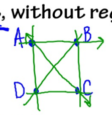
12-2 Combinations

A *combination* is a selection of objects in groups, without regard to order.

1. Given 4 points, no 3 of which are collinear. How many lines can be drawn?



$\overleftrightarrow{AC}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ — 6

$_4C_2$ (4 → things, C → combinations, 2 → at a time) $= \frac{4!}{2!(4-2)!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 2\cdot 1}$

Combinations of r objects, taken from a group of n objects is:

$$_nC_r = \frac{n!}{r!(n-r)!} \qquad \binom{n}{r}$$

(n = Total, r = at a time)

2. standard 52-card deck, p. 708

Standard Deck of 52 Playing Cards: (4 suits, 13 kinds)

Diamonds (Red):	2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦ A♦
Hearts (Red):	2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♥
Clubs (Black):	2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♣
Spades (Black):	2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♠

a) How many different 7-card hands are possible?

$_{52}C_7 = \frac{52!}{(7!\times 45!)} = 133{,}784{,}560$

b) How many 7-card hands with 4 kings and any other 3 cards are possible?

$17{,}296 = {_4C_4}$ (4 Kings) $\times$ ${_{48}C_3}$ (3 others) $= 1 \times \frac{48!}{3!\,45!} = \frac{48\cdot 47\cdot 46\cdot 45!}{3\cdot 2\cdot 1\cdot 45!}$

3. Twelve students apply for an ASB committee. In how many ways can committees of 2 or 3 or 4 students be chosen?

mutually exclusive events (no overlap)

$_{12}C_2 + {_{12}C_3} + {_{12}C_4}$

$\frac{12!}{2!\,10!} + \frac{12!}{3!\,9!} + \frac{12!}{4!\,8!}$

$\frac{12\cdot 11\cdot 10!}{2\cdot 1\cdot 10!} + \frac{12\cdot 11\cdot 10\cdot 9!}{3\cdot 2\cdot 1\cdot 9!} + \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2}$

$66 + 220 + 495 = 781$

286