ANGLES AND MEASUREMENT
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CHAPTER 2 Angles and Measurement

In Chapter 1, you studied many common geometric shapes and learned ways to describe a shape using its attributes. In this chapter, you will further **investigate** how to describe a complex shape by developing ways to accurately determine its angles, area, and perimeter. You will also use transformations from Chapter 1 to uncover special relationships between angles within a shape.

Throughout this chapter you will be asked to solve problems, such as those involving area or angles, in more than one way. This will require you to "see" shapes in multiple ways and to gain a broader understanding of problem solving.

Guiding Questions

Think about these questions throughout this chapter:

What is the relationship?

Can I generalize the process?

Is it always true?

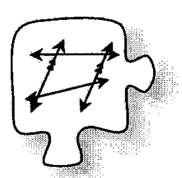
What information do I need?

Is there another way?

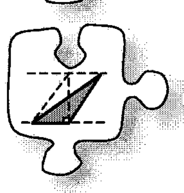
In this chapter, you will learn:

- the relationships between pairs of angles formed by transversals and the angles in a triangle.
- how to find the area and perimeter of triangles, parallelograms, and trapezoids.
- the relationship among the three side lengths of a right triangle (the Pythagorean Theorem).
- how to estimate the value of square roots.
- how to determine when the lengths of three segments can and cannot form a triangle.

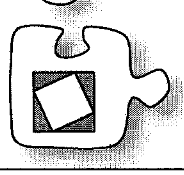
Chapter Outline



Section 2.1 You will broaden your understanding of angle, begun in Chapter 1, to include relationships between angles, such as those formed by intersecting lines or those inside a triangle.

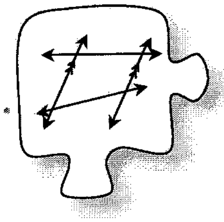


Section 2.2 After examining how units of measure work, you will develop methods to find the areas of triangles, parallelograms, and trapezoids as well as more complicated shapes.



Section 2.3 You will discover a relationship among the sides of a right triangle called the Pythagorean Theorem. This will allow you to find the perimeter of triangles, parallelograms, and trapezoids, and to find the distance between two points on a graph.

2.1.1 What's the relationship?



Complementary, Supplementary, and Vertical Angles

In Chapter 1, you compared shapes by looking at similarities between their parts. For example, two shapes might have sides of the same length or equal angles. In this chapter you will **examine** relationships between parts within a *single* shape or diagram. Today you will start by looking at angles to identify relationships in a diagram that make angle measures equal. As you **examine** angle relationships today, keep the following questions in mind to guide your discussion:

How can we name the angle?

What is the relationship?

How do we know?

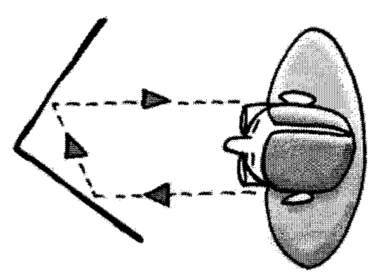
2-1. SOMEBODY'S WATCHING ME

Usually, to see yourself in a small mirror you have to be looking directly into it—if you move off to the side, you can't see your image any more. But Mr. Douglas knows a neat trick. He claims that if he makes a right angle with a hinged mirror, he can see himself in the mirror no matter from which direction he looks into it.

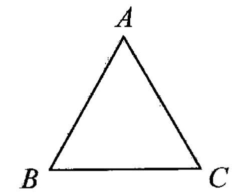
a. By forming a right angle with a hinged mirror, test Mr. Douglas's trick for yourself. Look into the place where the sides of the mirror meet. Can you see yourself? What if you look in the mirror from a different angle?

b. Does the trick work for *any* angle between the sides of the mirror? Change the angle between the sides of the mirror until you can no longer see your reflection where the sides meet.

c. At right is a diagram of a student trying out the mirror trick. What appears to be true about the lines of sight? Can you explain why Mr. Douglas's trick works? Talk about this with your team and be ready to share your ideas with the class.

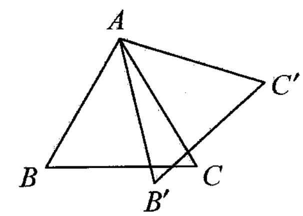


2-2. To completely understand how Mr. Douglas's reflection trick works, you need to learn more about the relationships between angles. But in order to clearly communicate relationships between angles, you will need a convenient way to refer to and name them. **Examine** the diagram of equilateral ΔABC at right.

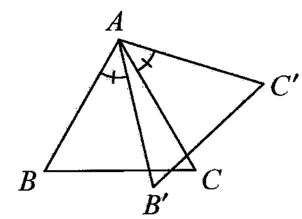


a. The "top" of this triangle is usually referred to as "angle A," written $\measuredangle A$. Point A is called the **vertex** of this angle. The *measure* of $\measuredangle A$ (the number of degrees in angle A) is written $m\measuredangle A$. Since ΔABC is equilateral, write an equation showing the relationship between its angles.

b. Audrey rotated ΔABC around point A to form $\Delta AB'C'$. She told her teammate Maria, "I think the two angles at A are equal." Maria did not know which angles she was referring to. How many angles can you find at A? Are there more than three?



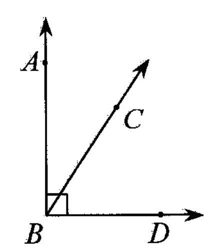
c. Maria asked Audrey to be more specific. She explained, "One of my angles is $\measuredangle BAB'$." At the same time, she marked her two angles with the same marking at right to indicate that they have the same measure.



Name her other angle. Be sure to use three letters so there is no confusion about which angle you mean.

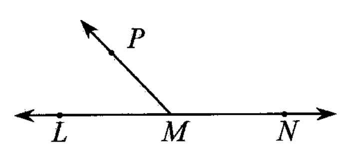
2-3. ANGLE RELATIONSHIPS

When you know two angles have a certain **relationship**, learning something about one of them tells you something about the other. Certain angle relationships come up often enough in geometry that we give them special names.



a. Two angles whose measures have a sum of 90° are called **complementary angles**. Since $\measuredangle ABD$ is a right angle in the diagram at right, angles $\measuredangle ABC$ and $\measuredangle CBD$ are complementary. If $m\measuredangle CBD = 76°$, what is $m\measuredangle ABC$? Show how you got your answer.

b. Another special angle is 180°. If the sum of the measures of two angles is 180°, they are called **supplementary angles**. In the diagram at right, $\measuredangle LMN$ is a straight angle. If $m\measuredangle LMP = 62°$, what is $m\measuredangle PMN$?



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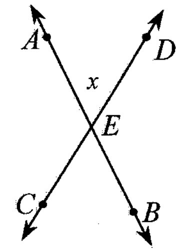
2-3. *Problem continued from previous page.*



c. Now consider the diagram at right, which shows $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersecting at E. If $x = 23°$, find $m\measuredangle AEC$, $m\measuredangle DEB$, and $m\measuredangle CEB$. Show all work.

d. When angles have equal measure, they are referred to as **congruent**. Which angle is congruent to $\measuredangle AED$?

e. When two lines intersect, the angles that lie on opposite sides of the intersection point are called **vertical angles**. For example, in the diagram above, $\measuredangle AED$ and $\measuredangle CEB$ are vertical angles. Find another pair of vertical angles in the diagram.

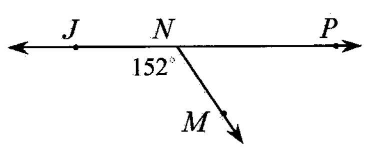
2-4. Travis noticed that the vertical angles in parts (c) and (d) of problem 2-3 are congruent and wondered if pairs of vertical angles always have the same measure.

a. Return to the diagram above and find $m\measuredangle CEB$ if $x = 54°$. Show all work.

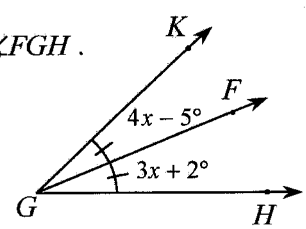
b. Based on your observations, write a **conjecture** (a statement based on an educated guess that is unproven). Start with, "*Vertical angles are ...*"

2-5. In the problems below, you will use geometric relationships to find angle measures. Start by finding a special relationship between some of the sides or angles, and use that relationship to write an equation. Solve the equation for the variable, then use that variable value to answer the original question.

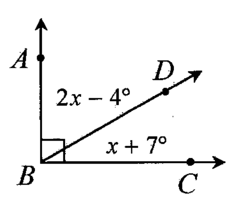
a. Find $m\measuredangle MNP$.



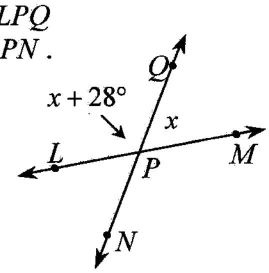
b. Find $m\measuredangle FGH$.



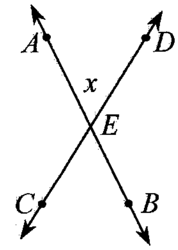
c. Find $m\measuredangle DBC$.



d. Find $m\measuredangle LPQ$ and $m\measuredangle LPN$.



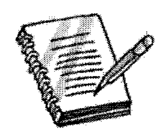
2-6. When Jao answered part (b) of problem 2-4, he wrote the conjecture: "*Vertical angles are equal.*" (Remember that a **conjecture** is an educated guess that has not yet been proven.)



a. **Examine** the diagram at right. Express the measures of every angle in the diagram in terms of x.

b. Do you think your vertical angle conjecture holds for *any* pair of vertical angles? Be prepared to convince the rest of the class.

2-7. Describe each of the angle relationships you learned about today in an entry in your Learning Log. Include a diagram, a description of the angles, and what you know about the relationship. For example, are the angles always equal? Do they have a special sum? Label this entry "Angle Relationships" and include today's date.

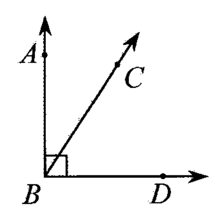


METHODS AND MEANINGS

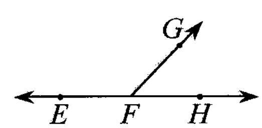
MATH NOTES

Angle Relationships

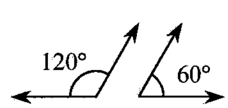
If two angles have measures that add up to 90°, they are called **complementary angles**. For example, in the diagram at right, $\measuredangle ABC$ and $\measuredangle CBD$ are complementary because together they form a right angle.



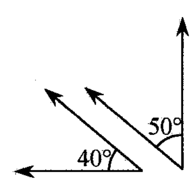
If two angles have measures that add up to 180°, they are called **supplementary angles**. For example, in the diagram at right, $\measuredangle EFG$ and $\measuredangle GFH$ are supplementary because together they form a straight angle.



Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.

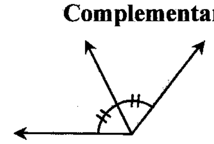


Supplementary



Complementary

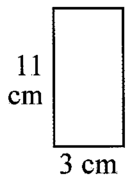
When two angles have equal measure, they are called **congruent**. Their equality can be shown with matching markings, as shown in the diagram at right.


Congruent

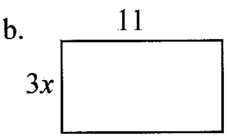
Review & Preview

2-8. Find the area of each rectangle below:

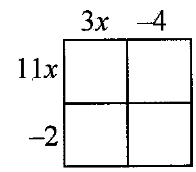
a.



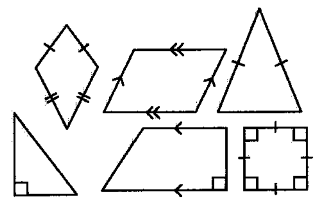
b.



c.



2-9. Mei puts the shapes at right into a bucket and asks Brian to pick one out.



a. What is the probability that he pulls out a quadrilateral with parallel sides?

b. What is the probability that he pulls out a shape with rotation symmetry?

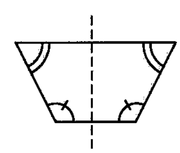
2-10. Camille loves guessing games. She is going to tell you a fact about her shape to see if you can guess what it is.

a. "My triangle has only one line of symmetry. What is it?"

b. "My triangle has three lines of symmetry. What is it?"

c. "My quadrilateral has no lines of symmetry but it does have rotation symmetry. What is it?"

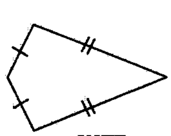
2-11. Jerry has an idea. Since he knows that an isosceles trapezoid has reflection symmetry, he **reasons**, "That means that it must have two pairs of congruent angles." He marks the congruent parts on his diagram at right.



ISOSCELES TRAPEZOID

Copy the shapes below onto your paper. Similarly mark which angles must be equal due to reflection symmetry.

a.

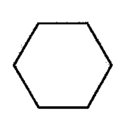


KITE

b.

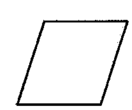
ISOSCELES TRIANGLE

c.



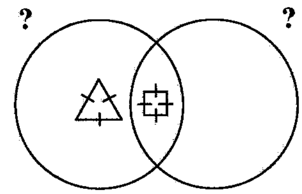
REGULAR HEXAGON

d.

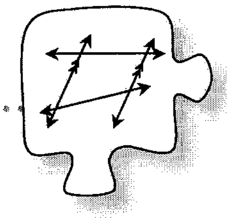


RHOMBUS

2-12. Larry saw Javon's incomplete Venn diagram at right, and he wants to finish it. However, he does not know the condition that each circle represents. Find a possible label for each circle, and place two more shapes into the diagram.



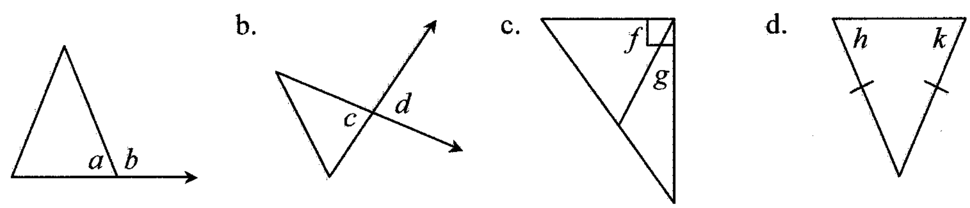
2.1.2 What's the relationship?



Angles Formed by Transversals

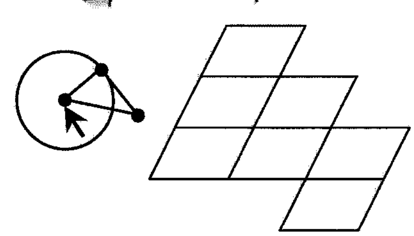
In Lesson 2.1.1, you **examined** vertical angles and found that vertical angles are always equal. Today you will look at another special relationship that guarantees angles are equal.

2-13. **Examine** the diagrams below. For each pair of angles marked on the diagram, quickly decide what relationship their measures have. Your responses should be limited to one of three relationships: congruent (equal measures), complementary (have a sum of 90°), and supplementary (have a sum of 180°).



2-14. Marcos was walking home after school thinking about special angle relationships when he happened to notice a pattern of parallelogram tiles on the wall of a building. Marcos saw lots of special angle relationships in this pattern, so he decided to copy the pattern into his notebook.

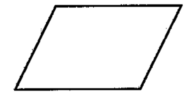
The beginning of Marcos's diagram is shown at right and provided on the Lesson 2.1.2 Resource Page. This type of pattern is sometimes called a **tiling**. In a tiling, a shape is repeated without gaps or overlaps to fill an entire page. In this case, the shape being tiled is a parallelogram.



Problem continues on next page →

2-14. *Problem continued from previous page.*

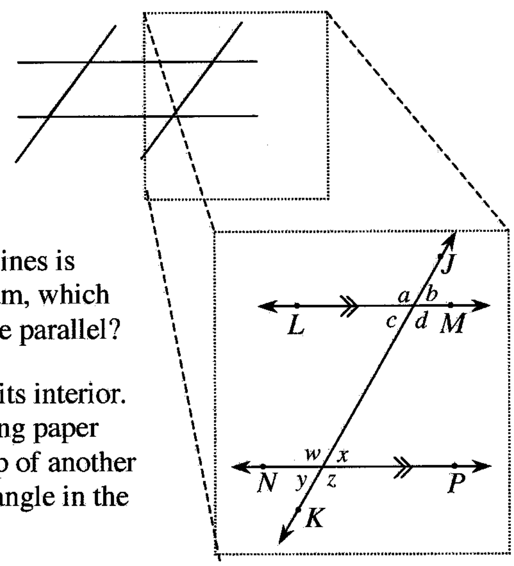
a. Consider the angles inside a single parallelogram. Are any angles congruent? On your resource page, use color to show which angles must have equal measure. If two angles are not equal, make sure they are shaded with different colors.



b. Since each parallelogram is a translation of another, what can be stated about the angles in the rest of Marcos' tiling? Use a dynamic geometry tool, transparencies on an overhead, or tracing paper to determine which angles must be congruent. Color all angles that must be equal the same color.

c. What about relationships between lines? Can you identify any lines that must be parallel? Mark all the lines on your diagram with the same number of arrows to show which lines are parallel.

2-15. Julia wants to learn more about the angles in Marcos's diagram and has decided to focus on just a part of his tiling. An enlarged view of that section is shown in the image below right, with some points and angles labeled.



a. A line that crosses two or more other lines is called a **transversal**. In Julia's diagram, which line is the transversal? Which lines are parallel?

b. Trace $\measuredangle x$ on tracing paper and shade its interior. Then translate $\measuredangle x$ by sliding the tracing paper along the transversal until it lies on top of another angle and matches it exactly. Which angle in the diagram corresponds with x?

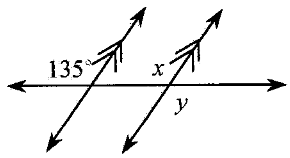
c. What is the relationship between the measures of angles x and b? Must one be greater than the other, or must they be equal? Explain how you know.

d. In this diagram, $\measuredangle x$ and $\measuredangle b$ are called **corresponding angles** because they are in the same position at two different intersections of the transversal. The corresponding angles in this diagram are equal because they were formed by translating a parallelogram. Name all the other pairs of equal corresponding angles you can find in Julia's diagram.

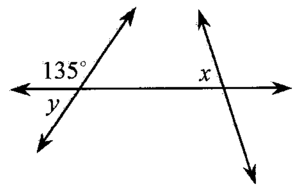
e. Suppose $b = 60°$. Use what you know about vertical, supplementary, and corresponding angle relationships to find the measures of all the other angles in Julia's diagram.

2-16. Frank wonders whether corresponding angles are *always* equal. For parts (a) through (d) below, decide whether you have enough information to find the measures of x and y. If you do, find the angle measures and state the relationship. Use tracing paper to help you find corresponding angles.

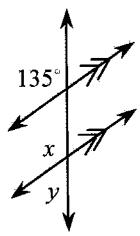
a.



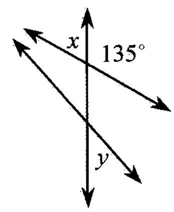
b.



c.



d.

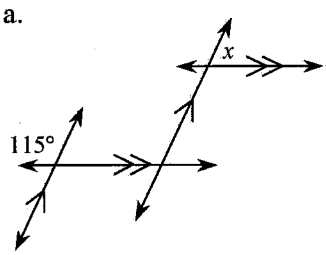


e. Answer Frank's question: Are corresponding angles always equal?

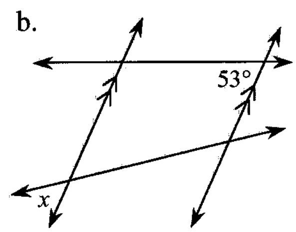
f. Conjectures are often written in the form, "*If..., then...*". A statement in if-then form is called a **conditional statement**. Make a conjecture about corresponding angles by completing this conditional statement: "If ..., then corresponding angles are equal."

2-17. For each diagram below, find the value of x *if possible*. If it is not possible, explain how you know. State the relationships you use. Be prepared to **justify** every measurement you find to other members of your team.

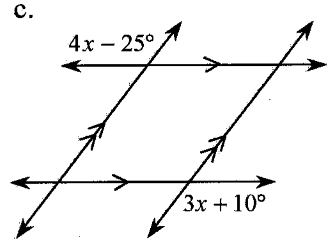
a.



b.



c.



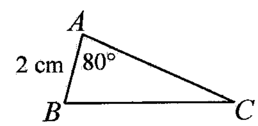
MATH NOTES

METHODS AND MEANINGS

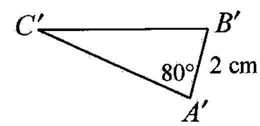
Naming Parts of Shapes

Part of geometry is the study of parts of shapes, such as points, line segments, and angles. To avoid confusion, standard notation is used to name these parts.

A point is named using a single capital letter. For example, the vertices (corners) of the triangle at right are named A, B, and C.

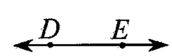


If a shape is transformed, the image shape is often named using **prime notation**. The image of point A is labeled A' (read as "A prime"), the image of B is labeled B' (read as "B prime"), etc. At right, $\Delta A'B'C'$ is the image of ΔABC.

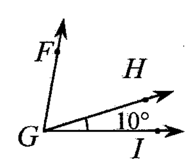


The side of a polygon is a **line segment**. A line segment is named by naming its endpoints and placing a bar above them. For example, one side of the first triangle above is named $\overline{AB}$. When referring to the length of a segment, the bar is omitted. In ΔABC above, $AB = 2$ cm.

A **line**, which differs from a segment in that it extends infinitely in either direction, is named by naming two points on the line and placing a bar with arrows above them. For example, the line below is named $\overleftrightarrow{DE}$. When naming a segment or line, the order of the letters is unimportant. The line below could also be named $\overleftrightarrow{ED}$.



An angle can be named by putting an angle symbol in front of the name of the angle's vertex. For example, the angle measuring 80° in ΔABC above is named $\measuredangle A$. Sometimes using a single letter makes it unclear which angle is being referenced. For example, in the diagram at right, it is unclear which angle is referred to by $\measuredangle G$. When this happens, the angle is named with three letters. For example, the angle measuring 10° is called $\measuredangle HGI$ or $\measuredangle IGH$. Note that the name of the vertex must be the second letter in the name; the order of the other two letters is unimportant.

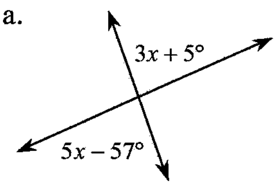


To refer to an angle's measure, an m is placed in front of the angle's name. For example, $m\measuredangle HGI = 10°$ means "the measure of $\measuredangle HGI$ is 10°."

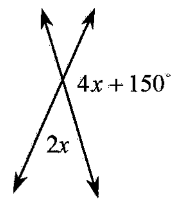
Review & Preview

2-18. **Examine** the diagrams below. What is the geometric relationship between the labeled angles? What is the relationship of their measures? Then, use the relationship to write an equation and solve for x.

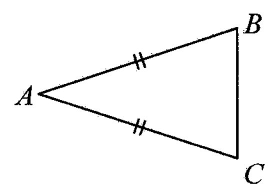
a.



b.



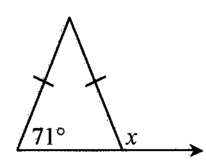
2-19. In problem 2-11, you determined that because an isosceles triangle has reflection symmetry, then it must have two angles that are congruent.



a. How can you tell which angles are congruent? For example, in the diagram at right, which angles must have equal measure? Name the angles and explain how you know.

b. **Examine** the diagram for part (a). If you know that $m\measuredangle B + m\measuredangle C = 124°$, then what is the measure of $\measuredangle B$? Explain how you know.

c. Use this idea to find the value of x in the diagram at right. Be sure to show all work.



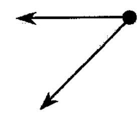
2-20. On graph paper, draw the quadrilateral with vertices (–1, 3), (4, 3), (–1, –2), and (4, –2).

a. What kind of quadrilateral is this?

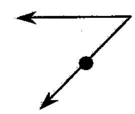
b. Translate the quadrilateral 3 units to the left and 2 units up. What are the new coordinates of the vertices?

2-21. Find the equation for the line that passes through (–1, –2) and (4, 3). Is the point (3, 1) on this line? Be sure to **justify** your answer.

2-22. Juan decided to test what would happen if he rotated an angle.

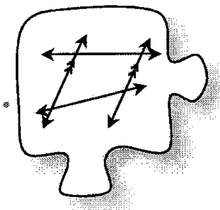


a. He copied the angle at right on tracing paper and rotated it 180° about its vertex. What type of angle pair did he create? What is the relationship of these angles?



b. Juan then rotated the same angle 180° through a different point (see the diagram at right). On your paper, draw Juan's angle and the rotated image. Describe the overall shape formed by the two angles.

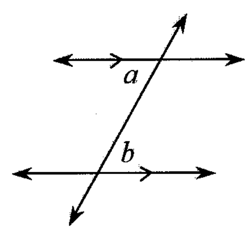
2.1.3 What's the relationship?



More Angles Formed by Transversals

In Lesson 2.1.2, you looked at corresponding angles formed when a transversal intersects two parallel lines. Today you will **investigate** other special angle relationships formed in this situation.

2-23. Suppose $\measuredangle a$ in the diagram at right measures 48°.

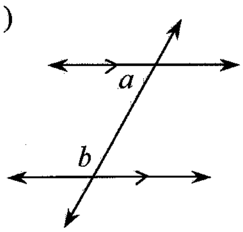


a. Use what you know about vertical, corresponding, and supplementary angle relationships to find the measure of $\measuredangle b$.

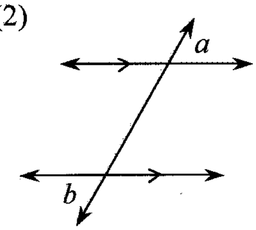
b. Julia is still having trouble seeing the angle relationships clearly in this diagram. Her teammate, Althea explains, "When I translate one of the angles along the transversal, I notice its image and the other given angle are a pair of vertical angles. That way, I know that *a* and *b* must be congruent."

Use Althea's method and tracing paper to determine if the following angle pairs are congruent or supplementary. Be sure to state whether the pair of angles created after the translation is a vertical pair or forms a straight angle. Be ready to **justify** your answer for the class.

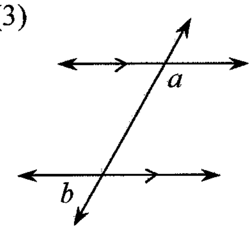
(1)



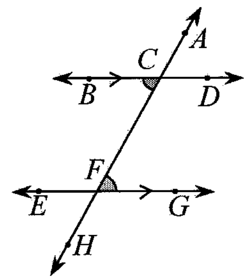
(2)



(3)



2-24. In problem 2-23, Althea showed that the shaded angles in the diagram are congruent. However, these angles also have a name for their geometric relationship (their relative positions on the diagram). These angles are called **alternate interior** angles. They are called "alternate" because they are on opposite sides of the transversal, and "interior" because they are both inside the parallel lines.



a. On tracing paper, trace and shade in $\measuredangle CFG$. How can you transform $\measuredangle CFG$ so that it lands on $\measuredangle BCF$? Be sure your team agrees.

b. Find another pair of alternate interior angles in this diagram.

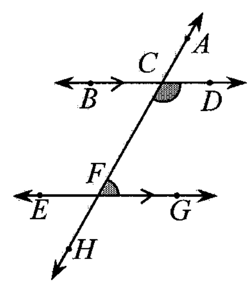
c. Think about the relationship between the measures of alternate interior angles. If the lines are parallel, are they always congruent? Are they always supplementary? Complete the conjecture, "*If lines are parallel, then alternate interior angles are…*".

d. Instead of writing conditional statements, Roxie likes to write **arrow diagrams** to express her conjectures. She expresses the conjecture from part (b) as

Lines are parallel → *alternate interior angles are congruent.*

This arrow diagram says the same thing as the conditional statement you wrote in part (c). How is it different from your conditional statement? What does the arrow mean?

2-25. The shaded angles in the diagram at right have another special angle relationship. They are called **same-side interior** angles.



a. Why do you think they have this name?

b. What is the relationship between the angle measures of same-side interior angles? Are they always congruent? Supplementary? Talk about this with your team.

Then write a conjecture about the relationship of the angle measures. Your conjecture can be in the form of a conditional statement or an arrow diagram. If you write a conditional statement, it should begin, "*If lines are parallel, then same-side interior angles are…*".

2-26. THE REFLECTION OF LIGHT

You know enough about angle relationships now to start analyzing how light bounces off mirrors. **Examine** the two diagrams below. Diagram A shows a beam of light emitted from a light source at A. In Diagram B, someone has placed a mirror across the light beam. The light beam hits the mirror and is reflected from its original path.

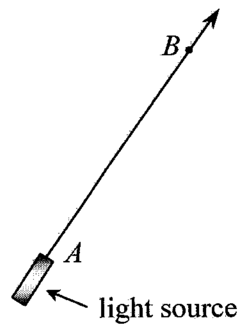


Diagram A

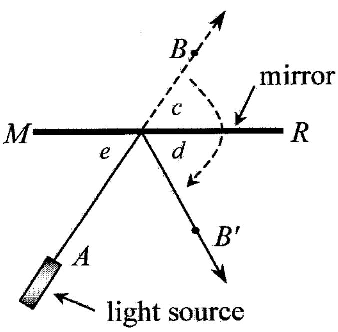


Diagram B

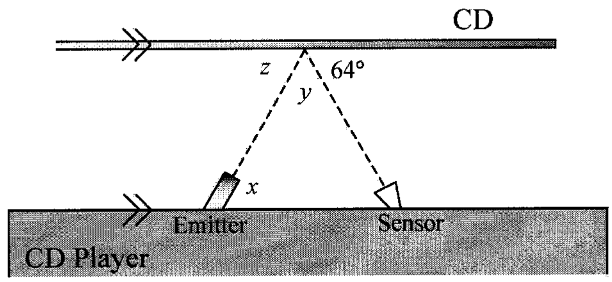
a. What is the relationship between angles *c* and *d*? Why?

b. What is the relationship between angles *c* and *e*? How do you know?

c. What is the relationship between angles *e* and *d*? How do you know?

d. Write a conjecture about the angle at which light hits a mirror and the angle at which it bounces off the mirror.

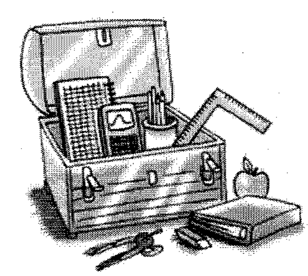
2-27. A CD player works by bouncing a laser off the surface of the CD, which acts like a mirror. An emitter sends out the light, which bounces off the CD and then is detected by a sensor. The diagram below shows a CD held parallel to the surface of the CD player, on which an emitter and a sensor are mounted.



a. The laser is supposed to bounce off the CD at a 64° angle as shown in the diagram above. For the laser to go directly into the sensor, at what angle does the emitter need to send the laser beam? In other words, what does the measure of angle x have to be? **Justify** your conclusion.

b. The diagram above shows two parts of the laser beam: the one coming out of the emitter and the one that has bounced off the CD. What is the angle ($\measuredangle y$) between these beams? How do you know?

2-28. ANGLE RELATIONSHIPS TOOLKIT



Obtain a Lesson 2.1.3 Resource Page ("Angle Relationships Toolkit") from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the new angle relationships you have studied so far in Chapter 2. Then, in the space provided, add a diagram and a description of the relationship for each special angle relationship you know. Be sure to specify any relationship between the measures of the angle (such as whether or not they are always congruent). In later lessons, you will continue to add relationships to this toolkit, so be sure to keep this resource page in a safe place. At this point, your toolkit should include:

- Vertical angles
- Straight angles
- Corresponding angles
- Alternate interior angles
- Same-side interior angles

METHODS AND MEANINGS

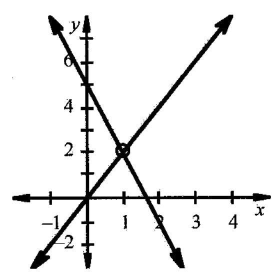
MATH NOTES

Systems of Linear Equations

In a previous course, you learned that a **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right. In a system, each variable represents the same quantity in both equations. For example, y represents the same quantity in *both* equations at right.

$$y = 2x$$
$$y = -3x + 5$$

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens when the graphs of two lines lie on top of each other. Such lines are said to **coincide**.

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable. For example, to solve the system at right:

$$x = -3y + 1$$
$$4x - 3y = -11$$

- Use substitution to rewrite the two equations as one. In other words, replace x with $(-3y+1)$ to get $4(-3y+1) - 3y = -11$. This equation can then be solved to find y. In this case, $y = 1$.
- To find the point of intersection, substitute to find the other value.
- Substitute $y = 1$ into $x = -3y + 1$ and write the answer for x and y as an ordered pair.
- To test the solution, substitute $x = -2$ and $y = 1$ into $4x - 3y = -11$ to verify that it makes the equation true. Since $4(-2) - 3(1) = -11$, the solution (–2, 1) must be correct.

$$x = -3y + 1$$
$$4(\quad) - 3y = -11$$
$$4(-3y+1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$

$$x = -3(1) + 1 = -2$$
$$(-2, 1)$$

Review & Preview

2-29. The set of equations at right is an example of a **system of equations**. Read the Math Notes box for this lesson on how to solve systems of equations. Then answer the questions below.

$$y = -x + 1$$
$$y = 2x + 7$$

a. Graph the system on graph paper. Then write its solution (the point of intersection) in (x, y) form.

b. Now solve the system using the Substitution Method. Did your solution match your result from part (a)? If not, check your work carefully and look for any mistakes in your algebraic process or on your graph.

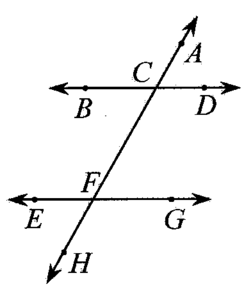
2-30. On graph paper, graph the rectangle with vertices at (2, 1), (2, 5), (7, 1), and (7, 5).

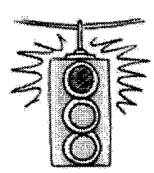
a. What is the area of this rectangle?

b. Shirley was given the following points and asked to find the area, but her graph paper is not big enough. Find the area of Shirley's rectangle, and explain to her how she can find the area without graphing the points.

Shirley's points: (352, 150), (352, 175), (456, 150), and (456, 175)

2-31. Looking at the diagram below, John says that $m\angle BCF = m\angle EFH$.



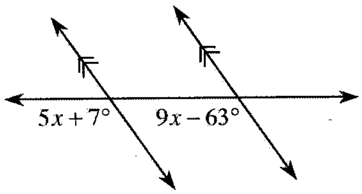


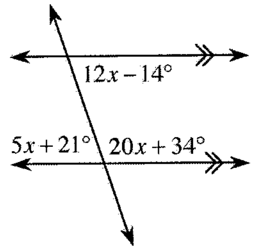
Note: This stoplight icon will appear periodically throughout the text. Problems with this icon display common errors that can be made. Be sure not to make the same mistakes yourself!

a. Do you agree with John? Why or why not?

b. Jim says, "You can't be sure those angles are equal. An important piece of information is missing from the diagram!" What is Jim talking about?

2-32. Use your knowledge of angle relationships to solve for x in the diagrams below. **Justify** your solutions by naming the geometric relationship.

a. 

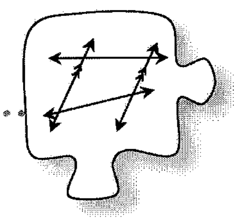
b. 

2-33. On graph paper, draw line segment $\overline{AB}$ if $A(6, 2)$ and $B(3, 5)$.

a. Reflect $\overline{AB}$ across the line $x = 3$ and connect points A and A'. What shape is created by this reflection? Be as specific as possible.

b. What polygon is created when $\overline{AB}$ is reflected across the line $y = -x + 6$ and all endpoints are connected to form a polygon?

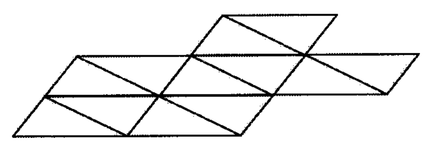
2.1.4 How can I use it?



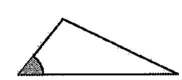
Angles in a Triangle

So far in this chapter, you have **investigated** the angle relationships created when two lines intersect, forming vertical angles. You have also **investigated** the relationships created when a transversal intersects two parallel lines. Today you will study the angle relationships that result when three non-parallel lines intersect, forming a triangle.

2-34. Marcos decided to study the angle relationships in triangles by making another tiling. Find his pattern, shown at right, on the Lesson 2.1.4 Resource Page.



a. Copy one of Marcos's triangles onto tracing paper. Use a colored pen or pencil to shade one of the triangle's angles on the tracing paper. Then use the same color to shade every angle on the resource page that is equal to the shaded angle.



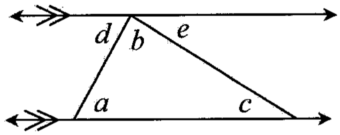
b. Repeat this process for the other two angles of the triangle, using a different color for each angle in the triangle. When you are done, every angle in your tiling should be shaded with one of the three colors.

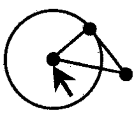
Problem continues on next page →

2-34. *Problem continued from previous page.*

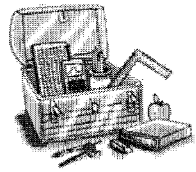
c. Now **examine** your colored tiling. What relationship can you find between the three different-colored angles? You may want to focus on the angles that form a straight angle. What does this tell you about the angles in a triangle? Write a conjecture in the form of a conditional statement or an arrow diagram. If you write a conditional statement, it should begin, "*If a polygon is a triangle, then the measure of its angles…*".

d. How can you convince yourself that your conjecture is true for all triangles? If a dynamic geometry tool is available, use it to convince yourself that your conjecture is always true. If technology is not available, use the diagram above and your Angle Relationships Toolkit to write a convincing argument that your conjecture is true.



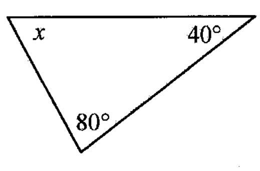


Then add this angle relationship to your Angle Relationships Toolkit from Lesson 2.1.3. This will be referred to as the **Triangle Angle Sum Theorem.** (A theorem is a statement that has been proven.)

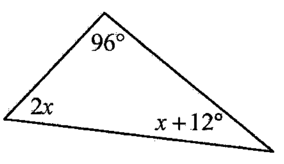


2-35. Use your conjecture from problem 2-34 about the angles in a triangle to find x in each diagram below. Show all work.

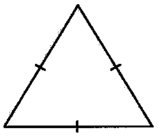
a.



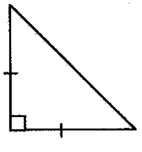
b.



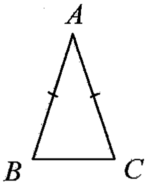
2-36. What can the Triangle Angle Sum Theorem help you learn about special triangles?



a. Find the measure of each angle in an equilateral triangle. **Justify** your conclusion.



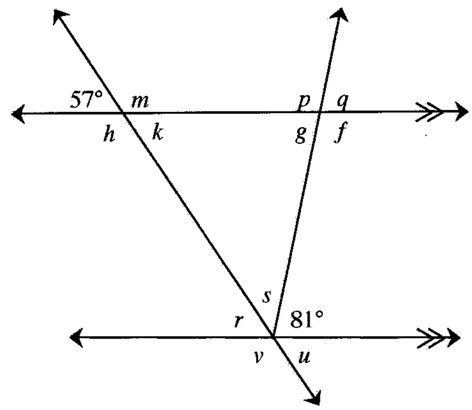
b. Consider the isosceles right triangle (also sometimes referred to as a "half-square") at right. Find the measures of all the angles in a half-square.



c. What if you only know one angle of an isosceles triangle? For example, if $m\measuredangle A = 34°$, what are the measures of the other two angles?

2-37. TEAM **REASONING** CHALLENGE

How much can you figure out about the figure at right using your knowledge of angle relationships? Work with your team to find the measures of all the labeled angles in the diagram at right. **Justify** your solutions with the name of the angle relationship you used. Carefully record your work as you go and be prepared to share your **reasoning** with the rest of the class.

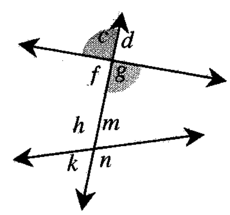


METHODS AND MEANINGS

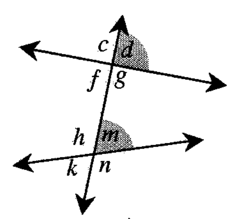
MATH NOTES

More Angle Pair Relationships

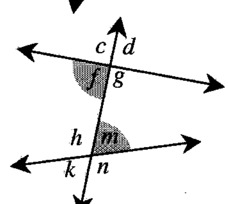
Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles $\measuredangle c$ and $\measuredangle g$ in the diagram at right. $\measuredangle c$ by itself is not a vertical angle, nor is $\measuredangle g$, although $\measuredangle c$ and $\measuredangle g$ together are a pair of vertical angles. Vertical angles always have equal measure.



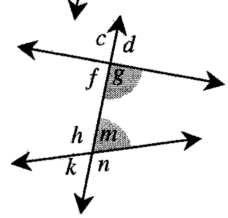
Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\measuredangle m$ and $\measuredangle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.



$\measuredangle f$ and $\measuredangle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.



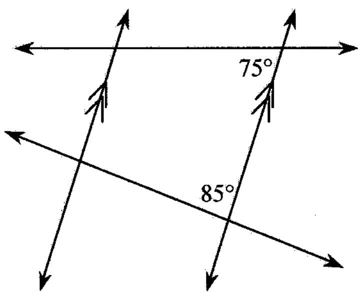
$\measuredangle g$ and $\measuredangle m$ are **same-side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same-side interior angles are supplementary when the lines intersected by the transversal are parallel.



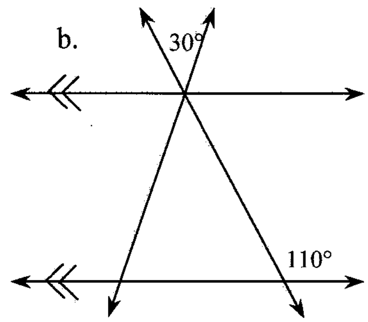
Review & Preview

2-38. Find all missing angles in the diagrams below.

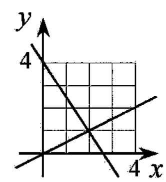
a.



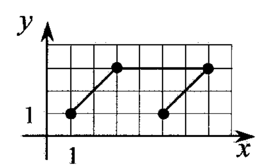
b.



2-39. Robert believes the lines graphed at right are perpendicular, but Mario is not convinced. Find the slope of each line, and explain how you know whether or not the lines are perpendicular.



2-40. The diagram at right represents only half of a shape that has the graph of $y = 1$ as a line of symmetry. Draw the completed shape on your paper, and label the coordinates of the missing vertices.

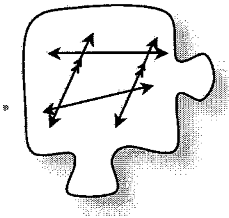


2-41. Janine measured the sides of a rectangle and found that the sides were 12 inches and 24 inches. Howard measured the same rectangle and found that the sides were 1 foot and 2 feet. When their math teacher asked them for the area, Janine said 288, and Howard said 2. Why did they get two different numbers for the area of the same rectangle?

2-42. Graph the system of equations at right on graph paper. Then state the solution to the system. If there is not a solution, explain why.

$$y = -\tfrac{2}{5}x + 1$$
$$y = -\tfrac{2}{5}x - 2$$

2.1.5 What's the relationship?



Applying Angle Relationships

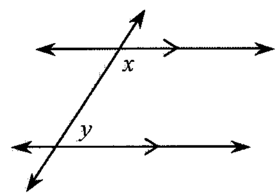
During Section 2.1, you have been learning about various special angle relationships that are created by intersecting lines. Today you will **investigate** those relationships a bit further, then apply what you know to explain how Mr. Douglas's mirror trick (from problem 2-1) works. As you work in your teams today, keep the following questions in mind to guide your discussion:

What's the relationship?

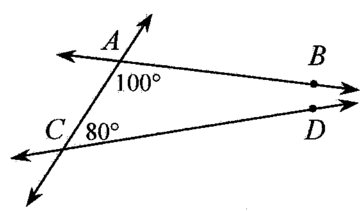
Are the angles equal? Are they supplementary?

How can I be sure?

2-43. Use your knowledge of angle relationships to answer the questions below.

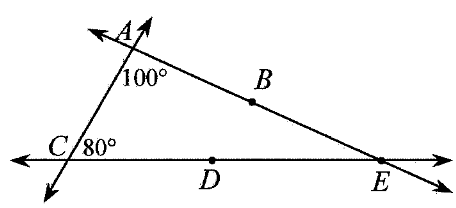


a. In the diagram at right, what is the sum of angles x and y? How do you know?



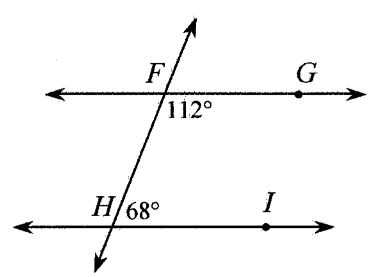
b. While looking at the diagram at right, Rianna exclaimed, "I think something is wrong with this diagram." What do you think she is referring to? Be prepared to share your thinking with the class.

2-44. Maria is still not convinced that the lines in part (b) of problem 2-43 *must* be parallel. She decides to assume that they are not parallel and draws the diagram at right.



a. Why must lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect in Maria's diagram?

b. What is $m\measuredangle BED$? Discuss this question with your team and explain what it tells you about $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

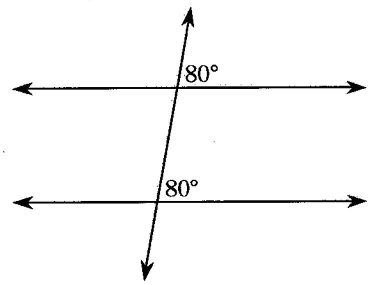


c. **Examine** the diagram at right. In this diagram, must $\overleftrightarrow{FG}$ and $\overleftrightarrow{HI}$ be parallel? Explain how you know.

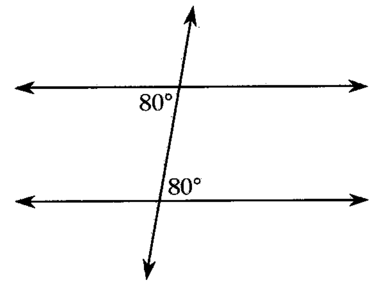
d. Write a conjecture based on your conclusion to this problem.

2-45. Use your conjecture from problem 2-44 to explain why lines must be parallel in the diagrams below.

a.



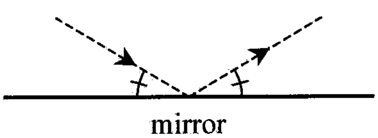
b.



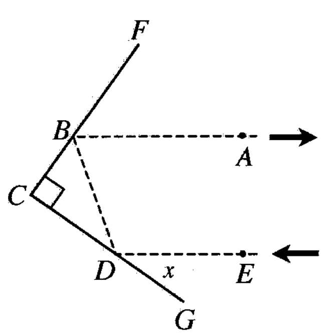
c. Looking back at the diagrams in parts (a) and (b), write two new conjectures that begin, "*If corresponding angles are equal, …*" and "*If the measures of alternate interior angles are equal, …*".

2-46. SOMEBODY'S WATCHING ME, Part Two

Remember Mr. Douglas' trick from problem 2-1? You now know enough about angle and line relationships to analyze why a hinged mirror set so the angle between the mirrors is 90° will reflect your image back to you from any angle. Since your reflection is actually light that travels from your face to the mirror, you will need to study the path of the light. Remember that a mirror reflects light, and that the angle the light hits the mirror will equal the angle it bounces off the mirror.



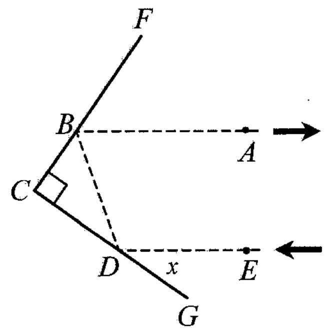
Your task: Explain why the mirror bounces your image back to you from any angle. Include in your analysis:



- Use angle relationships to find the measures of all the angles in the figure. (Each team member should choose a different x value and calculate all of the other angle measures using his or her selected value of x.)
- What do you know about the paths the light takes as it leaves you and as it returns to you? That is, what is the relationship between $\overline{BA}$ and $\overline{DE}$?
- Does Mr. Douglas' trick work if the angle between the mirrors is not 90°?

Further Guidance

2-47. Since you are trying to show that the trick works for *any* angle at which the light could hit the mirror, each team will work with a different angle measure for x in this problem. Your teacher will tell you what angle x your team should use in the diagram at right.



a. Using angle relationships and what you know about how light bounces off mirrors, find the measure of every other angle in the diagram.

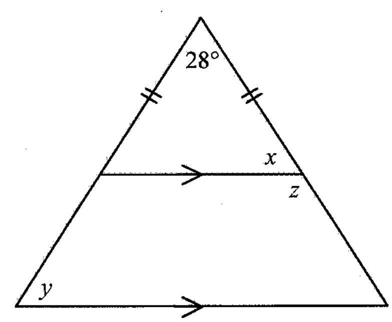
b. What is the relationship between $\measuredangle ABD$ and $\measuredangle EDB$? What does this tell you about the relationship between $\overline{BA}$ and $\overline{DE}$?

c. What if $m\measuredangle C = 89°$? Does the trick still work?

2-48. Explain why the 90° hinged mirror always sends your image back to you, no matter which angle you look into it from.

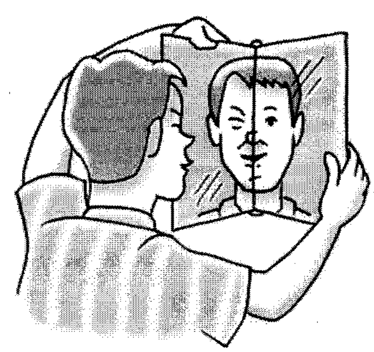
Further Guidance section ends here.

2-49. Use what you have learned in Section 2.1 to find the measures of x, y, and z at right. **Justify** each conclusion with the name of a geometric relationship from your Angle Relationships Toolkit.



2-50. EXTENSION

Hold a 90° hinged mirror at arm's length and find your own image. Now close your right eye. Which eye closes in the mirror? Look back at the diagram from problem 2-46. Can you explain why this eye is the one that closes?



MATH NOTES

METHODS AND MEANINGS

Proof by Contradiction

The kind of argument you used in Lesson 2.1.5 to **justify** "If same-side interior angles are supplementary, then lines are parallel" is sometimes called a **proof by contradiction**. In a proof by contradiction, you prove a claim by thinking about what the consequences would be if it were false. If the claim's being false would lead to an impossibility, this shows that the claim must be true.

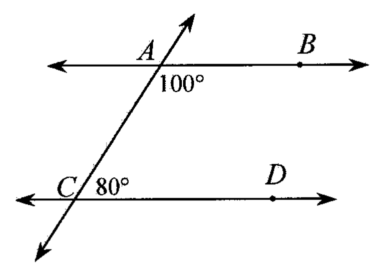
For example, suppose you know Mary's brother is seven years younger than Mary. Can you argue that Mary is at least five years old? A proof by contradiction of this claim would go:

Suppose Mary is less than five years old.

Then her brother's age is negative!

But this is impossible, so Mary must be at least five years old.

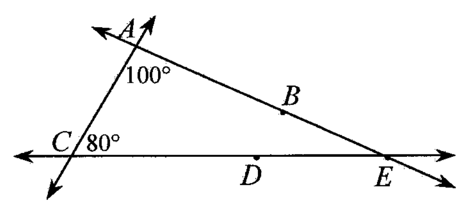
To show that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel in the diagram at right, you used a proof by contradiction. You argued:



Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at some point E.

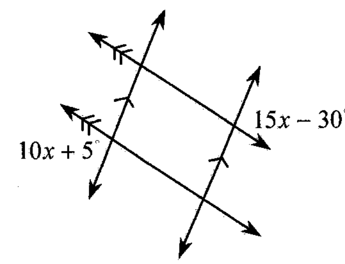
Then the angles in ΔAEC add up to more than 180°.

But this is impossible, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel.



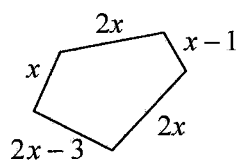
Review & Preview

2-51. Solve for x in the diagram at right.

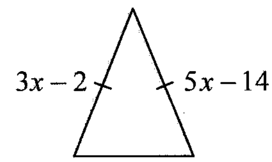


2-52. For each diagram below, set up an equation and solve for x.

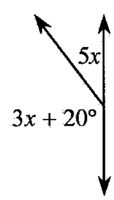
a. Perimeter = 76 units



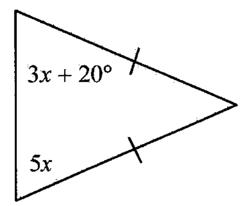
b.



c.



d.



2-53. Solve each system of equation below. Then verify that your solution makes each equation true. You may want to refer to the Math Notes box in Lesson 2.1.3.

a. $y = 5x - 2$
$y = 2x + 10$

b. $x = -2y - 1$
$2x + y = -20$

2-54. Graph the line $y = \frac{3}{4}x$ on graph paper.

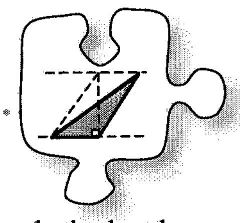
a. Draw a slope triangle.

b. Rotate your slope triangle 90° around the origin to get a new slope triangle. What is the new slope?

c. Find the equation of a line perpendicular to $y = \frac{4}{3}x$.

2-55. Mario has 6 shapes in a bucket. He tells you that the probability of pulling an isosceles triangle out of the bucket is $\frac{1}{3}$. How many isosceles triangles are in his bucket?

2.2.1 How can I measure an object?



Units of Measure

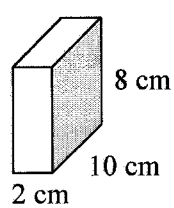
How tall are you? How large is the United States? How much water does your bathtub at home hold? All of these questions ask about the size of objects around you. How can we answer these questions more specifically than saying "big" or "small"? Today you will be **investigating** ways to answer these and other questions like them.

2-56. Your teacher will describe with words a figure he or she has drawn. Your job is to try to draw the **exact same figure** on your paper so that if you placed your drawing on top of your teacher's, the figures would match perfectly. Redraw your figure as many times as necessary.

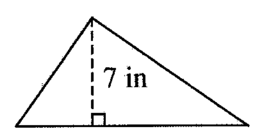
2-57. Length often provides a direct way to answer the question, "How big?" In this activity, your teacher will give your team rulers with a unit of length to use to measure distances. For instance, if you have an object that has the same length as three of your units placed next to one another, then the object's length is "3 units."

a. Common units you may have used before are inches or meters. However, your unit does not match any of the familiar units. Give your unit of measure a unique name. Then continue marking and labeling units on your ruler as accurately as possible.

b. The **dimensions** of a figure are its measurements of length. For example, the measurements in each figure below describe the relative size of each object.



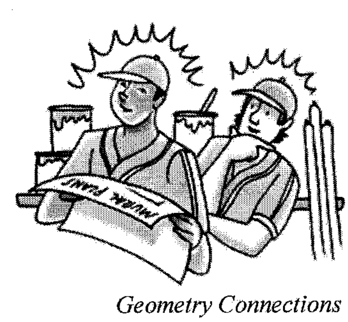
Dimensions can describe how long, wide, or tall the object is by measuring the lengths of the edges.



Or, dimensions can be found by measuring a length that is not a side.

Find the dimensions of the shape on the Lesson 2.2.1A Resource Page using your unit of length. That is, how wide is the shape? How tall? Compare your results with those of other teams. What happened?

c. The local baseball club is planning to make a mural that is the same general shape as the one you measured in part (b) (but fortunately a much larger version!). The club plans to frame the mural with neon tubes. Approximately how many units of neon tube will they need to do this?



2-58. To paint the mural, the wall must first be covered with a coat of primer. How much of the wall will need to be painted with the primer? Remember that the measurement of the region inside a shape is called the **area** of the shape.

a. Just as you were able to use your unit of length to measure distance, you need a unit of area to measure a surface. Use your team's ruler to make unit squares that are 1 unit long on each side. The area that your square unit covers is called "one square unit" and can be abbreviated as 1 sq. unit or 1 un^2. What would you call this unit of measure, given the name you chose in problem 2-57?

b. What is the approximate area of the mural? That is, how many of your square units fit within your shape?

2-59. Use your unit of measure to make a rectangle that has dimensions 3 units by 5 units.

a. What is the area of your rectangle? (That is, how many unit squares are there in your rectangle?)

b. When you answered part (a), did you count the squares? Did your team use a shortcut? If so, why does the shortcut work?

c. Compare your rectangle to rectangles that other teams made. What is the same about the rectangles and what is different?

2-60. If you found out that your gym teacher is going to make you run 31,680 inches next period, how useful is this information? Or, if you knew that you were 0.003977 miles long at birth, do you have any idea how long that is?

An important part of measurement is choosing an appropriate unit of measurement. With your team, suggest a unit of measurement that can best measure (and describe) each of these situations.

a. The distance you travel from home to get to school.

b. The surface of the school's soccer field.

c. The width of a strand of hair.

d. The length of your nose.

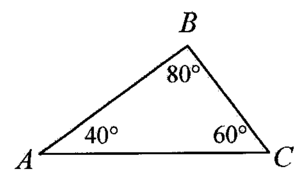
e. The amount of room in your locker.

MATH NOTES

METHODS AND MEANINGS

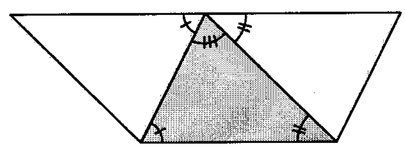
Triangle Angle Sum Theorem

The **Triangle Angle Sum Theorem** states that the measures of the angles in a triangle add up to 180°. For example, in $\triangle ABC$ at right:



$$m\measuredangle A + m\measuredangle B + m\measuredangle C = 180°$$

The Triangle Angle Sum Theorem can be verified by using a tiling of the given triangle (shaded at right). Because the tiling produces parallel lines, the alternate interior angles must be congruent. As seen in the diagram at right, the three angles of a triangle form a straight angle. Therefore, the sum of the angles of a triangle must be 180°.

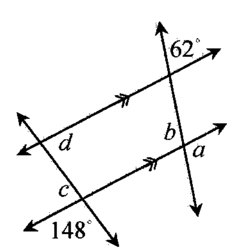


Review & Preview

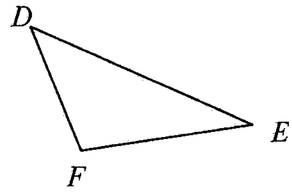
2-61. **Examine** the shapes in your Shape Toolkit. Then name at least three shapes in the Shape Toolkit that share the following qualities.

- They have only one line of symmetry.
- They have fewer than four sides.

2-62. **Examine** the diagram at right. Then use the information provided in the diagram to find the measures of angles *a*, *b*, *c*, and *d*. For each angle, name the relationship from your Angle Relationships Toolkit that helped **justify** your conclusion. For example, did you use vertical angles? If not, what type of angle did you use?



2-63. **Examine** the triangle at right.



a. If $m\measuredangle D = 48°$ and $m\measuredangle F = 117°$, then what is $m\measuredangle E$?

b. Solve for x if $m\measuredangle D = 4x + 2°$, $m\measuredangle F = 7x - 8°$, and $m\measuredangle E = 4x + 6°$. Then find $m\measuredangle D$.

c. If $m\measuredangle D = m\measuredangle F = m\measuredangle E$, what type of triangle is ΔFED?

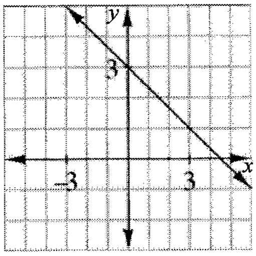
2-64. Plot ΔABC on graph paper if $A(6, 3)$, $B(2, 1)$, and $C(5, 7)$.

a. ΔABC is rotated about the origin 180° to become $\Delta A'B'C'$. Name the coordinates of A', B', and C'.

b. This time ΔABC is rotated 180° about point C to form $\Delta A''B''C''$. Name the coordinates of B''.

c. If ΔABC is rotated 90° clockwise (↻) about the origin to form $\Delta A'''B'''C'''$, what are the coordinates of point A'''?

2-65. **Examine** the graph at right.



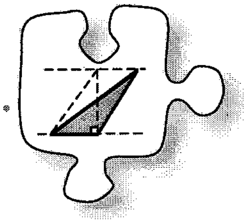
a. Find the equation of the line.

b. Is the line $y = \frac{3}{2}x + 1$ perpendicular to this line? How do you know?

c. On graph paper, graph $\overleftrightarrow{AB}$ if $A(-2, 4)$ and $B(4, 7)$. Then find the equation of $\overleftrightarrow{AB}$.

d. Find an equation of $\overleftrightarrow{AC}$ if $\overleftrightarrow{AC} \perp \overleftrightarrow{AB}$ from part (c).

2.2.2 How can I find the area?



Areas of Triangles and Composite Shapes

How much grass would it take to cover a football field? How much paint would it take to cover a stop sign? How many sequins does it take to cover a dress? Finding the area of different types of shapes enables us to answer many questions. However, different people will see a shape differently. Therefore, during this lesson, be especially careful to look for different **strategies** that can be used to find area.

As you solve these problems, ask yourself the following focus questions:

What shapes do I see in the diagram?

Does this problem remind me of one I have seen before?

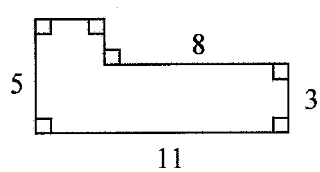
Is there another way to find the area?

2-66. **STRATEGIES** TO MEASURE AREA

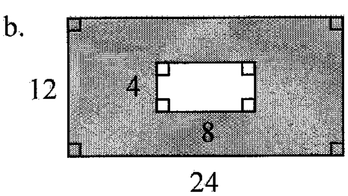
In Lesson 2.2.1 you used a grid to measure area. But what if a grid is not available? Or what if we want an exact measurement?

Examine the variety of shapes below. Work with your team to find the area of each one. If a shape has shading, then find the area of the shaded region. Be sure to listen to your teammates carefully and look for different **strategies**. Be prepared to share your team's method with the class.

a.



b.



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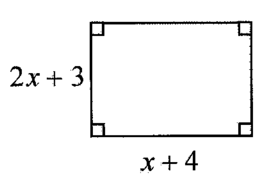

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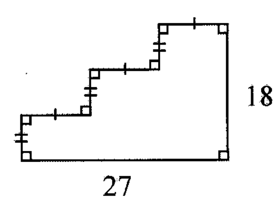
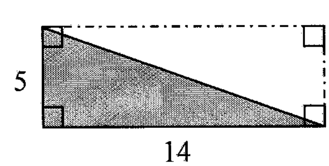
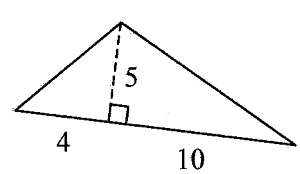


e.



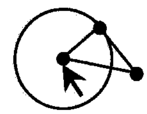
f.



2-67. Ismael claimed that he did not need to calculate the area for part (f) in problem 2-66 because it must be the same as the area for the triangle in part (e).

a. Is Ismael's claim correct? How do you know? Draw diagrams that show your thinking.

b. Do all triangles with the same base and height have the same area? Use your dynamic geometry tool to **investigate**. If no technology is available, obtain the Lesson 2.2.2 Resource Page and compare the areas of the given triangles.

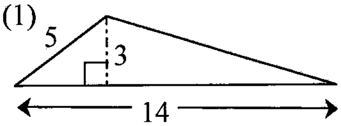


c. Explain why the area of any triangle is half the area of a rectangle that has the same base and height. That is, show that the area of a triangle must be $\frac{1}{2}bh$.

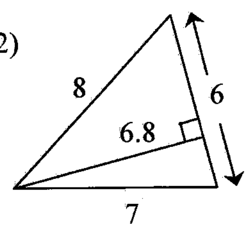
2-68. How do you know which dimensions to use when finding the area of a triangle?

a. Copy each triangle below onto your paper. Then find the area of each triangle. Draw any lines on the diagram that will help. Turning the triangles may help you discover a way to find their areas.

(1)



(2)



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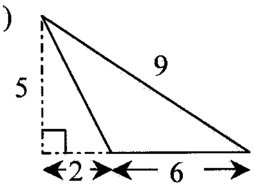
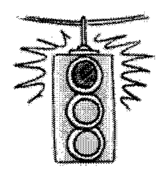
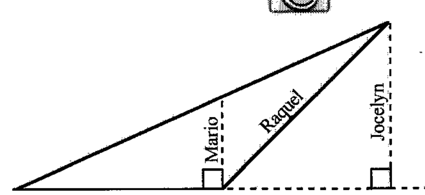


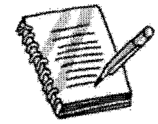
b. Look back at your work from part (a). Which numbers from each triangle did you use to find the area? For instance, in the center triangle, you probably used only the 6.8 and 6. Write an explanation and/or draw a diagram that would help another student understand how to **choose** which lengths to use when calculating the area.



c. Mario, Raquel, and Jocelyn are arguing about where the height is for the triangle at right. The three have written their names along the part they think should be the height. Determine which person is correct. Explain why the one you chose is correct and why the other two are incorrect.



2-69. In a Learning Log entry, describe at least two different **strategies** that were used today to find the area of irregular shapes. For each method, be sure to include an example. Title this entry "Areas of Composite Figures" and include today's date.

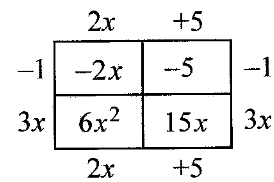


MATH NOTES

METHODS AND MEANINGS

Multiplying Binomials

One method for multiplying binomials is to use a generic rectangle. That is, use each factor of the product as a dimension of a rectangle and find its area. If $(2x + 5)$ is the base of a rectangle and $(3x - 1)$ is the height, then the expression $(2x+5)(3x-1)$ is the area of the rectangle. See the example below.



Multiply: $(2x+5)(3x-1) = 6x^2 - 2x + 15x - 5$
$= 6x^2 + 13x - 5$

Review & Preview

2-70. Review how to multiply binomials by reading the Math Notes box for this lesson. Then rewrite each of the expressions below by multiplying binomials and simplifying the resulting expression.

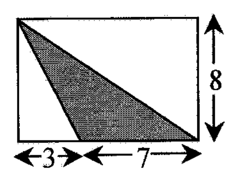
a. $(4x+1)(2x-7)$

b. $(5x-2)(2x+7)$

c. $(4x-3)(x-11)$

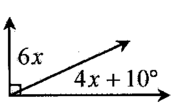
d. $(-3x+1)(2x-5)$

2-71. The shaded triangle at right is surrounded by a rectangle. Find the area of the triangle.

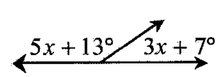


2-72. For each diagram below, solve for x. Explain what relationship from your Angle Relationships Toolkit you used for each problem.

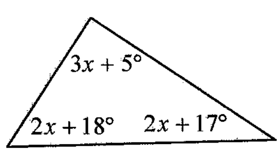
a.



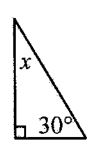
b.



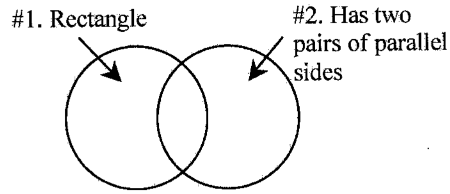
c.



d.



2-73. Daniel and Mike were having an argument about where to place a square in the Venn diagram at right. Daniel wants to put the square in the intersection (the region where the two circles overlap).



Mike doesn't think that's right. "*I think it should go in the right region because it is a square, not a rectangle.*"

"*But a square IS a rectangle!*" protests Daniel. Who is right? Explain your thinking.

2-74. What is the probability of drawing each of the following cards from a standard playing deck? Remember that a standard deck of cards includes: 52 cards of four suits. Two suits are black: clubs and spades; two are red: hearts and diamonds. Each suit has 13 cards: 2 through 10, Ace, Jack, Queen, and King.

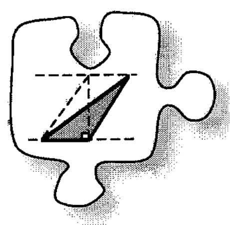
a. P(Jack)

b. P(spade)

c. P(Jack of spades)

d. P(not spade)

2.2.3 What's the area?



Areas of Parallelograms and Trapezoids

In Lesson 2.2.2, you used your knowledge of the area of a rectangle to develop a method to find the exact area of a triangle that works for all triangles. How can your understanding of the area of triangles and rectangles help with the study of other shapes? As you work today, ask yourself and your team members these focus questions:

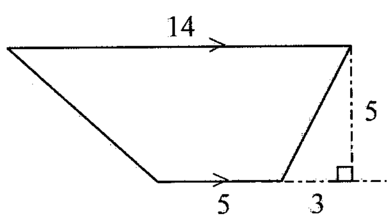
What do you see?

What shapes make up the composite figure?

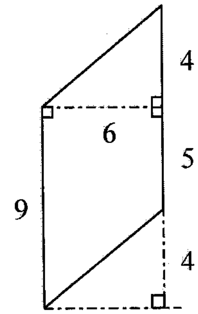
Is there another way?

2-75. Find the areas of the figures below. Can you find more than one method for each shape?

a.

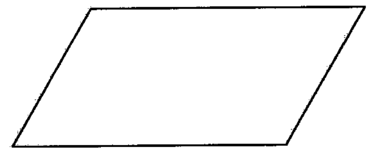


b.

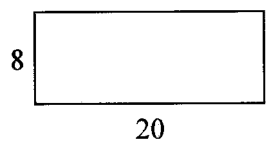


2-76. FINDING THE AREA OF A PARALLELOGRAM

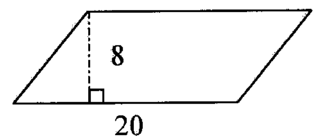
One of the shape in your Shape Bucket is shown at right. It is called a **parallelogram**: a four-sided shape with two pairs of parallel sides. How can you find the area of a parallelogram? Consider this question as you answer the questions below.



a. Kenisha thinks that the rectangle and parallelogram below have the same area. Her teammate Shaundra disagrees. Who is correct? **Justify** your conclusion.



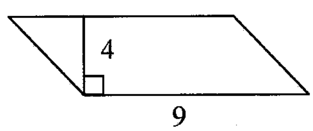
Rectangle



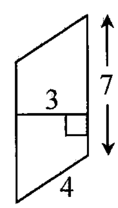
Parallelogram

b. In the parallelogram shown in part (a), the two lengths that you were given are often called the **base** and **height**. Several more parallelograms are shown below. In each case, find a related rectangle for which you know both the base and height. Rotating your book might help. Use what you know about rectangles to find the area of each parallelogram.

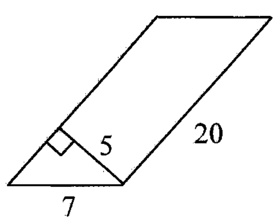
(1)



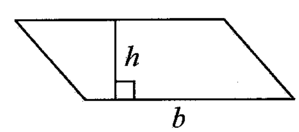
(2)



(3)



(4)

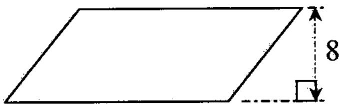


c. Describe how to find the area of a parallelogram when given its base and height.

d. Does the angle at which the parallelogram slants matter? Does every parallelogram have a related rectangle with equal area? Why or why not? Explain how you know.

2-77. Shaundra claims that the area of a parallelogram can be found by *only* using triangles.

a. Do you agree? Trace the parallelogram at right onto your paper. Then divide it into two triangles. (Do you see more than one way to do this? If you do, ask some team members to divide the parallelogram one way, and the others a second way.)

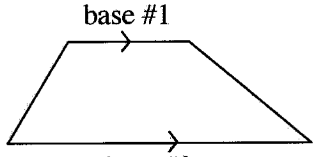


b. Use what you know about calculating the area of a triangle to find the area of the parallelogram. It may help to trace each triangle separately onto tracing paper so that you can rotate them and label any lengths that you know.

c. How does the answer to part (b) compare to the area you found in part (a) of problem 2-76?

2-78. FINDING THE AREA OF A TRAPEZOID

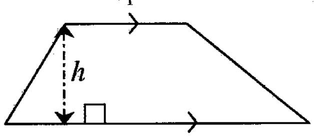
Another shape you will study from the Shape Bucket is a **trapezoid**: a four-sided shape that has at least one pair of parallel sides. The sides that are parallel are called **bases**, as shown in the diagram at right. Answer the questions below with your team to develop a method to find the area of a trapezoid.



Trapezoid

a. While playing with the shapes in her Shape Bucket, Shaundra noticed that two identical trapezoids can be arranged to form a parallelogram. Is she correct?

Trace the trapezoid shown at right onto a piece of tracing paper. Be sure to label its bases and height as shown in the diagram. Work with a team member to move and rearrange the trapezoid on each piece tracing paper so that they create a parallelogram.



b. Since you built a parallelogram from two trapezoids, you can use what you know about finding the area of a parallelogram to find the area of the trapezoid. If the bases of each trapezoid are b_1 and b_2 and the height of each is h, then find the area of the parallelogram. Then use this area to find the area of the original trapezoid.

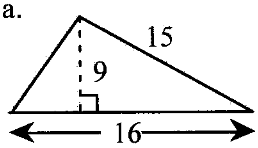
Problem continues on next page →

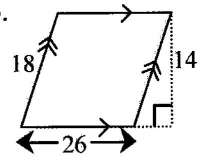
2-78. *Problem continued from previous page.*

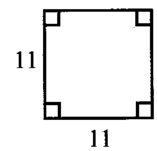
c. Kenisha sees it differently. She sees two triangles inside the trapezoid. If she divides a trapezoid into two triangles, what area will she get? Again assume that the bases of the trapezoid are b_1 and b_2 and the height is h.

d. Are the area expressions you created in parts (b) and (c) **equivalent**? That is, will they calculate the same area? Use your algebra skills to demonstrate that they are equivalent.

2-79. Calculate the exact areas of the shapes below.

a.


b.


c.


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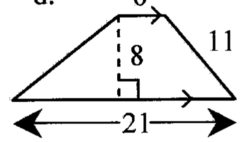
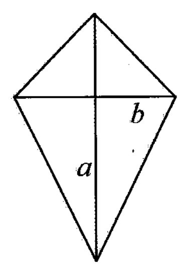


2-80. EXTENSION

Examine the diagram of the kite at right. Work with your team to find a way to show that its area must be half of the product of the diagonals. That is, if the length of the diagonals are a and b, provide a diagram or explanation of why the area of the kite must be $\frac{1}{2}ab$.



MATH NOTES

METHODS AND MEANINGS

Conditional Statements

A **conditional statement** is written in the form "**If ..., then ....**" Here are some examples of conditional statements:

If a shape is a rhombus, then it has four sides of equal length.

If it is February 14th, then it is Valentine's Day.

If a shape is a parallelogram, then its area is $A = bh$.

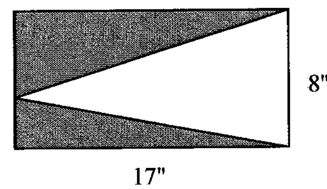
Review & Preview

2-81. Solve for y in terms of x. That is, rewrite each equation so that it starts "$y =$ ".

a. $6x + 5y = 20$

b. $4x - 8y = 16$

2-82. Calculate the area of the shaded region at right. Use the appropriate units.



2-83. How tall are you? How do you measure your height? Consider these questions as you answer the questions below.

a. Why do you stand up straight to measure your height?

b. Which diagram best represents how you would measure your height? Why?

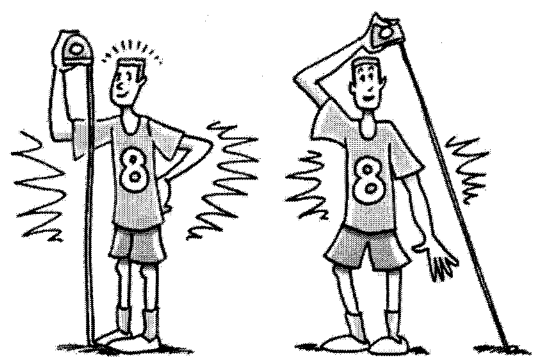


Diagram 1 **Diagram 2**

c. When you measure your height, do you measure up to your chin? Down to your knees? Explain.

2-84. Read the Math Notes box for this lesson. Then rewrite each of the following statements as a conditional statement.

a. Mr. Spelling is always unhappy when it rains.

b. The sum of two even numbers is always even.

c. Marla has a piano lesson every Tuesday.

2-85. Simplify the following expressions.

a. $2x + 8 + 6x + 5$

b. $15 + 3(2x - 4) - 4x$

c. $(x - 3)(3x + 4)$

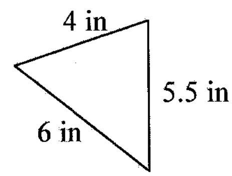
d. $5x(2x + 7) + x(3x - 5)$

2.2.4 How can I find the height?

Heights and Area

In Lesson 2.2.2, you learned that triangles with the same base and height must have the same area. But what if multiple dimensions of a shape are labeled? How can you determine which dimension is the height?

2-86. Candice missed the lesson about finding the area of the triangle. Not knowing where to start, she drew a triangle and measured its sides, as shown at right. After drawing her triangle, Candice said, "Well, I've measured all of the sides. I must be ready to find the area!"



If you think she is correct, write a description of how to use the side lengths to find the area. If you think she needs to measure anything else, copy the figure on your paper and add a line segment to represent a measurement she needs.

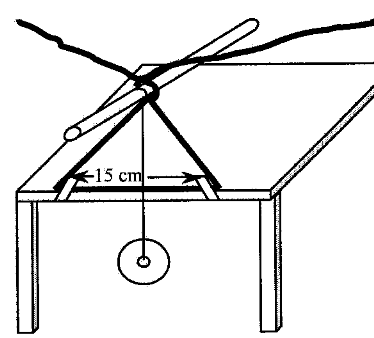
2-87. HEIGHT LAB

What is the height of a triangle? Is it like walking up any side of the triangle? Or is it like standing at the highest point and looking straight down? Today your team will build triangles with string and consider different ways height can be seen for triangles of various shapes.

a. Use the materials given to you by your teacher to make a triangle like the one in the diagram below.

(1) Tie one end of the short string to the weight and the other to the end of a pencil (or pen).

(2) Tape a 15 cm section of the long string along the edge of a desk or table. Be sure to leave long ends of string hanging off each side.



(3) Bring the loose ends of string up from the table and cross them as shown in the diagram. Then put the pencil with the weight over the crossing of the string. Cross the strings again on top of the pencil.

Problem continues on next page →

2-87. *Problem continued from previous page.*

b. Now, with your team, build and sketch triangles that meet the three conditions below. To organize your work, assign each team member one of the jobs described at right.

Student jobs:

- hold the pencil (or pen) with the weight.
- make sure that the weight hangs freely
- draw accurate sketches.
- hold the strings that make the sides of the triangles.

(1) The height of the triangle is inside the triangle.

(2) The height of the triangle is a side of the triangle.

(3) The height of the triangle is outside of the triangle.

c. Now make sure that everyone in your team has sketches of the triangles that you made.

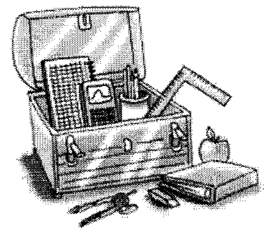
2-88. How can I find the height of a triangle if it is not a right triangle?

a. On the Lesson 2.2.4 Resource Page there are four triangles labeled (1) through (4). Assume you know the length of the side labeled "base." For each triangle, draw in the height that would enable you to find the area of the triangle. **Note:** You do not need to find the area.

b. Find the triangle for part (b) at the bottom of the same resource page. For this triangle, draw all three possible heights. First **choose** one side to be the base and draw in the corresponding height. Then repeat the process of drawing in the height for the other two sides, one at a time.

c. You drew in three pairs of bases and heights for the triangle in part (b). Using centimeters, measure the length of all three sides and all three heights. Find the area three times using all three pairs of bases and heights. Since the triangle remains the same size, your answers should match.

2-89. AREA TOOLKIT



Over the past several days, you have explored how to find the areas of triangles, parallelograms, and trapezoids. Today you will start a new page of your Geometry Toolkit, called the Area Toolkit. Keep this toolkit in a safe place. You will want it for reference in class and when doing homework.

At this time, describe what you know about finding the areas of triangles, rectangles, parallelograms, and trapezoids. Be sure to include an example for each shape.

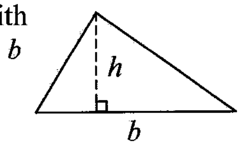
MATH NOTES

METHODS AND MEANINGS

Areas of a Triangle, Parallelogram, and Trapezoid

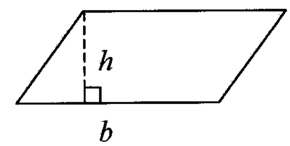
The area of a triangle is half the area of a rectangle with the same base and height. If the base of the triangle is length b and the height length h, then the area of the triangle is:

$$A = \frac{1}{2}bh.$$



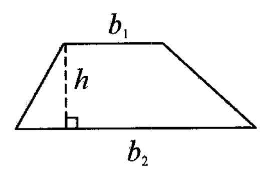
The area of a parallelogram is equal to a rectangle with the same base and height. If the base of the parallelogram is length b and the height length h, then the area of the parallelogram is:

$$A = bh.$$



Finally, the area of a trapezoid is found by averaging the two bases and multiplying by the height. If the trapezoid has bases b_1 and b_2 and height h, then the area is:

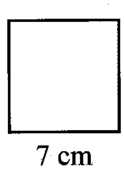
$$A = \frac{1}{2}(b_1 + b_2)h.$$



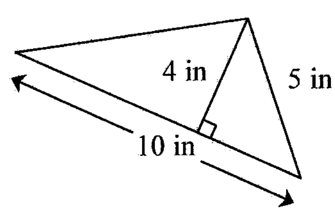
Review & Preview

2-90. Find the area of each figure below. Show all work. Remember to include units in your answer.

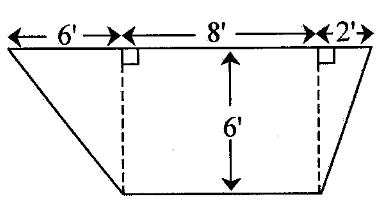
a. a square:



b.



c.



2-91. Multiply the expressions below. Then simplify the result, if possible.

a. $3x(5x + 7)$

b. $(x + 2)(x + 3)$

c. $(3x + 5)(x - 2)$

d. $(2x + 1)(5x - 4)$

2-92. Graph the following equations on the same set of axes. Label each line or curve with its equation. Where do the two curves intersect?

$$y = -x - 3 \qquad\qquad y = x^2 + 2x - 3$$

2-93. On graph paper, plot quadrilateral $ABCD$ if $A(2, 7)$, $B(4, 8)$, $C(4, 2)$, and $D(2, 3)$.

a. What is the best name for this shape? **Justify** your conclusion.

b. Quadrilateral $A'B'C'D'$ is formed by rotating $ABCD$ 90° clockwise about the origin. Name the coordinates of the vertices.

c. Find the area of $ABCD$. Show all work.

2-94. What is the probability of drawing each of the following cards from a standard playing deck? Refer to problem 2-74 if you need information about a deck of cards.

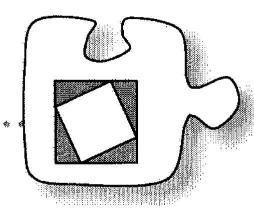
a. P(face card)

b. P(card printed with an even number)

c. P(red ace)

d. P(purple card)

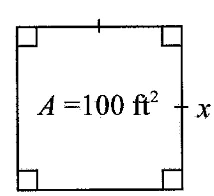
2.3.1 How can I use a square?



Squares and Square Roots

You now have the tools to find the area of many complex shapes and are also able to use transformations to create new shapes. In this lesson you will combine these skills to explore the area of a square and to develop a method for finding the length of the longest side of a right triangle.

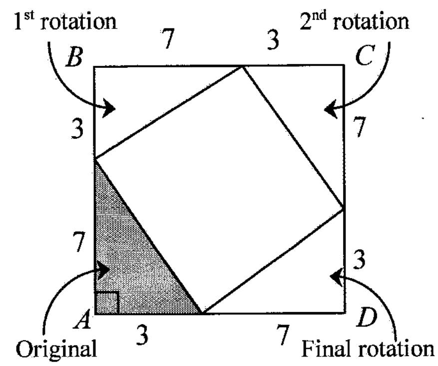
2-95. What do you notice about the rectangle at right?



a. Elyse does not know how to solve for x. Explain to her how to find the missing dimension.

b. What if the area of the shape above is instead 66 ft^2? What would x be in that case?

2-96. While Alexandria was doodling on graph paper, she made the design at right. She started with the shaded right triangle. She then rotated it 90° clockwise and translated the result so that the right angle of the image was at B. She continued this pattern until she completed the diagram.



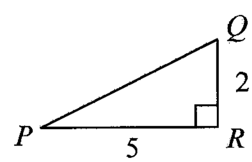
a. Draw Alexandria's design on graph paper. What is the shape of $ABCD$? How can you tell?

b. What is the shape of the inner quadrilateral? How do you know?

c. What is the area of the inner quadrilateral? Show all work that leads to your conclusion.

d. What's the length of the longest side of the shaded triangle?

2-97. DONNA'S DILEMMA



Donna needs help! She needs to find PQ (the length of $\overline{PQ}$) in $\triangle PQR$ shown at right.

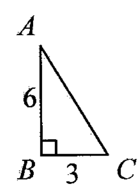
Alexandria realized that the triangle looked a lot like a piece of the square she drew earlier (shown in problem 2-96). Alexandria explained to Donna that if they could find the area of a square with side $\overline{PQ}$, then finding the length of $\overline{PQ}$ would be easy.

a. Draw $\triangle PQR$ on your paper.

b. Using the method from problem 2-96, help the students construct the square by rotating the triangle 90° and translating it. Then find the length of $\overline{PQ}$. Is the answer reasonable?

c. Find the perimeter of her triangle.

2-98. Use Donna and Alexandria's method to find the length of side $\overline{AC}$ in the diagram at right. That is, draw a square on $\overline{AC}$ using what you know about slope. Then use the area of the square to find $\overline{AC}$.



2-99. In a Learning Log entry, describe the process you developed today to find the length of the longest side of a right triangle when given the lengths of its legs. Label this entry “Finding the Length of the Hypotenuse” and include today’s date.

MATH NOTES

METHODS AND MEANINGS

Square Root

The **square root** of a number x (written $\sqrt{x}$) is defined as the length of a side of a square with area x. For example, $\sqrt{16} = 4$. Therefore if the side of a square has a length of 4 units, then its area is 16 square units.

When the area of a square is not a “perfect square” such as 49, 25, or 9 un^2 then the square root is not an integer, and instead is an **irrational number.** Note that an irrational number cannot be written as a fraction of two integers. For example, $\sqrt{17} = 4.123\ldots$ is an irrational number because 17 is not a perfect square. The symbol “≈” means “approximately equal to.” Since 4.123 is an approximate value of $\sqrt{17}$, it can be stated that $\sqrt{17} \approx 4.123$.

The square root of a number can often be estimated by comparing the number under the square root with its closest perfect squares. For example, since $\sqrt{16} = 4$, then $\sqrt{17}$ must be a little bigger than 4. Therefore, a good estimation of $\sqrt{17}$ is 4.1. Likewise, if you want to estimate $\sqrt{97}$, you may want to start with $\sqrt{100} = 10$. Since 97 is smaller than 100, a good estimate of $\sqrt{97}$ would be 9.8 or 9.9.

Review & Preview

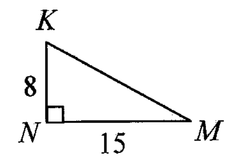
2-100. Read the Math Notes box for this lesson.

a. What would be a reasonable estimate of $\sqrt{68}$? Explain your thinking. After you have made an estimate, check your estimation with a calculator.

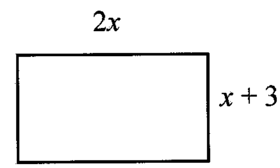
b. Repeat this process to estimate the values below.

(1) $\sqrt{5}$ (2) $\sqrt{85}$ (3) $\sqrt{50}$ (4) $\sqrt{22}$

2-101. Draw the triangle at right on graph paper. Then draw a square on $\overline{KM}$ and use it to find the length of $\overline{KM}$.



2-102. **Examine** the rectangle at right.



a. What is the perimeter in terms of x? In other words, find the perimeter.

b. If the perimeter is 78 cm, find the dimensions of the rectangle. Show all your work.

c. Verify that the area of this rectangle is 360 sq. cm. Explain how you know this.

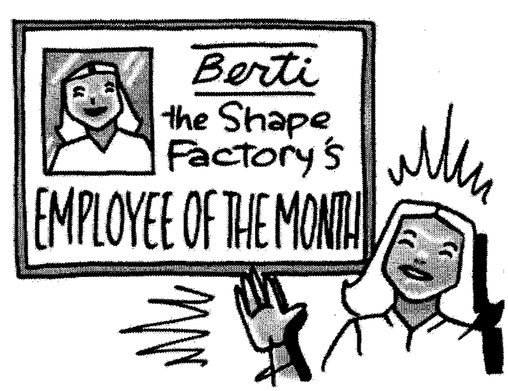
2-103. **Examine** the arrow diagram below.

Polygon is a parallelogram → *area of the polygon equals base times height.*

a. Write this conjecture as a conditional ("If, then") statement.

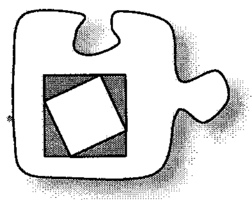
b. Write a similar conjecture about triangles, both as a conditional statement and as an arrow diagram.

2-104. Berti is the Shape Factory's top employee. She has received awards every month for having the top sales figures so far for the year. If she stays on top, she'll receive a $5000 bonus for excellence. She currently has sold 16,250 shapes and continues to sell 340 per month.



Since there are eight months left in the sales year, Sarita is working hard to catch up. While she has only sold 8,830 shapes, she is working overtime and on weekends so that she can sell 1,082 per month. Will Sarita catch up with Berti before the end of the sales year? If so, when?

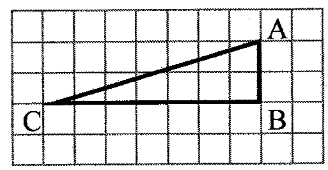
2.3.2 Is the answer reasonable?



Triangle Inequality

You now have several tools for describing triangles (lengths, areas, and angle measures), but can **any** three line segments create a triangle? Or are there restrictions on the side lengths of a triangle? And how can you know that the length you found for the side of a triangle is accurate? Today you will **investigate** the relationship between the side lengths of a triangle.

2-105. Roiri (pronounced "ROAR-ree") loves right triangles and has provided the one at right to analyze. He wants your team to find the length of the **hypotenuse** ($\overline{AC}$) using the method from problem 2-97.

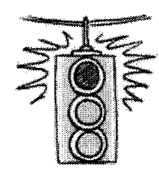


a. Estimate the length of $\overline{AC}$.

b. Draw Roiri's triangle on graph paper and then construct a square on the hypotenuse. Use the area of the square to find the length of the hypotenuse.

c. Was your result for the hypotenuse close to your estimate? Why or why not?

d. For a different triangle ΔABC where $AB = 3$ units and $BC = 4$ units, Roiri found that $AC = 25$. Donna is not sure that is possible. What do you think? **Visualize** this triangle and explain if you think this triangle is possible or not.

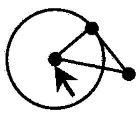


2-106. PINK SLIP

Oh no! During your last shift at the Shape Factory everything seemed to be going fine--until the machine that was producing triangles made a huge CLUNK and then stopped. Since your team was on duty, all of you will be held responsible for the machine's breakdown.

Luckily, your boss has informed you that if you can figure out what happened and how to make sure it will not happen again, you will keep your job. The last order the machine was processing was for a triangle with sides 3 cm, 5 cm, and 10 cm.

a. Use the manipulative (such as a dynamic geometry tool or pasta) provided by your teacher to **investigate** what happened today at the factory. Can a triangle be made with <u>any</u> three side lengths? If not, what condition(s) would make it impossible to build a triangle? Try building triangles with the side lengths listed below:



(1) 3 cm, 5 cm, and 10 cm (2) 4 cm, 9 cm, and 12 cm

(3) 2 cm, 4 cm, and 5 cm (4) 3 cm, 5 cm, and 8 cm

b. For those triangles that could not be built, what happened? Why were they impossible?

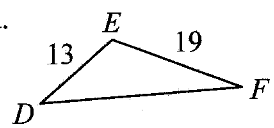
c. Use your dynamic geometry tool (or dry pasta) to **investigate** the restrictions on the three side lengths that can form a triangle. For example, if two sides of a triangle are 5 cm and 12 cm long, respectively, what is the longest side that could join these two sides to form a triangle? (Could the third side be 12 cm long? 19 cm long?) What is the shortest possible length that could be used to form a triangle? (Does 5 cm work? 9 cm?)

d. Write a memo to your boss explaining what happened. If you can convince your boss that the machine's breakdown was not your fault **and** show the company how to fix the machine so that this does not happen again, you might earn a promotion!

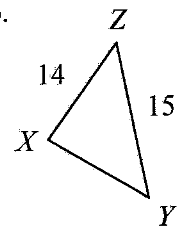
2-107. The values you found in parts (a) and (c) of problem 2-106 were the **minimum** and **maximum limits** for the length of the third side of any triangle with two sides of lengths 5 cm and 12 cm. The fact that there are restrictions on the side lengths that may be used to create a triangle is referred to as the **Triangle Inequality.**

Determine the minimum and maximum limits for each missing side length in the triangles below.

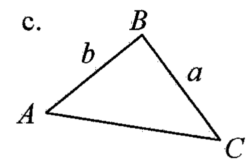
a.



b.



c.



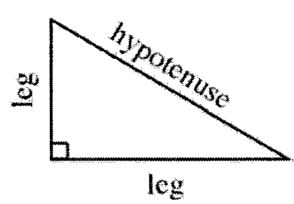
2-108. In a Learning Log entry, explain how you can tell if three sides will form a triangle or not. Draw diagrams to support your statements. Title this entry, "Triangle Inequality" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Right Triangle Vocabulary

All of the triangles that we have been working with in this section are right triangles, that is, triangles that contain a 90° angle. The two shortest sides of the right triangle (the sides that meet at the right angle) are called the **legs** of the triangle and the longest side (the side opposite the right angle) is called the **hypotenuse** of the triangle.



Review & Preview

2-109. Draw a right triangle with legs of length 6 and 8 units, respectively, onto graph paper. Construct a square on the hypotenuse and use the square's area to find the length of the hypotenuse.

2-110. Write the equation of each line described below in slope-intercept form ($y = mx + b$).

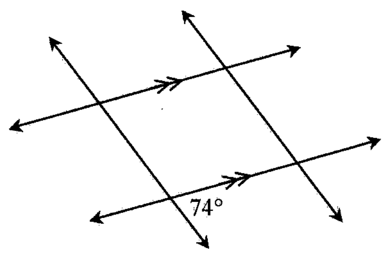
a. $m = \frac{6}{5}$ and $b = -3$

b. $m = -\frac{1}{4}$ and $b = 4.5$

c. $m = \frac{1}{3}$ and the line passes through the origin $(0, 0)$

d. $m = 0$ and $b = 2$

2-111. **Examine** the diagram at right. Based on the information in the diagram, which angles can you determine? Copy the diagram on your paper and find only those angles that you can **justify**.



2-112. Hannah's shape bucket contains an equilateral triangle, an isosceles right triangle, a regular hexagon, an isosceles trapezoid, a rhombus, a kite, a parallelogram and a rectangle. If she reaches in and selects a shape at random, what is the probability that that the shape will meet the criterion described below?

a. At least two sides congruent.

b. Two pairs of parallel sides.

c. At least one pair of parallel sides.

2-113. On graph paper, plot $ABCD$ if $A(-1, 2)$, $B(0, 5)$, $C(2, 5)$, and $D(6, 2)$.

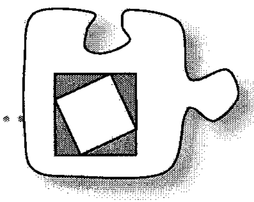
a. What type of shape is $ABCD$? **Justify** your answer.

b. If $ABCD$ is rotated 90° counterclockwise (↺) about the origin, name the coordinates of the image $A'B'C'D'$.

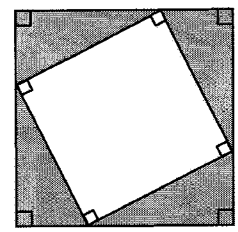
c. On your graph, reflect $ABCD$ across the y-axis to find $A''B''C''D''$. Name the coordinates of A'' and C''.

d. Find the area of $ABCD$. Show all work.

2.3.3 Is there a shortcut?



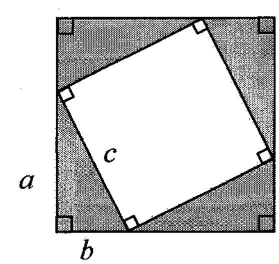
The Pythagorean Theorem



During Lessons 2.3.1 and 2.3.2, you have learned a method to find the length of a hypotenuse of a right triangle by finding the area of the square built on the hypotenuse, as shown in the diagram at right. However, what if the sides of the triangle make it difficult to draw (such as very large numbers or decimal values)? Or what if you do not even know one of the lengths of the legs?

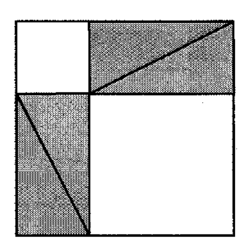
Today, you will work with your team to find the relationship between the legs and hypotenuse of a right triangle. By the end of this lesson, you should be able to find the side of any right triangle, when given the lengths of the other two sides.

2-114. Roiri complained that while the method from problem 2-97 works, it seems like too much work! He remembers that rearranging a shape does not change its area and thinks he can find a shortcut by rearranging Donna's square. Obtain a Lesson 2.3.3 Resource Page for your team and cut out the shaded triangles. Note that the lengths of the sides of the triangles are a, b, and c units respectively.



a. First, arrange the triangles to look like Donna's in the diagram at right. Draw this diagram on your paper. What is the area of the unshaded square?

b. Roiri claims that moving the triangles within the outer square won't change the area of the unshaded square. Is Roiri correct? Why or why not?

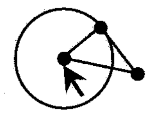


c. Move the shaded triangles to match the diagram at right. In this configuration, what is the total area that is unshaded?

d. Write an equation that relates the two ways that you found to represent the unshaded area in the figure.

2-115. The relationship you found in problem 2-114 (between the square of the lengths of the legs and the square of the length of the hypotenuse in a right triangle) is known as the **Pythagorean Theorem**. This relationship is a powerful tool because once you know the lengths of any two sides of a right triangle, you can find the length of the third side. But does this relationship always hold true?

a. Use a dynamic geometry tool to test whether the square of the hypotenuse always equals the sum of the squares of the legs of a right triangle.

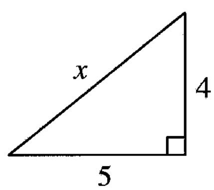


b. Add an entry in your Learning Log for the Pythagorean Theorem, explaining what it is and how to use it. Be sure to include a diagram.

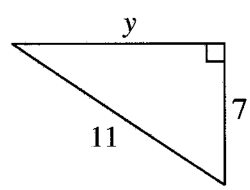
2-116. Apply the Pythagorean Theorem to answer the questions below.

a. For each triangle below, find the value of the variable.

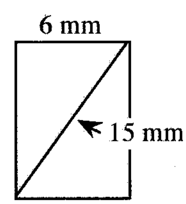
(1)



(2)



b. **Examine** the rectangle shown at right. Find its perimeter and area.



c. On graph paper, draw $\overline{AC}$ if $A(2, 6)$ and $C(5, -1)$. Then draw a slope triangle. Use the slope triangle to find the length of $\overline{AC}$.

2-117. The Garcia family took a day trip from Cowpoke Gulch. Their online directions told them to drive four miles north, six miles east, three miles north, and then one mile east to Big Horn Flat. Draw a diagram and calculate the direct distance (straight) from Cowpoke Gulch to Big Horn Flat.

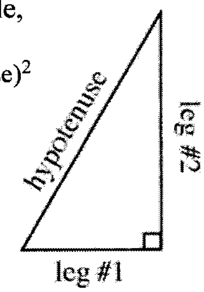
METHODS AND MEANINGS

MATH NOTES

The Pythagorean Theorem

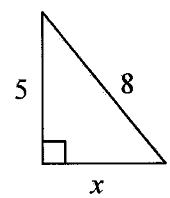
The **Pythagorean Theorem** states that in a right triangle,

(length of leg #1)^2 + (length of leg #2)^2 = (length of hypotenuse)^2



The Pythagorean Theorem can be used to find a missing side length in a right triangle. See the example below.

$$5^2 + x^2 = 8^2$$
$$25 + x^2 = 64$$
$$x^2 = 39$$
$$x = \sqrt{39} \approx 6.24$$



In the example above, $\sqrt{39}$ is an example of an **exact** answer, while 6.24 is an **approximate** answer.

Review & Preview

2-118. One of the algebra topics you have reviewed during this chapter is solving systems of equations. Assess what you know about solving systems as you answer the questions below.

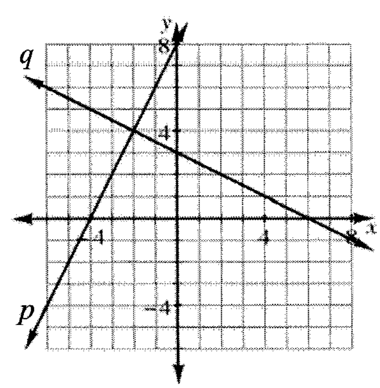
a. Find the points of intersection of the lines below using any method. Write your solutions as a point (x, y).

(1) $y = -x + 8$
$y = x - 2$

(2) $2x - y = 10$
$y = -4x + 2$

b. Find the equation for each line on the graph at right. Remember, the general form of any line in the **slope-intercept form** is $y = mx + b$.

c. Solve the system of equations you found in part (b) algebraically. Verify that your solution matches the one shown in the graph at right.



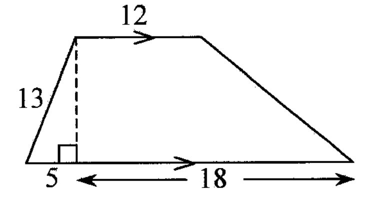
2-119. Lines p and q graphed in problem 2-118 form a triangle with the x-axis.

a. How can you describe this triangle? In other words, what is the most appropriate name for this triangle? How do you know?

b. Find the area of the triangle.

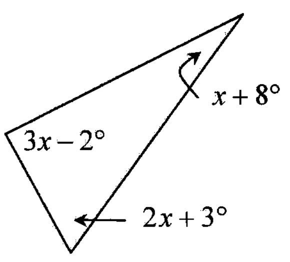
c. What is the perimeter?

2-120. Find the area of the trapezoid at right. What **strategies** did you use?

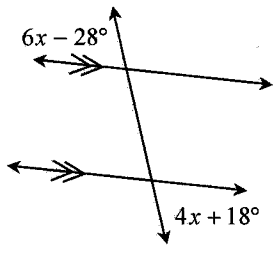


2-121. Use the relationships in the diagrams below to solve for x, if possible. If it is not possible, state how you know. If it is possible, **justify** your solution by stating which geometric relationships you use.

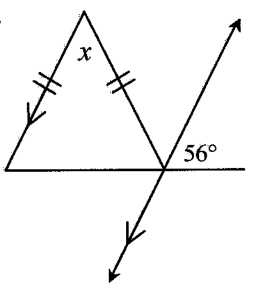
a.



b.



c.



2-122. Find the minimum and maximum limits for the length of a third side of a triangle if the other two sides are 8" and 13".

Chapter 2 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned in this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following two subjects. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Connections: How are the topics, ideas, and words that you learned in previous courses connected to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

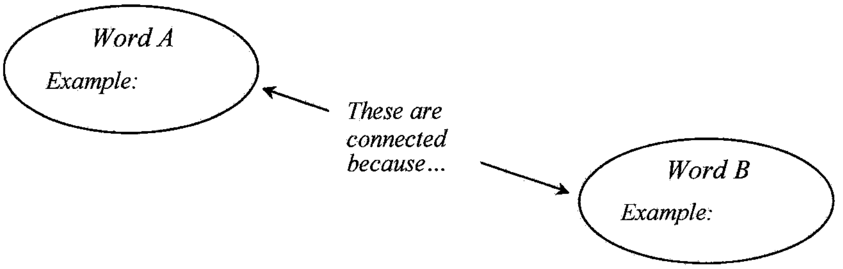
The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

alternate interior angles	area	**arrow diagram**
base	**complementary angles**	**conditional statement**
conjecture	**corresponding angles**	**dimension**
height	**hypotenuse**	**leg**
line	**parallelogram**	perimeter
proof by contradiction	**Pythagorean Theorem**	**rectangle**
right angle	**same-side interior angles**	**square**
square root	straight angle	**supplementary angles**
theorem	**transversal**	**trapezoid**
triangle inequality	**unit of measure**	**vertical angles**

Problem continues on next page →

② *Problem continued from previous page.*

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the example below. A word can be connected to any other word as long as there is a justified connection. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure to either re-draw your concept map on your paper or glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will direct you how to do this.

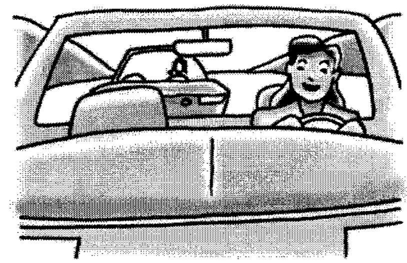
④ WHAT HAVE I LEARNED?

This section will help you recognize those types of problems you feel comfortable with and those you need more help with. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on similar problems.

CL 2-123. Sandra's music bag contains:

3 CDs of country music
6 CDs of rock and roll
4 CDs of rap music
5 cassette tapes of country music
1 cassette tape of rock and roll
3 cassette tapes of rap music



a. As Sandra drives, she often reaches into her bag and randomly pulls out some music (she does not want to take her eyes off the road). What is the probability that she will select some rap music?

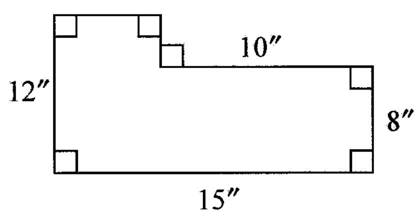
b. Find P(CD), that is, the probability that she will randomly select a CD (of any type of music).

c. Find P(cassette of country music).

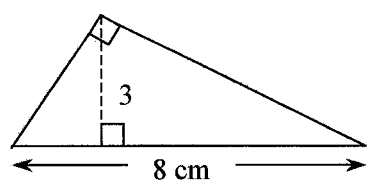
d. Find P(not rock and roll), the probability that the music she randomly selects is **not** rock and roll.

CL 2-124. Find the area of each figure.

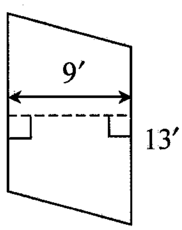
a.



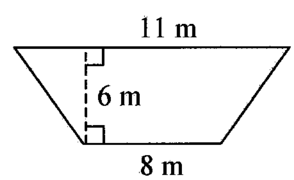
b.



c.

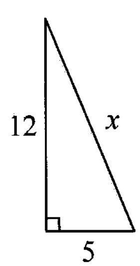


d.

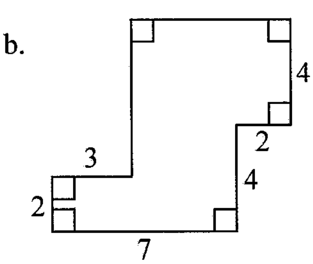


CL 2-125. Find the perimeter of each figure.

a.



b.



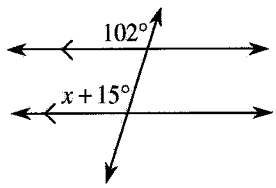
CL 2-126. Graph the segment that connects the points $A(-4,8)$ and $B(7,3)$.

a. What is the slope of $\overline{AB}$?

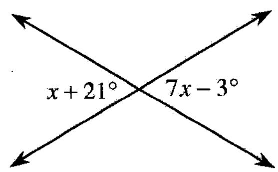
b. How long is $\overline{AB}$?

CL 2-127. Identify the geometric angle relationship(s) in each diagram. Use what you know about those relationships to write an equation and solve for x.

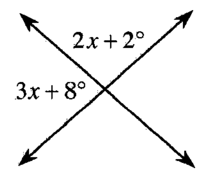
a.



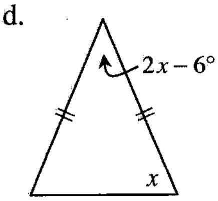
b.



c.



d.



CL 2-128. **Examine** the system of equations at right.

$$y = -2x + 6$$
$$y = \frac{1}{2}x - 9$$

a. Solve the system below <u>twice</u>: graphically and algebraically. Verify that your solutions from the different methods are the same.

b. What is the relationship between the two lines? How can you tell?

CL 2-129. Charlotte was transforming the hexagon *ABCDEF*.

a. What single transformation did she perform in Diagram #1?

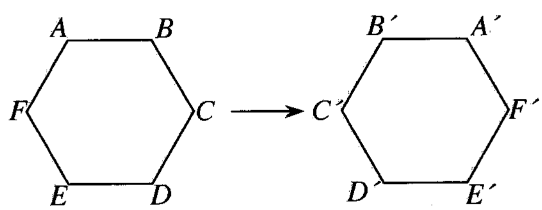


Diagram #1

b. What single transformation did she perform in Diagram #2?

c. What transformation didn't she do? Write directions for this type of transformation for hexagon *ABCDEF* and perform it.

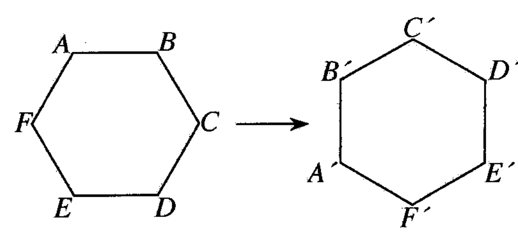


Diagram #2

CL 2-130. Explain what you are doing when you find the perimeter of a shape. How is that different than finding the area?

CL 2-131. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

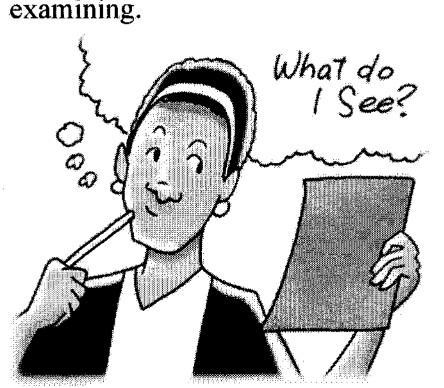
⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **examining**. Read the description of this Way of Thinking at right.

Examining

To **examine** means to look at a diagram and recognize its geometric properties. As you develop this way of thinking you will learn what specifically to look for in a diagram depending on the problem you are solving. Often, examining is what you do to answer to the question, "How does this diagram help me?" When you catch yourself thinking, "I can see from the diagram that... ", you are examining.

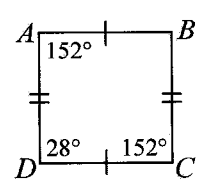


Think about the diagrams that you have looked at or drawn during this chapter. When have you had to understand information from a diagram or picture? What helps you to see what is in the diagram? What diagrams or parts of diagrams have you seen in a previous math class? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any of the methods you have developed to **examine** the problem in order to identify what is important or what information is conveyed in a diagram.

Once your discussion is complete, think about the way you think as you answer the following problems.

a. Sometimes, **examining** a shape means you have to disregard how it *looks* and concentrate on the information provided by the markings.

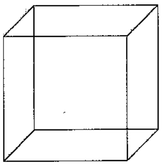
   For example, **examine** the shape at right. This shape looks like a square, but is it? Make as many statements as you can state about this shape based on the markings. What shape is *ABCD?* Which statements are obvious from the diagram and which did you have to think about?



Problem continues on next page →

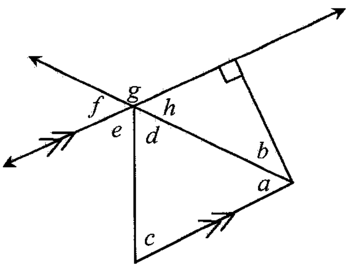
⑤ *Problem continued from previous page.*

b. At other times, examining a diagram suggests that you notice different parts that contribute to the entire diagram. For example, look at the diagram at right. Assuming that the diagram is drawn to scale, what shapes can you find in the diagram?



c. In part (a), you looked for attributes of a shape, while in part (b), you looked for shapes within a shape.

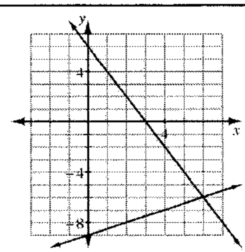
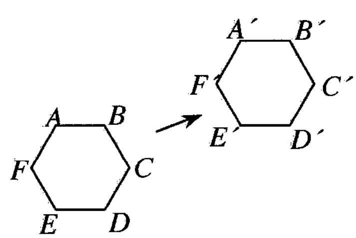
However, another aspect of examining a diagram is to look for relationships between parts of the diagram. For example, **examine** the diagram at right. What relationships do you see between the angles in the diagram? What relationships can you find between the lines and/or line segments? List as many relationships as you can based on the information in the diagram.



Answers and Support for Closure Activity #4

What Have I Learned?

Problem	Solutions	Need Help?	More Practice
CL 2-123.	a. $\frac{7}{22} \approx 0.32$ b. $\frac{13}{22} \approx 0.59$ c. $\frac{5}{22} \approx 0.23$ d. $\frac{15}{22} \approx 0.68$	Lesson 1.3.3 Math Notes box	Problems 2-9, 2-55, 2-74, 2-94, 2-112
CL 2-124.	a. 140 sq. in. b. 12 sq. cm. c. 117 sq. ft. d. 57 sq. m.	Lesson 2.2.4 Math Notes box	Problems 2-66, 2-68, 2-71, 2-75, 2-76, 2-78, 2-79, 2-82, 2-83, 2-90, 2-120
CL 2-125.	a. $x = 13$ units; $p = 30$ units b. $p = 34$ units	Lessons 1.1.3 and 2.3.3 Math Notes boxes	Problems 1-44, 1-65, 1-93, 2-52(a), 2-57, 2-97, 2-102, 2-116(b), 2-119

Problem	Solutions	Need Help?	More Practice
CL 2-126.	a. $m=-\frac{5}{11}$ b. length = $\sqrt{146}\approx 12.1$ units	Lessons 1.2.5 and 2.3.3 Math Notes boxes	Problems 1-36, 1-76, 2-116, 2-117
CL 2-127.	a. $x+15°=102°$; $x=87°$; corresponding angles b. $7x-3°=x+21°$; $x=4°$; vertical angles c. $2x+2°+3x+8°=180°$; $x=34°$; supplementary angles d. $x+x+2x-6°=180°$; $x=46.5°$; sum of angles in a triangle is 180°	Lessons 2.1.1 and 2.1.4 Math Notes boxes	Problems 2-13, 2-16, 2-17, 2-18, 2-19, 2-23, 2-31, 2-32, 2-35, 2-36, 2-37, 2-38, 2-49, 2-51, 2-52, 2-62, 2-63, 2-72, 2-111, 2-121
CL 2-128.	a. (6, –6) b. They are perpendicular because the slopes are opposite reciprocals. 	Lessons 2.1.3, 1.2.1, and 1.2.5 Math Notes boxes	Problems 1-62, 1-92, 2-29, 2-39, 2-42, 2-53, 2-54(c), 2-65, 2-92, 2-118
CL 2-129.	a. Reflection (flip) b. Rotation (turn counterclockwise 90°) c. Translation (slide). Answers will vary, and example is provided below: 	Lessons 1.2.2 Math Notes box	Problems 1-50, 1-51, 1-59, 1-60, 1-61, 1-64, 1-69, 1-73, 1-85, 1-96, 2-11, 2-19, 2-20, 2-33, 2-64, 2-113
CL 2-130.	The perimeter of a shape is the length of that shape's boundary. While to find the area, consider everything *inside* the boundary.	Lesson 1.1.3 Math Notes box	Problems 1-65, 1-66, 1-74, 1-84, 2-57, 2-116