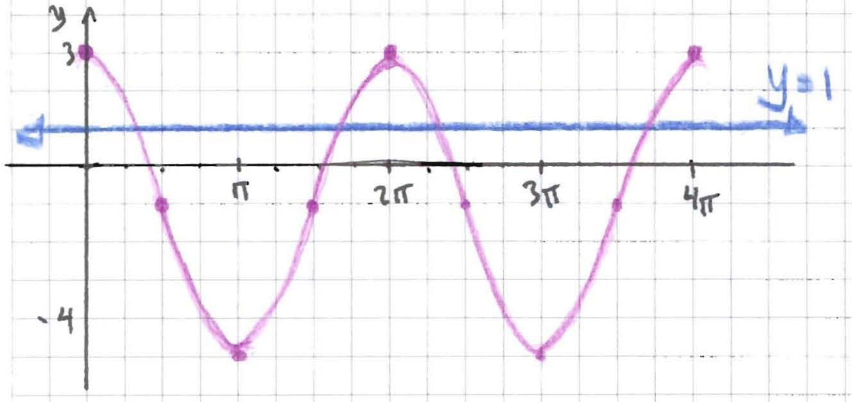
3. (a) $y = 4\sin\left(\frac{\pi}{2} - x\right) - 1$

$= 4\cos x - 1$



(b) $4\sin\left(\frac{\pi}{2} - x\right) - 1 = 1$

(c) $4\sin\left(\frac{\pi}{2} - x\right) = 2$

$\sin\left(\frac{\pi}{2} - x\right) = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$.

(d) THE SOLUTIONS TO THE EQUATION IN PART (C) ARE THE x-COORDINATES OF THE POINTS OF INTERSECTION OF THE GRAPHS.

4. (a) Prove that $\cos\theta + 2\sin^2\frac{\theta}{2} = 1$

$$\cos\theta + 2\sin^2\frac{\theta}{2} = \cos\theta + 2\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2$$

$$= \cos\theta + 2\left(\frac{1-\cos\theta}{2}\right)$$

$$= \cos\theta + 1 - \cos\theta$$

$$= 1.$$

(b) We must show that

$$2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta = \sin 2\theta$$

$$2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta$$

$$= 2\sin\theta\cos\theta\left(\cos^2\theta + \sin^2\theta\right)$$

$$= 2\sin\theta\cos\theta$$

$$= \sin 2\theta.$$

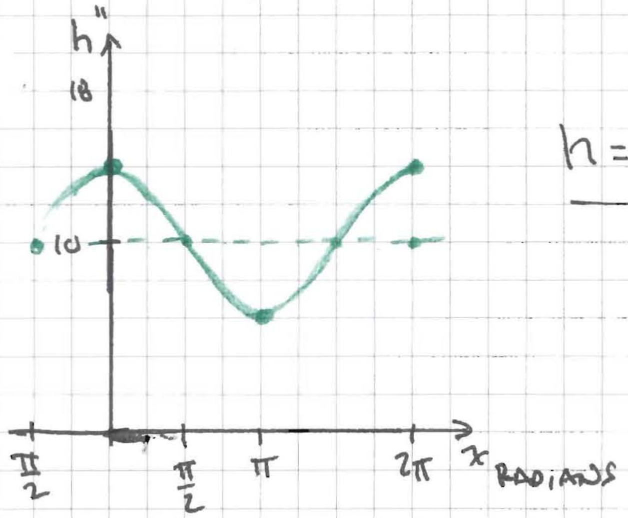
5. (a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{4\pi}{3}, \frac{5\pi}{3} + 2\pi n, n \in I$.

(c) $\tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3} + \pi n, n \in I$

(b) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n, n \in I$.

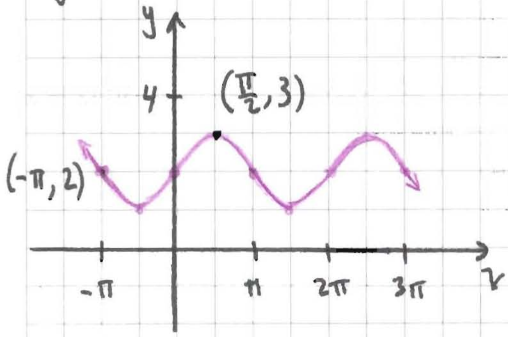
(d) $\csc^{-1}(\sqrt{2}) = \frac{\pi}{4}, \frac{3\pi}{4} + 2\pi n, n \in I$

6.



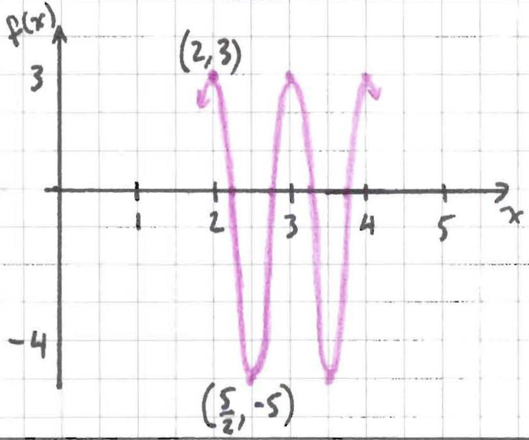
$h = 4\cos x + 10 = 4\sin\left(\frac{\pi}{2} - x\right) + 10$.

1. (a) $y = -\sin(x+\pi)+2$

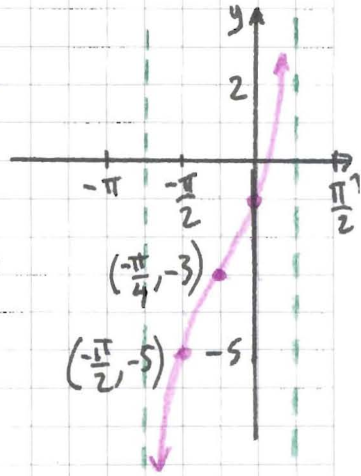


(b) $f(x) = 4\cos(2\pi x - 4\pi) - 1$

$= 4\cos 2\pi(x-2) - 1$



(c) $y = 2\tan\left(x+\frac{\pi}{4}\right) - 3$



2. (a) $2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$2x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \dfrac{\sin^{-1}\left(\frac{1}{2}\right)}{2}$

$x = \dfrac{\frac{\pi}{6} + 2\pi n}{2}, \dfrac{\frac{5\pi}{6} + 2\pi n}{2}, n \in \mathbb{Z}$

$x = \frac{\pi}{12} + \pi n, \frac{5\pi}{12} + \pi n, n \in \mathbb{Z}.$

(b) $3 - e^{(x-5)} = -7$

$-e^{(x-5)} = -10$

$e^{(x-5)} = 10$

$x - 5 = \ln(10)$

$x = 5 + \ln(10) \approx 7.303$

(c)

$2\cos^2 x - \sin x - 1 = 0$

$2(1-\sin^2 x) - \sin x - 1 = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$(-2\sin x + 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}.$

(d) $\sin\frac{x}{2} + \cos x = 1$

$\pm\sqrt{\dfrac{1-\cos x}{2}} + \cos x = 1$

$\left(\pm\sqrt{\dfrac{1-\cos x}{2}}\right)^2 = (1-\cos x)^2$

$\dfrac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$

$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$

$0 = 2\cos^2 x - 3\cos x + 1$

$0 = (2\cos x - 1)(\cos x - 1)$

$\cos x = \frac{1}{2}$ or $\cos x = 1$

$x = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n, 0 + 2\pi n$