-3)^2\,dx = \dfrac{1}{3}\left[\dfrac{1}{3}(x-3)^3\right]_2^5$

$= \frac{1}{9}\left[2^3 - (-1)^3\right] = \frac{1}{9}(8+1) = 1$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow (c-3)^2 = 1 \Leftrightarrow$

$c - 3 = \pm 1 \Leftrightarrow c = 2$ or 4

(c)

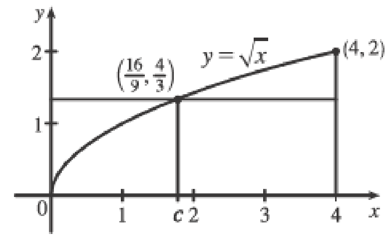


10. (a) $f_{\text{ave}} = \dfrac{1}{4-0}\displaystyle\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{3/2}\right]_0^4$

$= \frac{1}{6}\left[x^{3/2}\right]_0^4 = \frac{1}{6}[8-0] = \frac{4}{3}$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow \sqrt{c} = \frac{4}{3} \Leftrightarrow c = \frac{16}{9}$

(c)



11. (a) $f_{\text{ave}} = \dfrac{1}{\pi - 0}\displaystyle\int_0^\pi (2\sin x - \sin 2x)\,dx$

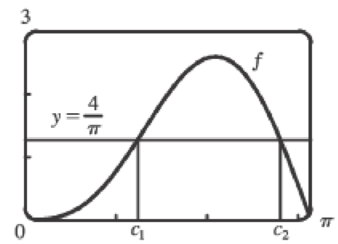
$= \frac{1}{\pi}\left[-2\cos x + \frac{1}{2}\cos 2x\right]_0^\pi$

$= \frac{1}{\pi}\left[\left(2+\frac{1}{2}\right) - \left(-2+\frac{1}{2}\right)\right] = \frac{4}{\pi}$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow 2\sin c - \sin 2c = \frac{4}{\pi} \Leftrightarrow$

$c_1 \approx 1.238$ or $c_2 \approx 2.808$

(c)



12. (a) $f_{\text{ave}} = \dfrac{1}{2-0}\displaystyle\int_0^2 \dfrac{2x}{(1+x^2)^2}\,dx$

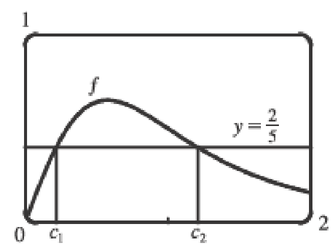
$= \dfrac{1}{2}\displaystyle\int_1^5 \dfrac{1}{u^2}\,du \qquad [u = 1 + x^2,\ du = 2x\,dx]$

$= \dfrac{1}{2}\left[-\dfrac{1}{u}\right]_1^5 = -\dfrac{1}{2}\left(\dfrac{1}{5} - 1\right) = \dfrac{2}{5}$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow \dfrac{2c}{(1+c^2)^2} = \dfrac{2}{5} \Leftrightarrow 5c = (1+c^2)^2 \Leftrightarrow$

$c_1 \approx 0.220$ or $c_2 \approx 1.207$

(c)



13. f is continuous on $[1, 3]$, so by the Mean Value Theorem for Integrals there exists a number c in $[1, 3]$ such that

$\int_1^3 f(x)\,dx = f(c)(3-1) \Rightarrow 8 = 2f(c)$; that is, there is a number c such that $f(c) = \frac{8}{2} = 4$.

14. The requirement is that $\frac{1}{b-0}\int_0^b f(x)\,dx = 3$. The LHS of this equation is equal to

$\frac{1}{b}\int_0^b (2+6x-3x^2)\,dx = \frac{1}{b}\left[2x+3x^2-x^3\right]_0^b = 2+3b-b^2$, so we solve the equation $2+3b-b^2=3 \quad \Leftrightarrow$

$b^2-3b+1=0 \quad \Leftrightarrow \quad b=\frac{3\pm\sqrt{(-3)^2-4\cdot 1\cdot 1}}{2\cdot 1}=\frac{3\pm\sqrt{5}}{2}$. Both roots are valid since they are positive.