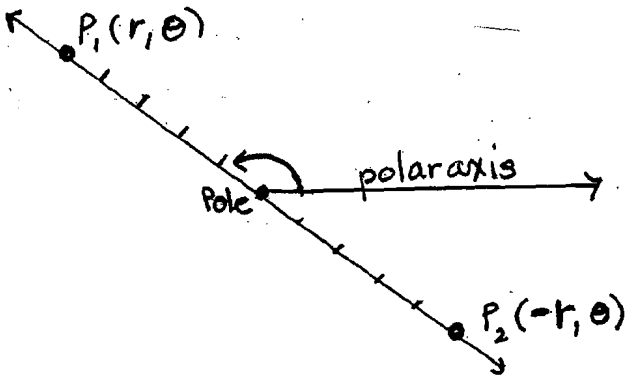
Notes 5|27|09

TRIGONOMETRY

POLAR COORDINATES; POLAR AND RECTANGULAR COORDINATES

Polar coordinates are used to simplify work with complex numbers. When we write $P(r,\theta)$, we are describing point P by its polar coordinates r and θ, where r is the distance from the pole to P, and θ is the angle measured from the polar axis to the terminal side of the angle.

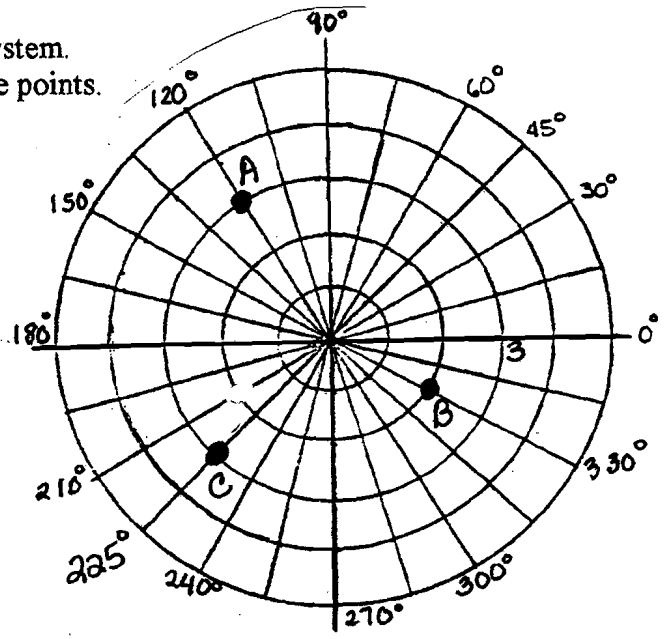


Example 1. Locate the points in the polar coordinate system. Give two other sets of coordinates for these points.

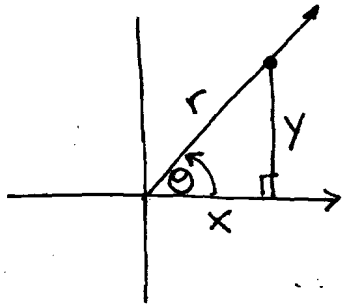
A (3, 120°) (3, -240°) (-3, 300°)

B (2, -30°) (2, 330°) (-2, 150°)

C (-3, 45°) (3, 225°) (3, -135°)



By using a right triangle in standard position in quadrant 1, SOH-CAH-TOA, and the Pythagorean Theorem, we can obtain "formulas" that can be used to convert rectangular coordinates (x , y) to polar coordinates (r, θ), or polar coordinates to rectangular coordinates.



$\cos\theta = \frac{x}{r} \Rightarrow \boxed{x = r\cos\theta}$

$\sin\theta = \frac{y}{r} \Rightarrow \boxed{y = r\sin\theta}$

$\boxed{r = \pm\sqrt{x^2+y^2}}$

(r, θ)

Example 2. Convert $(3, 240°)$ to rectangular coordinates.

Q3

$x = r\cos\theta$

$x = 3\cos 240°$

$x = 3\left(-\frac{1}{2}\right)$

$x = -\frac{3}{2}$

$y = r\sin\theta$

$y = 3\sin 240°$

$y = 3\left(-\frac{\sqrt{3}}{2}\right)$

$y = \frac{-3\sqrt{3}}{2}$

$\left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

Q4

(x, y)

Example 3. Convert $(3, -3)$ to polar coordinates.

$r = \sqrt{3^2+(-3)^2} = 3\sqrt{2}$

$x = r\cos\theta$

$3 = 3\sqrt{2}\cos\theta$

$\frac{1}{\sqrt{2}} = \cos\theta$

$315° = \theta$

because $(3,-3)$ is in Q4

Example 4. Find a polar equation for $x^2 + y^2 = 2x$.

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$r \neq 0$, so you can divide Σ by r!

Example 5. Find a rectangular equation for the polar equation $r\sin\theta + r = 2$. Identify the graph.

$y + r = 2$

$y + \left(\pm\sqrt{x^2+y^2}\right) = 2$

$\left(\pm\sqrt{x^2+y^2}\right)^2 = (2-y)^2$

$x^2 + y^2 = 4 - 4y + y^2$

$x^2 - 4 = -4y$

$-\frac{1}{4}x^2 + 1 = y$

parabola!!