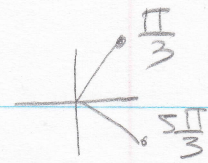
W/s 10.4



1. $\sin 2x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ $\qquad \boxed{2\cos x - 1 = 0}$

$\rightarrow \cos x = \frac{1}{2}$

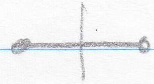
$x = \frac{\pi}{3} + 2n\pi$ $\qquad x = \frac{5\pi}{3} + 2n\pi$

$n=0 \rightarrow x = \boxed{\frac{\pi}{3}\ ;\ \frac{5\pi}{3}}$

$n=1 \rightarrow x = \cancel{\frac{7\pi}{3}}\ ;\ \cancel{\frac{11\pi}{3}}$

$(1,0)$

$(-1,0)$



$x = 0 + n\pi$

$n=0 \rightarrow \boxed{x=0}$

$n=1 \rightarrow \boxed{x=\pi}$

$n=2 \rightarrow x = \cancel{2\pi}$

2. $\tan 2x + \tan x = 0$

$$\frac{2\tan x}{1-\tan^2 x} + \frac{\tan x}{1}\frac{(1-\tan^2 x)}{(1-\tan^2 x)} = 0$$

$$\frac{2\tan x + \tan x - \tan^3 x}{\cancel{1-\tan^2 x}} = \frac{0}{\cancel{1-\tan^2 x}}$$

You can clear the fractions because of $=$

$3\tan x - \tan^3 x = 0$

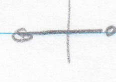
$\tan x\,(3 - \tan^2 x) = 0$

$\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$

$(1,0)$

$(-1,0)$



$x = 0 + n\pi$

$n=0 \rightarrow x = \boxed{0}$

$n=1 \rightarrow x = \boxed{\pi}$

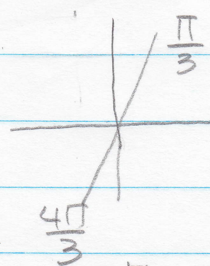
$n=2 \rightarrow \cancel{2\pi}$

$3 - \tan^2 x = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \sqrt{3}$



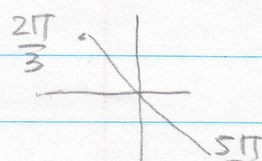
$x = \frac{\pi}{3} + n\pi$

$n=0 \rightarrow x = \boxed{\frac{\pi}{3}}$

$n=1 \rightarrow x = \boxed{\frac{4\pi}{3}}$

$n=2 \rightarrow x = \cancel{\frac{7\pi}{3}}$

$\tan x = -\sqrt{3}$



$x = \frac{2\pi}{3} + n\pi$

$n=0 \rightarrow x = \boxed{\frac{2\pi}{3}}$

$n=1 \rightarrow x = \boxed{\frac{5\pi}{3}}$

$n=2 \rightarrow x = \cancel{\frac{8\pi}{3}}$

3. $3\sin x = 1 + \cos 2x$

$3\sin x = 1 + (1 - 2\sin^2 x)$ → everything in terms of $\sin x$

$3\sin x = 2 - 2\sin^2 x$

$2\sin^2 x + 3\sin x - 2 = 0$

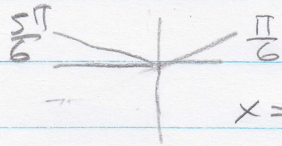
$2\sin x \quad -1$

$\sin x \quad +2$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -2$

not poss.



$x = \pi/6 + 2n\pi$ $\qquad$ $x = \frac{5\pi}{6} + 2n\pi$

$n = 0 \rightarrow x = \boxed{\pi/6}$ $\qquad$ $x = \boxed{5\pi/6}$

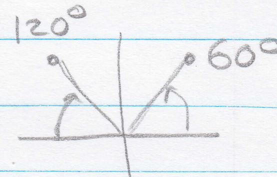
$n = 1$ too big.

4. $4\sin x \cos x = \sqrt{3}$ → Divide both sides by 2

$2\sin x \cos x = \sqrt{3}/2$

$\sin 2x = \sqrt{3}/2$ → $\theta = 2x$

$\sin\theta = \sqrt{3}/2$



$2x = 60° + 360° n$ $\qquad$ $2x = 120° + 360° \cdot n$

$x = 30° + 180° n$ $\qquad$ $x = 60° + 180° \cdot n$

$n=0$ $\quad x = \boxed{30°}$ $\qquad$ $x = \boxed{60°}$

$n=1$ $\quad x = \boxed{210°}$ $\qquad$ $x = \boxed{240°}$

$n=2$ $\quad x = 390°$ → too big $\qquad$ $x = 420°$ → too big

10.4#3

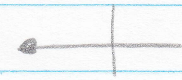
cos formula ; change sign

5 $\cos 8x \cos 5x + \sin 8x \sin 5x = -1$

$\cos(8x - 5x) = -1$

$\cos 3x = -1$ $\quad \theta = 3x$

$\cos\theta = -1 \rightarrow (-1, 0)$



$\theta = 180^\circ + 360^\circ \cdot n$

$3x = 180^\circ + 360^\circ \cdot n$

$x = 60^\circ + 120^\circ \cdot n$

$n=0 \rightarrow x = \boxed{60^\circ}$

$n=1 \rightarrow x = \boxed{180^\circ}$

$n=2 \rightarrow x = \boxed{300^\circ}$

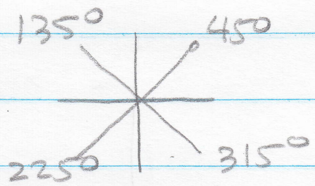
$n=3 \rightarrow x = \cancel{420^\circ}$

6 $\sin 2x \tan x = 1$

$\frac{2\sin x \cancel{\cos x}}{1} \cdot \frac{\sin x}{\cancel{\cos x}} = 1 \rightarrow 2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$



$x = 45^\circ + n \cdot 90^\circ$

$n=0 \rightarrow x = \boxed{45^\circ}$

$n=1 \rightarrow x = \boxed{135^\circ}$

$n=2 \rightarrow x = \boxed{225^\circ}$

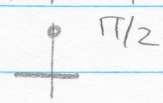
$n=3 \rightarrow x = \boxed{315^\circ}$

$n=4 \rightarrow x = \cancel{405^\circ}$

7 $\sin x \cos x = \frac{1}{2}$ → multiply both sides by 2

$2\sin x \cos x = 1 \rightarrow \sin 2x = 1$; $\theta = 2x$

$\sin\theta = 1 \quad (0, 1)$



$2x = \frac{\pi}{2} + 2n\pi$

$x = \frac{\pi}{4} + n\pi$

$n=0 \rightarrow x = \boxed{\pi/4}$

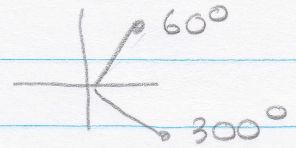
$n=1 \rightarrow x = \boxed{5\pi/4}$

$n=2 \rightarrow x = \cancel{9\pi/4}$

8 $\quad 2\cos(x+45^\circ)=1$

$\cos(x+45^\circ) = \frac{1}{2} \quad \rightarrow \theta = x+45^\circ$

$\cos\theta = \frac{1}{2}$



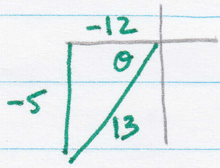
$\theta = 60^\circ + 360^\circ \cdot n$ $\qquad$ $\theta = 300^\circ + 360^\circ \cdot n$

$x+45^\circ = 60^\circ + 360^\circ n$ $\qquad$ $x+45^\circ = 300^\circ + 360^\circ \cdot n$

$n=0 \rightarrow x+45^\circ = 60^\circ \rightarrow x = \boxed{15^\circ}$ $\qquad$ $x+45^\circ = 300^\circ \rightarrow \boxed{x = 255^\circ}$

$n=1 \rightarrow x+45^\circ = 420^\circ \rightarrow x = 3\cancel{75^\circ}$ $\qquad$ $x+45^\circ = 660^\circ \rightarrow x = \cancel{615^\circ}$

9.



$\sin\theta = -5/13$

$\cos\theta = -12/13$

$\tan\theta = 5/12$

a $\quad \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(5/12)}{1-(5/12)^2} = \dfrac{\frac{10}{12}}{1-\frac{25}{144}} = \dfrac{10}{12} \cdot \dfrac{144}{119}$

$= \boxed{\dfrac{120}{119}}$

b. $\sin 2\theta = 2\sin\theta\cos\theta = 2(-5/13)(-12/13) = \boxed{120/169}$

c $\quad \cos\frac{1}{2}\theta$

$\pi < \theta < \frac{3\pi}{2}$

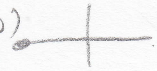
$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \rightarrow \frac{\theta}{2}$ is in Q II so $\cos\frac{\theta}{2}$ is neg

$\cos\frac{\theta}{2} = -\sqrt{\dfrac{1+\cos\theta}{2}} = -\sqrt{\dfrac{1+(-12/13)}{2}} = -\sqrt{\dfrac{\frac{1}{13}}{2}}$

$= -\sqrt{\dfrac{1}{13}\cdot\dfrac{1}{2}} = -\sqrt{\dfrac{1}{26}\cdot\dfrac{26}{26}} = \boxed{-\dfrac{\sqrt{26}}{26}}$

10-4 #6

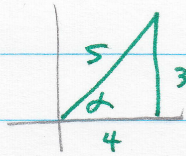
(-1,0)
c, s



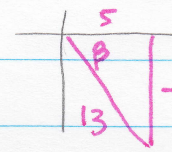
10 $\csc(180^\circ - \theta) = \dfrac{1}{\sin(180^\circ - \theta)} = \dfrac{1}{\sin 180^\circ \cos\theta - \cos 180^\circ \sin\theta}$

$= \dfrac{1}{(0)\cos\theta - (-1)\sin\theta} = \dfrac{1}{\sin\theta} = \boxed{\csc\theta}$

11



$\tan\alpha = \dfrac{3}{4}$



$\tan\beta = -12/5$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{\frac{5}{5}\cdot\frac{3}{4} + \frac{-12}{5}\cdot\frac{4}{4}}{1 - \left(\frac{3}{4}\right)\left(-\frac{12}{5}\right)} = \dfrac{\frac{15-48}{20}}{1 + \frac{36}{20}}$

$= -\dfrac{33}{20} \div \dfrac{56}{20} = \dfrac{-33}{20} \cdot \dfrac{20}{56} = \boxed{\dfrac{-33}{56}}$

12. $\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2\sin\alpha\sin\beta$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta - (\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$\cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta$

$2\sin\alpha\sin\beta$

13 $\sin(\alpha+\beta) - \sin(\alpha - \beta) = 2\sin\beta\cos\alpha$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta - (\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$\cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta - \cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta$

$= 2\cos\alpha\sin\beta$

14 $\sin 15^\circ \cos 15^\circ \rightarrow \theta = 15^\circ$

$\sin\theta\cos\theta = y$

$2\sin\theta\cos\theta = 2y$

$\sin 2\theta = 2y$

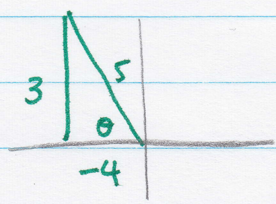
$y = \dfrac{\sin 2\theta}{2} = \dfrac{\sin 2(15^\circ)}{2} = \dfrac{\sin 30^\circ}{2} = \dfrac{\frac{1}{2}}{2} = \boxed{\dfrac{1}{4}}$

15. $\sin 112.5^\circ$ is + because 112.5° is in Q II

$\theta = 225^\circ$

$\sin 112.5^\circ = \sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\cos 225^\circ}{2}}$

$= +\sqrt{\frac{1-(-\sqrt{2}/2)}{2}} = +\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \boxed{+\frac{\sqrt{2+\sqrt{2}}}{2}}$



$\sin\theta = 3/5$

$\cos\theta = -4/5$

16. $\sin 2\theta = 2\sin\theta\cos\theta = 2(3/5)(-4/5) = \boxed{-24/25}$

17. $\cos 2\theta = 2\cos^2\theta - 1 = 2(-4/5)^2 - 1 = 2(\frac{16}{25}) - 1 = \boxed{7/25}$

18. $\cos\frac{\theta}{2} \rightarrow$ must find what Quad $\frac{\theta}{2}$ is in

$90^\circ < \theta < 180^\circ \rightarrow 45^\circ < \frac{\theta}{2} < 90^\circ \rightarrow$ Q I

so $\cos\frac{\theta}{2}$ is $\boxed{\text{pos}}$

$\cos\frac{\theta}{2} = +\sqrt{\frac{1+\cos\theta}{2}} = +\sqrt{\frac{1+(-4/5)}{2}} = +\sqrt{\frac{\frac{1}{5}}{2}}$

$= +\sqrt{\frac{1}{5}\cdot\frac{1}{2}\cdot\frac{10}{10}} = \boxed{+\frac{\sqrt{10}}{10}}$

19. $\sin\frac{\theta}{2}$ is pos because $\frac{\theta}{2}$ is in Q I

$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-(-4/5)}{2}} = +\sqrt{\frac{9/5}{2}} = +\sqrt{\frac{9}{10}\cdot\frac{10}{10}}$

$\boxed{\frac{+3\sqrt{10}}{10}}$

20 $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{\frac{3}{5}}{1+(-\frac{4}{5})} = \frac{\frac{3}{5}}{\frac{1}{5}} = \frac{3}{5}\cdot\frac{5}{1} = \boxed{3}$

OR $\tan\theta = +\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = +\sqrt{\frac{1-(-4/5)}{1+(-4/5)}} = +\sqrt{\frac{9/5}{1/5}} = \boxed{3}$