

Ch 9-10 Review

① $A = 17^\circ$

$B = 120^\circ$

$a = 6$

$C = 180^\circ - 120^\circ - 17^\circ = 43^\circ$

Law of Sines: $\frac{6}{\sin 17^\circ} = \frac{c}{\sin 43^\circ}$

$c = \frac{6 \cdot \sin 43^\circ}{\sin 17^\circ} \approx 13.996$

② $a = 94$; $b = 73$; $c = 52$

$b^2 = a^2 + c^2 - 2ac \cos B$

$73^2 = 94^2 + 52^2 - 2(94)(52) \cos B$

$-6211 = -9776 \cos B \rightarrow \cos B = \frac{-6211}{-9776}$

$B = \cos^{-1} \left(\frac{6211}{9776} \right) \approx 50.6^\circ$

③ $A = 32^\circ$

$b = 10$

$c = 15$

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 10^2 + 15^2 - 2(10)(15) \cos 32^\circ$

$a^2 \approx 70.586$

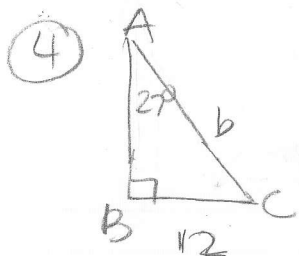
$a \approx 8.4$

$\frac{8.4}{\sin 32^\circ} = \frac{10}{\sin B}$

$8.4 \sin B = 10 \sin 32^\circ$

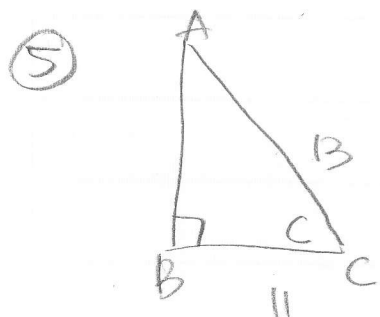
$\sin B = \frac{10 \sin 32^\circ}{8.4} \rightarrow B = \sin^{-1} \left(\frac{10 \sin 32^\circ}{8.4} \right) \approx 39.1^\circ$

Since b is not the longest side; $\angle B$ can't be obtuse.



$$\sin 27^\circ = \frac{12}{b} \rightarrow b \sin 27^\circ = 12$$

$$b = \frac{12}{\sin 27^\circ} = 26.432$$



$$\cos C = \frac{11}{13} \rightarrow C = \cos^{-1}\left(\frac{11}{13}\right) \approx 32.2^\circ$$

⑥

$$A = 36^\circ$$

$$b = 15$$

$$a = 10$$

$$\frac{10}{\sin 36^\circ} = \frac{15}{\sin B}$$

$$10 \sin B = 15 \sin 36^\circ$$

$$B = \sin^{-1}\left(\frac{15 \sin 36^\circ}{10}\right) \approx 61.845^\circ$$

since we don't know if b is the longest side or not B can be obtuse

$$A = 36^\circ$$

$$B = 61.845^\circ$$

$$C = 82.155^\circ$$

$$\frac{c}{\sin 82.155^\circ} = \frac{10}{\sin 36^\circ}$$

$$c = \frac{10 \sin 82.155^\circ}{\sin 36^\circ}$$

$$c \approx 16.854$$

$$A = \frac{1}{2} ab \sin C$$

$$\frac{1}{2} (10)(15) \sin 82.155^\circ$$

$$74.3 \text{ u}^2$$

$$A = 36^\circ$$

$$B = 180^\circ - 61.845^\circ = 118.155^\circ$$

$$C = 25.845^\circ$$

$$\frac{c}{\sin 25.845^\circ} = \frac{10}{\sin 36^\circ}$$

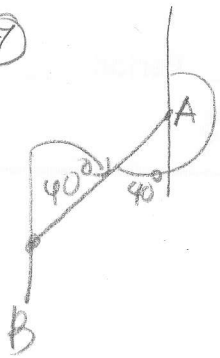
$$c = \frac{10 \sin 25.845^\circ}{\sin 36^\circ}$$

$$c \approx 7.417$$

$$A = \frac{1}{2} (10)(15) \sin(25.845^\circ)$$

$$32.7 \text{ u}^2$$

⑦

B to A = 040° 3 digits

⑧

$$2 \sin 5x \cos 5x$$

$$\theta = 5x$$

$$2 \sin \theta \cos \theta = \sin 2\theta = \sin 2(5x) = \sin 10x$$

⑨

$$\frac{\tan 195^\circ + \tan 15^\circ}{1 - \tan 195^\circ \tan 15^\circ} = \tan(195^\circ + 15^\circ) = \tan 210^\circ$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

⑩

$$\sin 165^\circ \cos 15^\circ + \cos 165^\circ \sin 15^\circ = \sin(165^\circ + 15^\circ) = \sin 180^\circ = 0$$

⑪

$$\cos^2 75^\circ - \sin^2 75^\circ$$

$$\theta = 75^\circ \rightarrow \cos^2 \theta - \sin^2 \theta = \cos 2\theta$$

$$= \cos 2(75^\circ) = \cos 150^\circ$$

$$= -\frac{\sqrt{3}}{2}$$

⑫

$$\cos(45^\circ - x) - \cos(45^\circ + x)$$

$$\cos 45^\circ \cos x + \sin 45^\circ \sin x - (\cos 45^\circ \cos x - \sin 45^\circ \sin x)$$

$$\cos 45^\circ \cos x + \sin 45^\circ \sin x - \cos 45^\circ \cos x + \sin 45^\circ \sin x$$

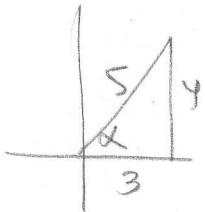
$$2 \sin 45^\circ \sin x$$

$$\left(2\right) \left(\frac{\sqrt{2}}{2}\right) (\sin x)$$

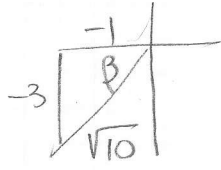
$$= \sqrt{2} \sin x$$

(13) $\csc(90^\circ - x) = \frac{1}{\sin 90^\circ - x}$ (0,1)
 $\sin(90^\circ - x) = \sin 90^\circ \cos x - \cos 90^\circ \sin x$
 $1 \cdot \cos x - 0 \cdot \sin x = \cos x$
 $\csc 90^\circ - x = \frac{1}{\cos x} = \sec x$

(14)



$\cos \alpha = \frac{3}{5}$
 $\sin \alpha = \frac{4}{5}$



$\sin \beta = -\frac{3}{\sqrt{10}}$
 $\cos \beta = -\frac{1}{\sqrt{10}}$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\frac{3}{5} \cdot \frac{-1}{\sqrt{10}} - \frac{4}{5} \cdot \frac{-3}{\sqrt{10}}$$

$$\frac{-3}{5\sqrt{10}} + \frac{12}{5\sqrt{10}} = \frac{9\sqrt{10}}{5\sqrt{10}\sqrt{10}}$$

$$= \frac{9\sqrt{10}}{50}$$

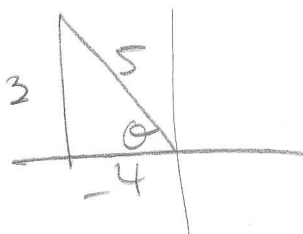
(15) $\cos 15^\circ = \cos(45^\circ - 30^\circ)$
 $\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$
 $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

(16) $\cos 15^\circ = \cos\left(\frac{30^\circ}{2}\right) = \oplus \sqrt{\frac{1 + \cos 30^\circ}{2}}$

\oplus because 15° is in Q I

$$\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

(17)



$\sin \theta = \frac{3}{5}$
 $\cos \theta = -\frac{4}{5}$

$\sin 2\theta = 2 \sin \theta \cos \theta$
 $= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

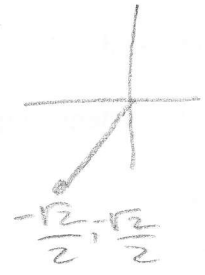
(18) $90^\circ < \theta < 180^\circ$
 $45^\circ < \frac{\theta}{2} < 90^\circ$
 $\frac{\theta}{2}$ is in Q I

$$\sin \frac{\theta}{2} = \oplus \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - (-\frac{4}{5})}{2}}$$

$$\sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

(19) $\tan \theta = -\frac{1}{3}$

$$\tan\left(\frac{5\pi}{4} - \theta\right) = \frac{\tan \frac{5\pi}{4} - \tan \theta}{1 + \tan \frac{5\pi}{4} \tan \theta}$$



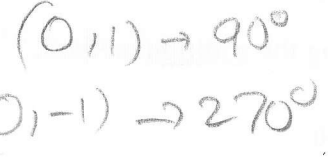
$$= \frac{+1 - \left(-\frac{1}{3}\right)}{1 + (+1)\left(-\frac{1}{3}\right)} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{4}{3} \cdot \frac{3}{2} = 2$$

(20) $\sin 2x + \cos x = 0$ $0 \leq x < 360^\circ$

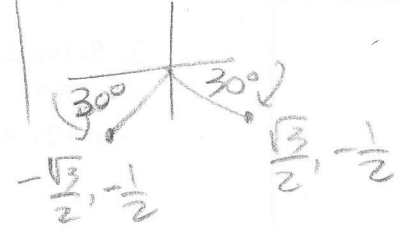
$$2\sin x \cos x + \cos x = 0$$

$$\cos x (2\sin x + 1) = 0$$

$\cos x = 0$



$\sin x = -\frac{1}{2}$



$180^\circ + 30^\circ = 210^\circ$

$360^\circ - 30^\circ = 330^\circ$

(21) $\cos 2x \cos x - \sin 2x \sin x = -1$

$$\cos(2x + x) = -1$$

$$\cos 3x = -1$$

$$\cos \theta = -1 \rightarrow (-1, 0); \theta = 180^\circ$$




$$3x = 180^\circ + n \cdot 360^\circ$$

$$x = 60^\circ + n \cdot 120^\circ$$

$$60^\circ, 180^\circ, 300^\circ$$

$$\textcircled{22} \quad \frac{1}{\sin(\frac{\pi}{2}-x)} ; \quad \sin(\frac{\pi}{2}-x) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$$

$$= \cos x - 0 \sin x = \cos x$$


$$\frac{1}{\sin(\frac{\pi}{2}-x)} = \frac{1}{\cos x} = \sec x$$

$$\textcircled{23} \quad (1 + \csc \theta)(1 - \csc \theta) = 1 - \csc^2 \theta$$

$$\csc^2 \theta = \cot^2 \theta + 1$$

$$1 - \csc^2 \theta = 1 - (\cot^2 \theta + 1) = 1 - \cot^2 \theta - 1 = -\cot^2 \theta$$

$$\textcircled{24} \quad \sec x \tan(-x) \cos x$$

$$\left(\frac{1}{\cos x}\right) (\ominus \tan x) \cos x = \ominus \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = -\frac{\sin x}{\cos x}$$

$$= -\tan x$$

$$\textcircled{25} \quad 1 + \cos 4x = 0$$

$$\cos 4x = -1 ; \quad \theta = 4x$$

$$\cos \theta = -1 \rightarrow (-1, 0)$$


$$\theta = \pi$$

$$4x = \pi + 2n\pi$$

$$x = \frac{\pi}{4} + \frac{2n\pi}{4}$$

$$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$(26) \quad 2 \tan x \csc x + 3 \tan x = 0$$

$$\tan x (2 \csc x + 3) = 0$$

$$\tan x = 0$$

$$x = 0, \pi$$

$$\csc x = -\frac{3}{2}$$

$$\sin x = -\frac{2}{3}$$

$$x = \pi + .7297 \approx 3.871$$

$$x = 2\pi - .7297 \approx 5.553$$

$$(27) \quad 8 \sin x \cos x = 8 \sin x$$

$$8 \sin x \cos x - 8 \sin x = 0$$

$$8 \sin x (\cos x - 1) = 0$$

$$8 \sin x = 0 \quad \cos x = 1$$

$$(1, 0) \rightarrow 0$$

$$(1, 0) = 0$$

$$(-1, 0) \rightarrow \pi$$



$$(28) \quad \cos^2 \theta = 8 \sin^2 \theta + 1$$

$$\cos^2 \theta = (1 - \cos^2 \theta) + 1$$

$$\cos^2 \theta - 1 + \cos^2 \theta - 1 = 0$$

$$2 \cos^2 \theta - 2 = 0$$

$$\cos^2 \theta = 1$$

$$\cos \theta = \pm 1$$

$$(1, 0) \quad (-1, 0)$$

$$0, \pi$$



$$(29) \quad 2 \csc \frac{x}{3} = 5$$

$$\csc \frac{x}{3} = \frac{5}{2}$$

$$\csc \theta = \frac{5}{2}$$

$$\sin \theta = \frac{2}{5}$$

$$\theta = \frac{x}{3}$$

$\sin \theta$ pos in Q I & Q II

Q I

$$\theta = \sin^{-1} \frac{2}{5}$$

$$\theta = 23.6^\circ$$

Q II

$$\theta = 180^\circ - \sin^{-1} \left(\frac{2}{5} \right)$$

$$= 156.4^\circ$$

$$\frac{x}{3} = 23.6^\circ + n \cdot 360$$

$$x = 70.8^\circ + 1080n$$

$$70.8^\circ$$

$$\frac{x}{3} = 156.4^\circ + n \cdot 360$$

$$x = 469.2 + 1080n$$

too big

$$\textcircled{30} \quad \sin^2 x - \sin x = 0$$

$$\sin x (\sin x - 1) = 0$$

$$\sin x = 0 \quad \left| \quad \sin x = 1 \right.$$

$$\begin{array}{l} 1, 0 \\ -1, 0 \\ 0, \pi \end{array} \quad \left(0, 1 \right)$$

$$\begin{array}{l} \frac{\pi}{2} \end{array}$$

$$\textcircled{31} \quad 2 \sin 2x + 1 = 0$$

$$\sin 2x = -\frac{1}{2}$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

$$2x = \frac{7\pi}{6} + 2n\pi \quad \left| \quad 2x = \frac{11\pi}{6} + 2n\pi \right.$$

$$x = \frac{7\pi}{12} + n\pi$$

$$x = \frac{11\pi}{12} + n\pi$$

$$\frac{7\pi}{12}, \frac{19\pi}{12}$$

$$\frac{11\pi}{12}, \frac{23\pi}{12}$$

$$\textcircled{32} \quad \tan^3 \theta = 3 \tan \theta$$

$$\tan^3 \theta - 3 \tan \theta = 0$$

$$\tan \theta (\tan^2 \theta - 3) = 0$$

$$\tan \theta = 0$$

$$\theta = 0; \pi$$

$$\tan^2 \theta = 3$$

$$\tan \theta = \pm \sqrt{3}$$

$$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$