Chapter 2 "FRAPPY"

{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Men's shirt sizes are determined by neck sizes. Suppose that men's neck sizes are approximately normally distributed with mean 15.7 inches and standard deviation 0.7 inch. A retailer sells men's shirts in sizes S, M, L, XL, where the shirt sizes are defined in the table below.

Shirt Size	Neck Size
S	14 ≤ neck size < 15
M	15 ≤ neck size < 16
L	16 ≤ neck size < 17
XL	17 ≤ neck size < 18

Scoring:

E P I (a) Because the retailer only stocks the sizes listed above, what proportion of customers will find the retailer does not carry any shirts in their sizes? Show your work.

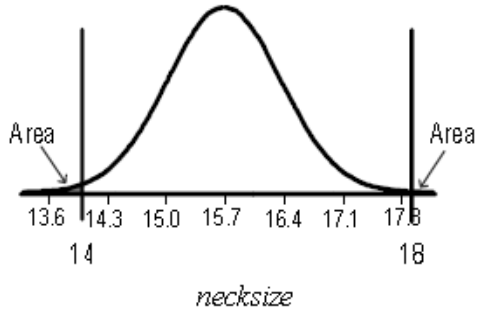
E P I (b) Using a sketch of a normal curve, illustrate the proportion of men whose shirt size is M. Calculate this proportion.

E P I (c) Of 12 randomly selected customers, what is the probability that exactly 4 will request size M? Show your work.

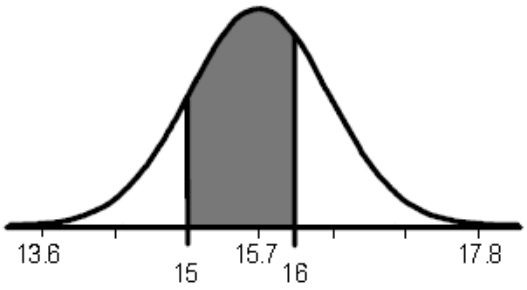
Total:__/4

Part (a):

$$
\begin{aligned}
&P(necksize < 14 \ \ or \ \ necksize \geq 18) \\
&= P(necksize < 14) + P(necksize \geq 18) \\
&= P\left(z < \frac{14 - 15.7}{0.7}\right) + P\left(z \geq \frac{18 - 15.7}{0.7}\right) \\
&= P(z < -2.429) + P(z \geq 3.286) \\
&= 0.00758 + 0.00051 \\
&= 0.00809
\end{aligned}
$$



Part (b):



necksize

$$
\begin{aligned}
&P(15 \leq necksize < 16) \\
&= P\left(\frac{15 - 15.7}{0.7} \leq z < \frac{16 - 15.7}{0.7}\right) \\
&= P(-1.000 \leq z < 0.429) \\
&= 0.50723
\end{aligned}
$$

Part (c):

X = number of customers who request size M
X is binomial with $n = 12$ customers and $p = 0.5072$

$$P(X = 4) = {}_{12}C_4 (0.5072)^4 (0.4928)^8 = 495(0.06618)(0.00348) = 0.1139$$

Part (a) is essentially correct (E) if the response

1. recognizes the need to look at neck sizes below 14 and above 18
2. correctly computes the two tail probabilities (except for minor arithmetic or transcription errors) and adds those probabilities

Part (a) is partially correct (P) if the response

considers only neck sizes below 14 (or above 18) but computes that corresponding tail area correctly
OR
recognizes the need to look at neck sizes below 14 and above 18 but does not compute both tail probabilities correctly
OR
recognizes the need to look at neck sizes below 14 and above 18 but approximates tail probabilities using the Empirical Rule
OR
computes the proportion of customers that will find the store carries their size
(i.e., 1 – correct answer)
OR
States the correct answer (0.0081) without supporting work

NOTE: A normal curve with correct regions shaded showing both correct end points (14 and 18) and the mean and the standard deviation may be used for element 1.

Part (b) is essentially correct (E) if

1. the appropriate probability is illustrated using a normal curve in which the end points are identified and the mean and standard deviation are implied
2. the required probability is correctly computed (except for minor arithmetic errors)

Part (b) is partially correct (P) if only one of the above elements is correct.

NOTES:

(1) If part (a) was not essentially correct because the student interchanged the mean and standard deviation, and the same values for mean and standard deviation are used in part (b), then part (b) can be considered essentially correct if the probability calculated is correct for the mean and standard deviation used.

(2) A reasonable approximation using the Empirical Rule in part (b) is only acceptable if the computation in part (a) is done correctly (i.e., without using the Empirical Rule).

Part (c) is essentially correct (E) if

1. the student recognized the setting as binomial
2. the probability calculated in part (b) is used for p
3. work is shown — that is, the correct values for n and x are given and the desired probability calculated, or the binomial formula is correctly evaluated.

Part (c) is partially correct (P) if

the student recognizes the situation as binomial and identifies p from part (b) but does not compute the desired probability

OR

the student computes the probability as either $(0.5072)^4\,(0.4928)^8$ or $\binom{12}{4}(0.5072)^4$

OR

the student gives the correct probability of 0.1139 but work is not shown

NOTE: Rounding the probability in part (b) for use in part (c) is acceptable.

4 Complete Response (3E)

All three parts essentially correct

3 Substantial Response (2E 1P)

Two parts essentially correct and one part partially correct

2 Developing Response (2E 0P or 1E 2P or 3P)

Two parts essentially correct and no parts partially correct
OR
One part essentially correct and two parts partially correct
OR
Three parts partially correct

1 Minimal Response (1E 1P or 1E 0P or 0E 2P)

One part essentially correct and either zero or one parts partially correct
OR
No parts essentially correct and two parts partially correct

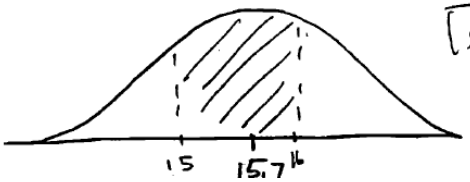
Sample 4:

(a) Because the retailer only stocks the sizes listed above, what proportion of customers will find that the retailer does not carry any shirts in their sizes? Show your work.

$1 - \text{normalcdf}(14, 18, 15.7, 0.7) \approx \boxed{0.0081} = 0.81\%$

I found the proportion of people within the necksize 14 to 18 because that is what retailers carry. Subtracting from one to obtain the proportion that retailers do not carry, I found that .008l of population of customers are out of the range of shirt sizes that retailer carries.

(b) Using a sketch of a normal curve, illustrate the proportion of men whose shirt size is M. Calculate this proportion.



z-scores −1 0 .429

normalcdf(15, 16, 15.7, 0.7) = $\boxed{.5072}$

OR

$P(x<16) = P\left(z < \frac{16-15.7}{.7}\right) = P(z < .429) = .6664$

$P(x \le 15) = P\left(z < \frac{15-15.7}{.7}\right) = P(z < -1) = .1587$

$P(x<16) - P(x<15) = P(15 \le x < 16)$

$.6664 - .1587 = \boxed{.5077}$

(c) Of 12 randomly selected customers, what is the probability that exactly 4 will request size M? Show your work.

${}_{12}C_4 (.5072)^4 (.4928)^8 \approx \boxed{.11394} = 11.394\%$

This is a binomial distribution with success probability of .5072, 12 trials, and expect exactly 4 successes.

binompdf(12, .5072, 4) = .11394