

SECTION 9.7: CIRCLES & LENGTHS OF SEGMENTS

Standards:

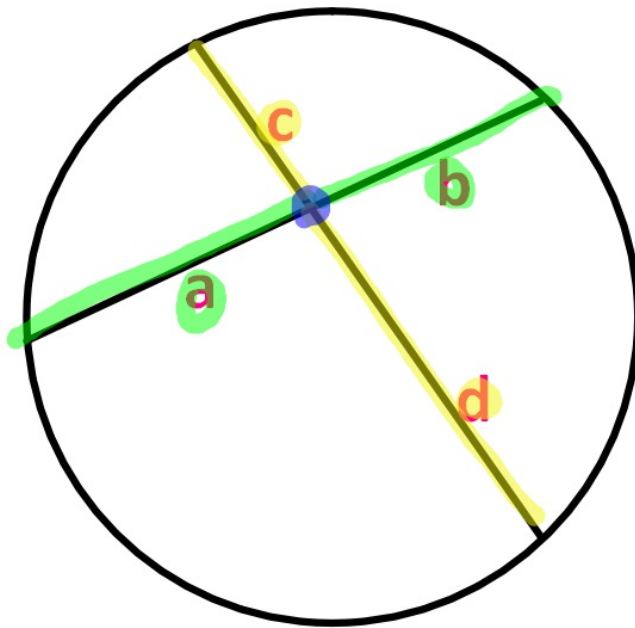
7.0 - Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.

17.0 - Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

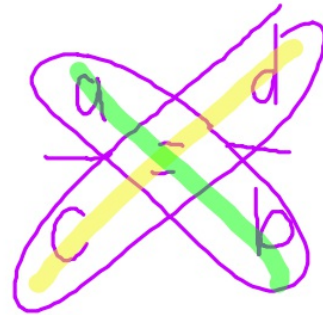
21.0 - Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

THEOREM

When 2 chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord.

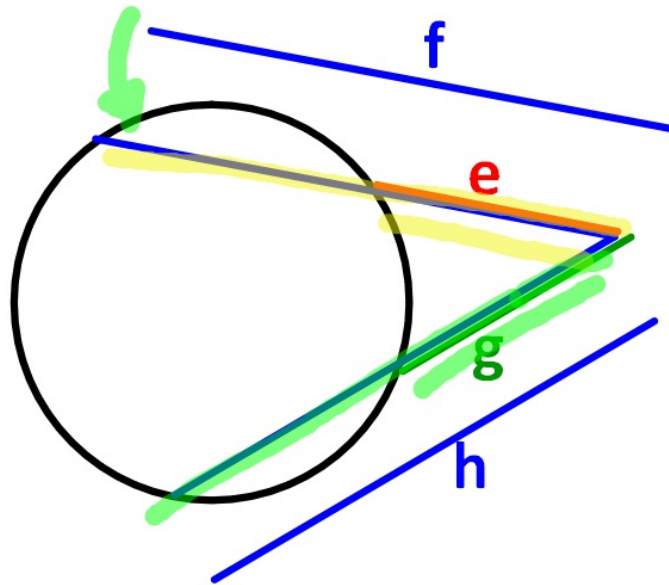


$$a \cdot b = c \cdot d$$



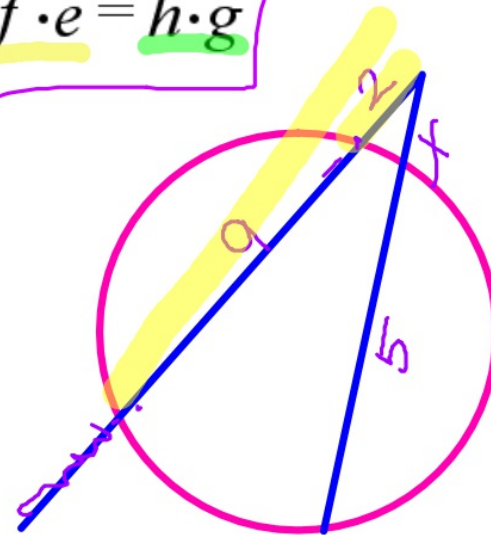
THEOREM

When 2 secant segments are drawn to a circle from an external point, the product of one secant segment and its external segment = the product of the other secant segment and its external segment.



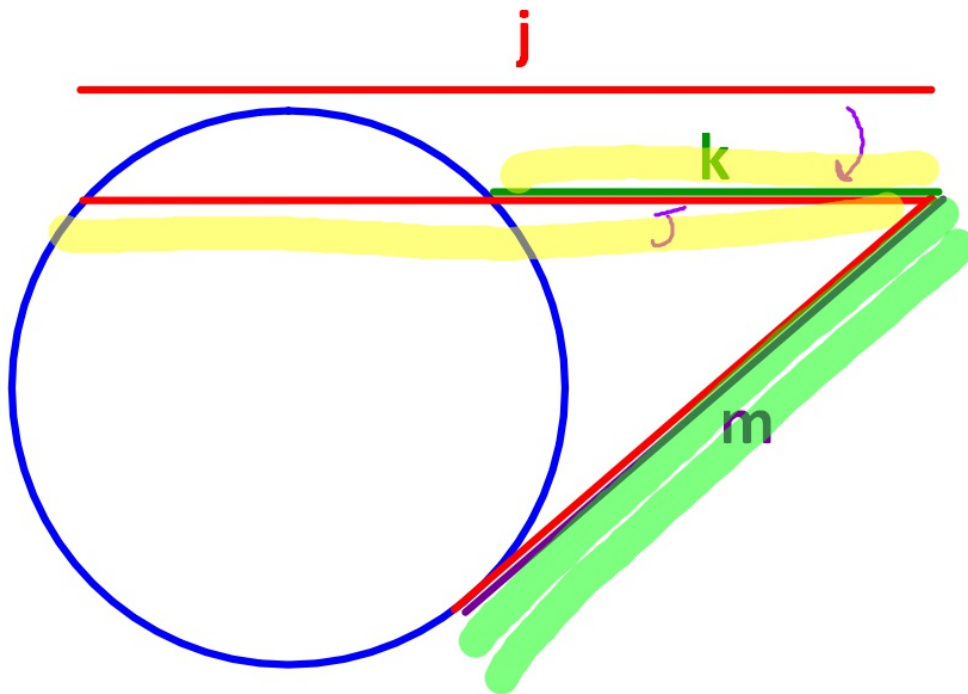
$$f \cdot e = h \cdot g$$

$$(2)(9+2) = (5+x)(x)$$



THEOREM

When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the secant segment and its external segment = the square of the tangent segment.

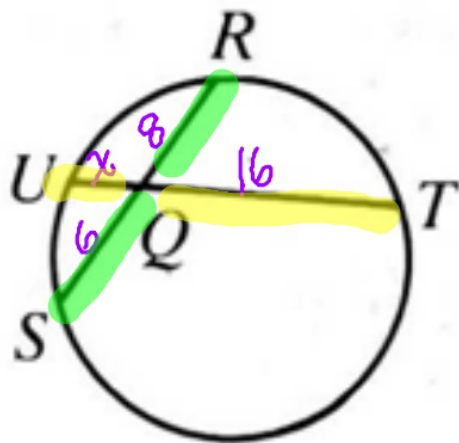


$$j \cdot k = m^2$$

$$\text{sec(whole)} \cdot \text{sec(outside)} = \text{tan} \cdot \text{tan} \text{ or } \text{tan}^2$$

1) If $RQ = 8$, $QS = 6$, and $TQ = 16$, then

✦ $QU = \underline{3}$.

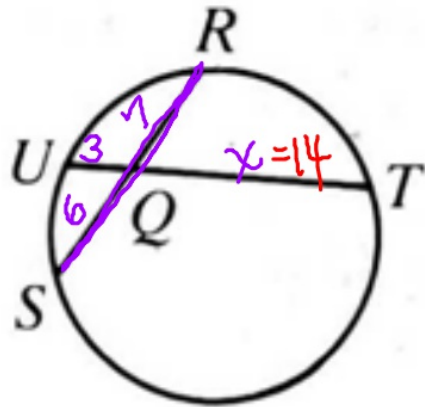


$$6 \cdot \cancel{8} = \cancel{16} \cdot x$$

$$3 = x$$

2) If $RS = 13$, $RQ = 7$, and $QU = 3$, then

★ $TQ = \underline{14}$.

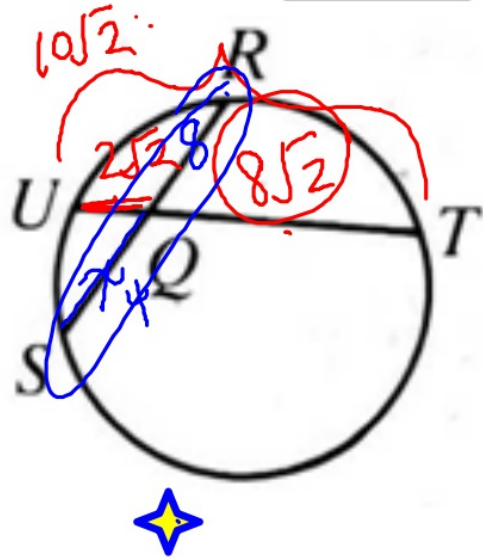


$$7 \cdot 6 = 3 \cdot x$$

$$\frac{42}{3} = \frac{3x}{3}$$

$$14 = x$$

3) If $TU = 10\sqrt{2}$, $QU = 2\sqrt{2}$, and $RQ = 8$, then
 $RS = \underline{12}$.

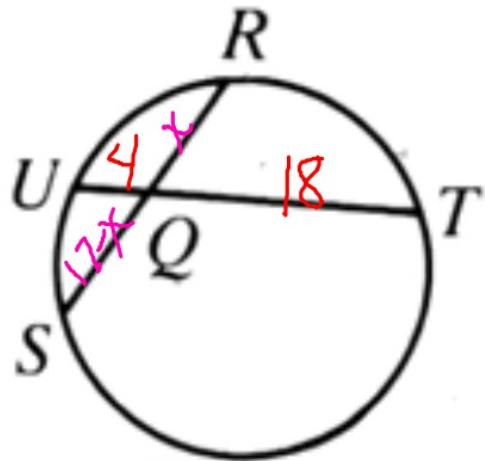


$$8x = (2\sqrt{2})(8\sqrt{2})$$

$$\frac{8x}{8} = \frac{16 \cdot 2}{8}$$

4

3) If $RQ = ??$, $TQ = 18$, and $QU = 4$ then
 $RS = 17$



$$x = 9 \text{ or } 8$$

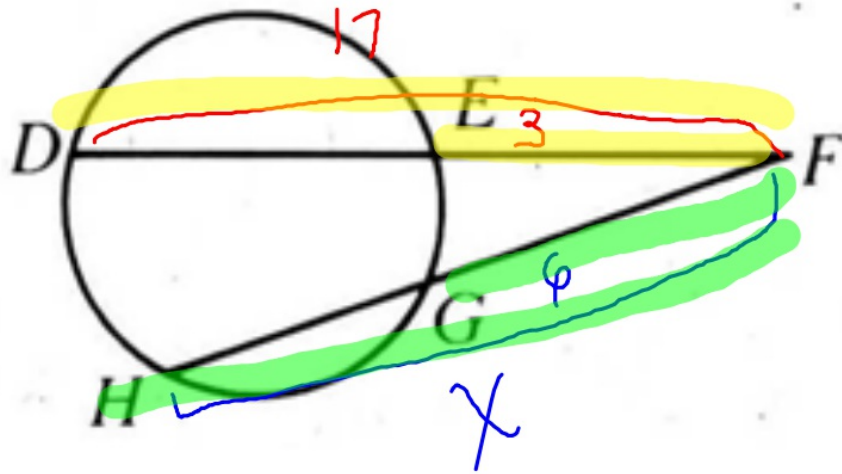
$$(17-x)(x) = 4 \cdot 18$$
$$17x - x^2 = 72$$

$$0 = x^2 - 17x + 72$$

$$(x-9)(x-8)$$

5) If $DF = 17$, $EF = 3$, and $GF = 6$, then

✦ $HF = \underline{8.5}$.



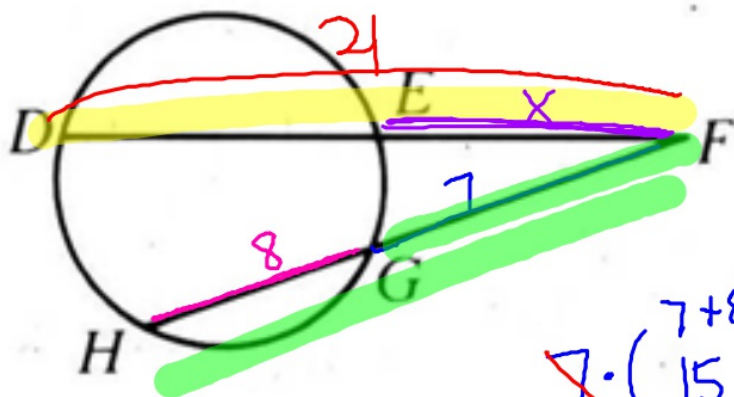
$$(2x)(17) = 6 \cdot x$$

$$17 = 2x$$

$$8.5 = x$$

6) If $HG = 8$, $GF = 7$, and $DF = 21$, then

✦ $EF = \underline{\hspace{2cm}}$.



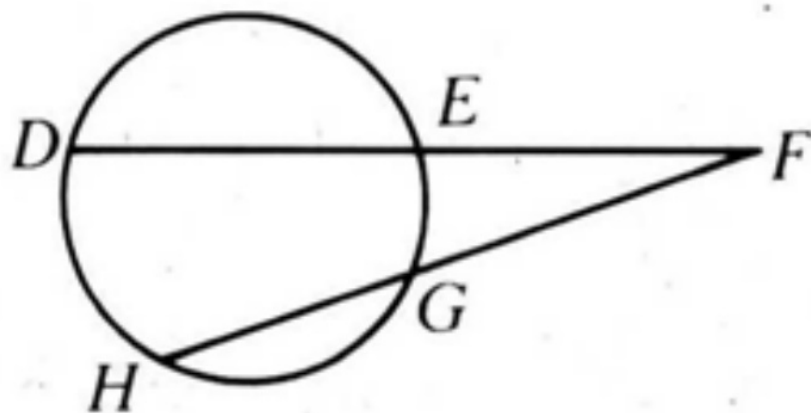
$$7 \cdot (7+8) = 21x$$

$$15 = 3x$$

$$5 = x$$

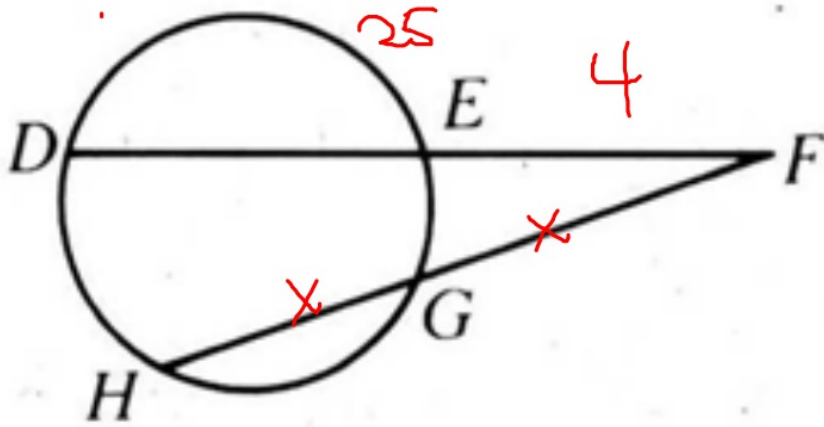
7) If $GF = 8$, $HG = 10$, and $DE = 18$, then

✦ $EF = \underline{6}$.



8) If $DF = 25$, $EF = 4$, and $HG = GF = x$, then

★ $x = \underline{5\sqrt{2}}$.



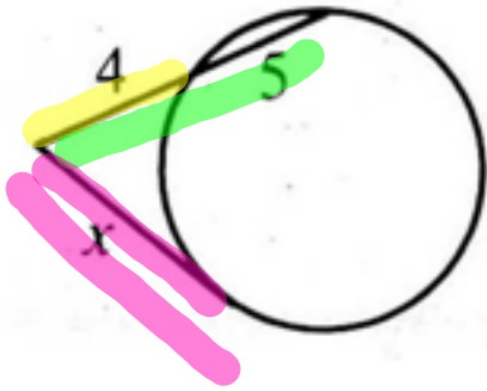
$$4 \cdot 25 = x(\cancel{x} + x)$$

$$100 = 2x^2$$

$$\sqrt{50} = \sqrt{x^2}$$

$$5\sqrt{2} = x$$

9)



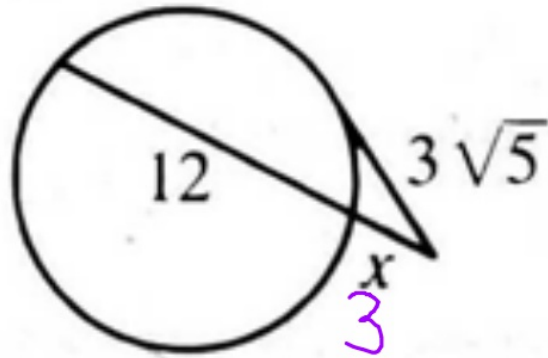
$$x^2 = 4 \cdot 9 \quad (4+5)$$

$$x^2 = 36$$

$$x = 6$$



10)



3

$$(3\sqrt{5})(3\sqrt{5}) = x(x+12)$$

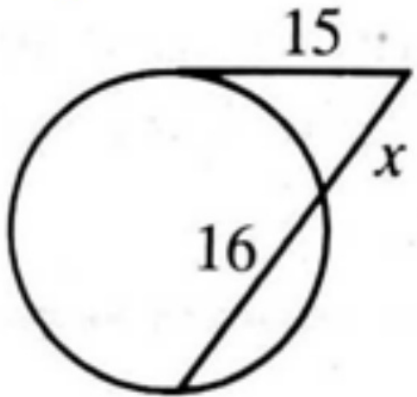
$$45 = x^2 + 12x$$

$$0 = x^2 + 12x - 45$$

$$0 = (x+15)(x-3)$$

$$x = -15 \text{ or } 3$$

11)



✦ 9

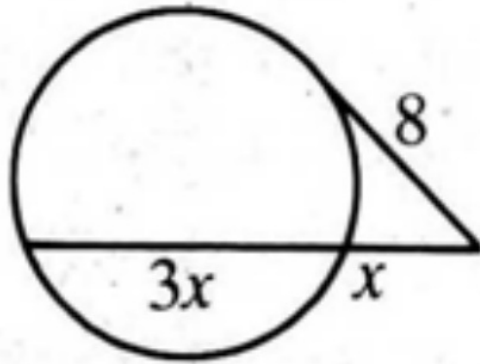
$$15^2 = x(x + 16)$$

$$225 = x^2 + 16x$$

$$0 = x^2 + 16x - 225$$

$$(x + 25)(x - 9)$$

12)



4

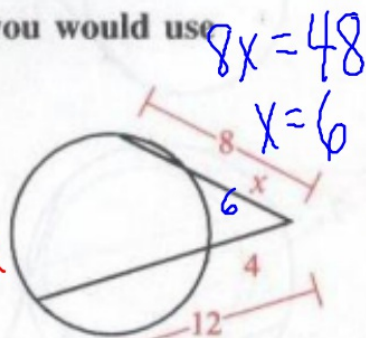
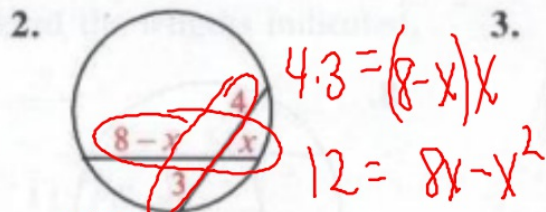
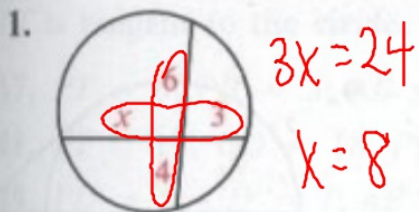
$$\frac{\text{sec}}{\text{ext}} = \frac{\text{sec}}{\text{ext}} \Rightarrow (\text{ext})(\text{sec}) = (\text{ext})(\text{sec})$$

$$\frac{\text{tan}}{\text{ext}} = \frac{\text{whole}}{\text{tan}} = \frac{\text{ext}}{\text{sec}} \Rightarrow x \cdot 8 = 4 \cdot 12$$

■ Pg. 363 (CE):
1-6

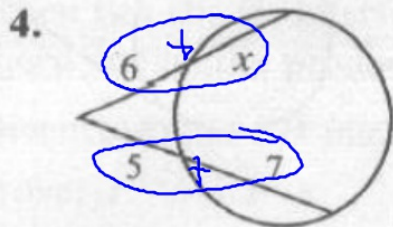
Classroom Exercises

Chords, secants, and tangents are shown. State the equation you would use to find the value of x . Then solve for x .



(6, 2) $x^2 - 8x + 12 = 0$
 $(x-6)(x-2) = 0$

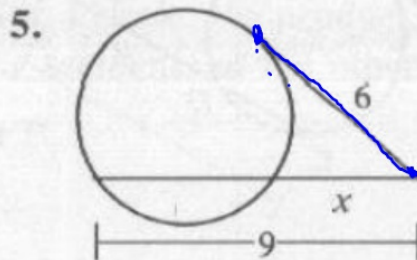
Chords, secants, and tangents are shown. State the equation you would use to find the value of x . Then solve for x .



$$6(6+x) = 5 \cdot 12$$

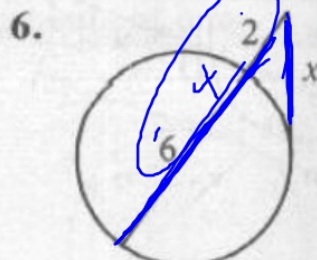
$$36 + 6x = 60$$

$$\begin{array}{r} 36 + 6x = 60 \\ -36 \quad -36 \\ \hline 6x = 24 \end{array}$$



$$6 \cdot 6 = x \cdot 9$$

$$36 = 9x$$



$$x^2 = 2(8)$$

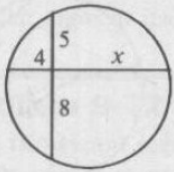
$$\sqrt{x^2} = \sqrt{16}$$

$$x = 4$$

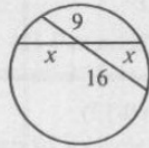
Written Exercises

Chords, secants, and tangents are shown. Find the value of x .

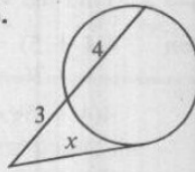
1.



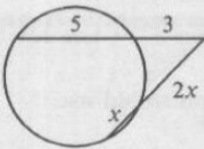
2.



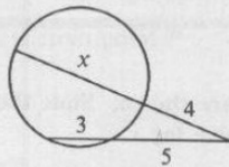
3.



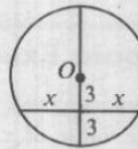
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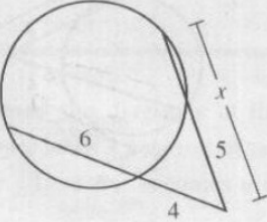
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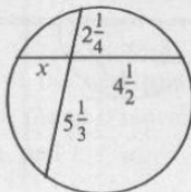
6.



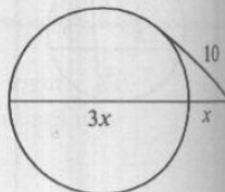
7.



8.



9.



HW 12.7(9.7)

■ Pg. 364 (WE):

#1-9, 13-20

Chords \overline{AB} and \overline{CD} intersect at P . Find the lengths indicated.

Example $AP = 5; BP = 4; CD = 12; CP = \underline{\quad?}$

Solution Let $CP = x$. Then $DP = 12 - x$.

$$x(12 - x) = 5 \cdot 4$$

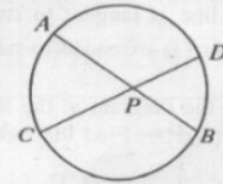
$$12x - x^2 = 20$$

$$x^2 - 12x + 20 = 0$$

$$(x - 2)(x - 10) = 0$$

$$x = 2 \text{ or } x = 10$$

$$CP = 2 \text{ or } 10$$



13. $AP = 6; BP = 8; CD = 16; DP = \underline{\quad?}$

14. $CD = 10; CP = 6; AB = 11; AP = \underline{\quad?}$

15. $AB = 12; CP = 9; DP = 4; BP = \underline{\quad?}$

16. $AP = 6; BP = 5; CP = 3 \cdot DP; DP = \underline{\quad?}$

\overline{PT} is tangent to the circle. Find the lengths indicated.

17. $PT = 6; PB = 3; AB = \underline{\quad?}$

18. $PT = 12; CD = 18; PC = \underline{\quad?}$

19. $PD = 5; CD = 7; AB = 11; PB = \underline{\quad?}$

20. $PB = AB = 5; PD = 4; PT = \underline{\quad?}$ and $PC = \underline{\quad?}$

