Algebra 2H **Mathematical Induction** **Date**____________________

Mathematical Induction **is a method of proof used to show that an algebraic statement is true for all positive integers.**

To prove a statement is true for all positive integers n:

1. **Show the statement is true for $n = 1$.**
2. **Assume the statement is true for $n = k - 1$, where $k - 1$ is any positive integer.**
3. **Show the statement is true for the next positive integer, $n = k$.**

Ex 1: ~~Proof~~ Prove by M.I.: $\sum_{i=1}^{n} (3i-2) = \frac{n(3n-1)}{2}$

1. For $n=1$: $3 \cdot 1 - 2 = \frac{1(3 \cdot 1 - 1)}{2}$

$3 - 2 = \frac{1(2)}{2}$

$1 = 1$ ✓

2. Assume that $1 + 4 + 7 + \cdots + (3(k-1)-2) = \dfrac{(k-1)(3(k-1)-1)}{2}$.

3. Show that $1 + 4 + 7 + \cdots + (3k-5) + (3k-2) = \dfrac{k(3k-1)}{2}$

Proof:

$1 + 4 + 7 + \cdots + (3k-5) = \dfrac{(k-1)(3k-4)}{2}$

$1 + 4 + 7 + \cdots + (3k-5) + (3k-2) = \dfrac{3k^2-7k+4}{2} + 3k-2$

$= \dfrac{3k^2-7k+4+6k-4}{2}$

$= \dfrac{3k^2-k}{2}$

$= \dfrac{k(3k-1)}{2}$

Ex 2: Prove by M.I. $\sum_{i=1}^{n} 2^i = 2(2^n-1)$

1. For $n=1$: $2^1 = 2(2^1-1)$

$2 = 2(1)$

$2 = 2$ ✓

2. Assume that $2 + 4 + 8 + \cdots + 2^{K-1} = 2(2^{K-1} - 1)$

3. Show that $2 + 4 + 8 + \cdots + \underline{2^{K}} = 2(2^{K} - 1)$

Proof:

$$
\begin{aligned}
2 + 4 + 8 + \cdots + 2^{K-1} &= 2^{1}(2^{K-1} - 1) \\
2 + 4 + 8 + \cdots + 2^{K-1} \underline{+ 2^{K}} &= 2^{K} - 2 \underline{+ 2^{K}} \\
&= 2 \cdot 2^{K} - 2 \\
&= 2(2^{K} - 1) \checkmark
\end{aligned}
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