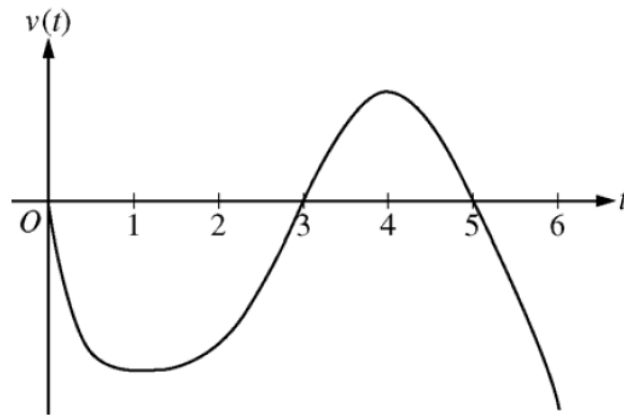


PRACTICE TEST 4 FREE RESPONSE QUESTIONS #4-6 – NO CALCULATOR

Spend approximately 15 minutes per question. Complete each question on a separate sheet of paper. Do not use any outside resources while working on the questions, just do the best you can.



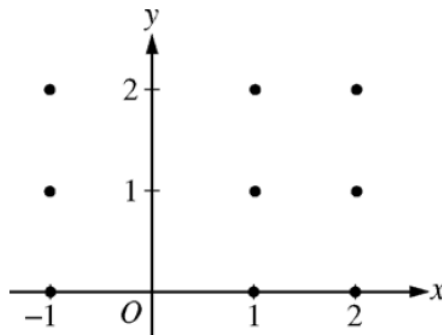
Graph of v

4. A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.
- For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
 - For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
 - On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
 - During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \rightarrow \infty} f(x)$.

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.

(d) Find $\lim_{x \rightarrow 0^+} f(x)$.