① $\cos^2\theta + \sin^2\theta = 1$

1c $(\sin\theta - 1)(\sin\theta + 1) =$

$= \sin^2\theta + \sin\theta - \sin\theta - 1$

$= \sin^2\theta - 1$

$= \sin^2\theta - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta - \sin^2\theta - \cos^2\theta$

$= -\cos^2\theta$

3a $1 + \cot^2 A = \csc^2 A$

3c $\dfrac{1}{\sin^2 A} - \dfrac{1}{\tan^2 A} =$

$\csc^2 A - \cot^2 A$

$1 + \cot^2 A - \cot^2 A$

$= 1.$

5 $\cos\theta \cdot \cot(90^\circ - \theta)$

$\cos\theta \cdot \tan\theta$

$\cos\theta \cdot \dfrac{\sin\theta}{\cos\theta}$

$\sin\theta$

5c $\cos\theta(\sec\theta - \cos\theta)$

$\cos\theta\left(\dfrac{1}{\cos\theta} - \cos\theta\right)$

$1 - \cos^2\theta$

$\sin^2\theta + \cos^2\theta - \cos^2\theta$

$\sin^2\theta$

6 $(1 - \cos\theta)(1 + \cos\theta)$

$= 1 + \cos\theta - \cos\theta - \cos^2\theta$

$= 1 - \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) - \cos^2\theta$

$= \sin^2\theta$

3b $(\csc A - 1)(\csc A + 1)$

$= \csc^2 A + \csc A - \csc A - 1$

$= \csc^2 A - 1$

$= 1 + \cot^2 A - 1 = \cot^2 A.$

5b $\csc^2 x\,(1 - \cos^2 x)$

$\csc^2 x\,(\sin^2 x + \cos^2 x - \cos^2 x)$

$\csc^2 x\,(\sin^2 x)$

$\dfrac{1}{\sin^2 x} \cdot \sin^2 x = 1$

7 $\sin A \tan A + \sin(90^\circ - A)$

$\sin A \tan A + \cos A$

$\dfrac{\sin A}{1} \cdot \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{1}$

$\dfrac{\sin^2 A}{\cos A} + \dfrac{\cos A}{1} \cdot \dfrac{\cos A}{\cos A}$

$\dfrac{\sin^2 A + \cos^2 A}{\cos A} = \dfrac{1}{\cos A} = \sec A$

(9) $(\sec B - \tan B)(\sec B + \tan B)$

$\sec^2 B + \sec B \tan B - \sec B \tan B - \tan^2 B$

$\sec^2 B - \tan^2 B$

$(\tan^2 B + 1) - \tan^2 B = 1$

(11) $(\csc x - \cot x)(\sec x + 1) =$

$\csc x \sec x + \csc x - \cot x \sec x - \cot x$

$\frac{1}{\sin x}\cdot\frac{1}{\cos x} + \frac{1}{\sin x} - \frac{\cos x}{\sin x}\cdot\frac{1}{\cos x} - \frac{\cos x}{\sin x}$

$\frac{1}{\sin x \cos x} + \frac{1}{\sin x} - \frac{1}{\sin x} - \frac{\cos x}{\sin x}$

$\frac{1}{\sin x \cos x} - \frac{\cos x \cos x}{\sin x \cos x}$

$\frac{1-\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} =$

$\frac{\sin x}{\cos x} = \tan x$

(13) $\frac{\sin x \cos x}{1-\cos^2 x} = \frac{\sin x \cos x}{\sin^2 x + \cos^2 x - \cos^2 x} = \frac{\sin x \cos x}{\sin^2 x} = \frac{\cos x}{\sin x} = \cot x$

(15) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$(\sin^2 x + 2\sin x \cos x + \cos^2 x) + (\sin^2 x - 2\sin x \cos x + \cos^2 x)$

$2\sin^2 x + 2\cos^2 x = 2(\sin^2 x + \cos^2 x) = 2(1) = 2$

(17) $\frac{\cot^2 \theta}{1+\csc\theta} + \sin\theta \csc\theta =$

$1 + \cot^2\theta = \csc^2\theta \rightarrow \cot^2\theta = \csc^2\theta - 1$

$\frac{\csc^2\theta - 1}{1+\csc\theta} + \sin\theta \cdot \frac{1}{\sin\theta} = \frac{(\csc\theta + 1)(\csc\theta - 1)}{(1+\csc\theta)} + 1$

$= \csc\theta - 1 + 1 = \csc\theta$

p 321

(19) $\cos^3 y + \cos y \sin^2 y$

Factor $\cos y \rightarrow \cos y(\cos^2 y + \sin^2 y)$

$\cos y \quad (1) = \cos y$

(21) $\frac{\cos\theta}{\cos\theta} \cdot \frac{1+\sin\theta}{\cos\theta} + \frac{1+\sin\theta}{\cos\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} = \frac{(\cos^2\theta + \sin^2\theta) + 2\sin\theta + 1}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)} = \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta$$

(23) $$\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta} = \frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{(\sin^2\theta - \cos^2\theta)}$$

$$= \sin^2\theta + \cos^2\theta = 1$$

p 32a #8

a. $\cot A(\sec A - \cos A)$

$$\frac{\cos A}{\sin A}\left(\frac{1}{\cos A} - \cos A\right)$$

$$\frac{\cos A}{\sin A \cos A} - \frac{\cos^2 A}{\sin A}$$

$$\frac{1}{\sin A} - \frac{\cos^2 A}{\sin A}$$

$$\frac{1-\cos^2 A}{\sin A} = \frac{\sin^2 A + \cos^2 A - \cos^2 A}{\sin A}$$

$$\frac{\sin^2 A}{\sin A} = \sin A$$

(b) $$\frac{\cot\theta}{\sin(90^\circ - \theta)} = \frac{\cot\theta}{\cos\theta}$$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\rightarrow \frac{\cos\theta}{\sin\theta} \div \frac{\cos\theta}{1}$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = \frac{1}{\sin\theta}$

$= \csc\theta$

p329

(8c) $(\sec x+\tan x)(1-\sin x) =$

$\sec x - \sec x \sin x + \tan x - \tan x \sin x$

$\frac{1}{\cos x} - \frac{1}{\cos x}\cdot\frac{\sin x}{1} + \frac{\sin x}{\cos x} - \frac{\sin x}{\cos x}\cdot\frac{\sin x}{1}$

$\frac{1}{\cos x} - \frac{\sin x}{\cos x} + \frac{\sin x}{\cos x} - \frac{\sin^2 x}{\cos x}$

① $\frac{1-\sin^2 x}{\cos x} = \frac{\sin^2 x+\cos^2 x-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

(d) $\frac{\cot\alpha+\tan\alpha}{\csc^2\alpha} = \frac{\frac{\cos\alpha\cos\alpha}{\sin\alpha\cos\alpha}+\frac{\sin\alpha\sin\alpha}{\cos\alpha\sin\alpha}}{\frac{1}{\sin^2\alpha}} = \frac{\frac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha\cos\alpha}}{\frac{1}{\sin^2\alpha}} \overset{\text{①}}{=} \frac{1}{\sin\alpha\cos\alpha}\cdot\frac{\sin^2\alpha}{1} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

p299

(10) $2\tan\theta+1=0$

$\tan\theta = -\frac{1}{2}$

Ref ∠: $\tan^{-1}(\frac{1}{2})$

tan is ⊖ in Q II & IV

II $\theta = 180^\circ - \tan^{-1}(\frac{1}{2}) \approx 153.4^\circ$

IV $\theta = 360^\circ - \tan^{-1}(\frac{1}{2}) \approx 333.4^\circ$

(18) $3\sin x+2=4$

$3\sin x = 2$

$\sin x = \frac{2}{3}$

Ref ∠ $= \sin^{-1}(\frac{2}{3})$

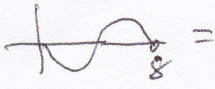
sine is ⊕ in Q I & II

I: $\theta = \sin^{-1}(\frac{2}{3}) \approx 0.73$

II $\theta = \pi - \sin^{-1}(\frac{2}{3}) \approx 2.41$

p305 #14

Amplitude = 2 ; AOW: $y=0$

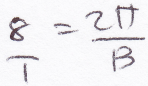
Per  $= \frac{8}{T} = \frac{2\pi}{B}$

$8B = 2\pi \rightarrow B = \frac{\pi}{4}$

neg sin : no phase shift

$y = -2\sin\frac{\pi}{4}x$

(16) Amp: 5 ; AOW: $y=0$

Per:  $\rightarrow$ Per $= \frac{2}{T} = \frac{2\pi}{B}$

$2B = 2\pi \rightarrow B = \pi$

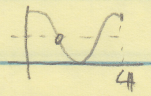
neg cos: no phase shift

$y = -5\cos\pi x$

p313 ① Max = 4, Min = 0 → Amplitude $= \frac{4-0}{2} = 2$

AOW: $y = 2$ $\left(\frac{4+0}{2}\right)$

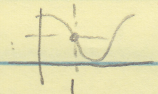
Period $= 4 = {}^{2\pi}/_{B} \rightarrow 4B = 2\pi \rightarrow B = {}^{2\pi}/_{4} = {}^{\pi}/_{2}$



Cos graph → no phase shift

$y = 2\cos {}^{\pi}/_{2}x + 2$

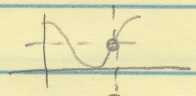
neg sine graph:



phase shift: R+1

$y = -2\sin\frac{\pi}{2}(x+1)+2$

; pos sine graph



phase shift: R+3

$y = 2\sin\frac{\pi}{2}(x-3)+2$

9. $y = 3 + 5\sin 2x$

Amp = 5

AOW: $y = 3$

$B = 2$

$per = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

Scale: ${}^{\pi}/_{4}$

phase shift: none