7-2 THEORETICAL AND EXPERIMENTAL PROBABILITY

CHECK IT OUT!

1a. There are 36 possible outcomes, and 5 outcomes with the sum of 6.
$P(\text{sum is 6}) = \frac{5}{36}$

b. There are 36 possible outcomes, and 0 outcomes with a difference of 6.
$P(\text{difference is 6}) = \frac{0}{36} = 0$

c. There are 36 possible outcomes, and 15 outcomes where the red cube is greater.
$P(\text{red cube is greater}) = \frac{15}{36} = \frac{5}{12}$

2. There are 100 possible outcomes.
The number of possible outcomes where both numbers are less than 9 is
$(\text{first number} < 9)(\text{second number} < 9)$
$= 8 \times 8 = 64.$
$P(\text{both numbers less than 9}) = \frac{64}{100} = \frac{16}{25}$
The probability that both numbers are less than 9 is $\frac{16}{25}$.

3. Order is not important. It is a combination.
The number of possible outcomes is
${}_8C_2 = \frac{8!}{2!6!} = \frac{8 \cdot 7}{2 \cdot 1} = 28.$
There is only 1 way to play both of the retailer's ads.
$P(\text{play both of the retailer's ads}) = \frac{1}{28}$

4. Area of large triangle is $A_t = \frac{1}{2}(15)(15) = 112.5.$
Area of small triangle is $A_s = \frac{1}{2}(4)(4) = 8.$
$\frac{A_s}{A_t} = \frac{8}{112.5} = \frac{16}{225}$
The probability that the point is in the small triangle is $\frac{16}{225}$.

5a. $P(\text{diamond}) = \frac{9}{26}$

b. $P(\text{not club}) = 1 - P(\text{club})$
$= 1 - \frac{7}{26} = \frac{19}{26}$

THINK AND DISCUSS

1. No, the probability of an event cannot exceed 1.

2. sum of 5 and sum of 9

3. experimental: $\frac{8}{20} = \frac{2}{5}$; theoretical: $\frac{1}{2}$

4.

Experimental The experimental probability of rolling a 5 on a number cube that was rolled 30 times and landed on 5 7 times is $\frac{7}{30}$.	**Theoretical** There are 6 equally-likely outcomes when rolling a number cube, so each has a theoretical probability of $\frac{1}{6}$
Probability	
Complement The complement of the experimental probability above is $1 - \frac{7}{30}$, or $\frac{23}{30}$, which is the experimental probability of not rolling a 5.	**Geometric** If a circle of radius 3 is inside a square with side length 10 and any point inside the square is equally likely, the probability of a random point being in the circle is $\frac{\pi(3)^2}{10^2} \approx 0.28$

EXERCISES

GUIDED PRACTICE

1. theoretical probability

2. There are 8 possible outcomes, and 4 outcomes where the quarter shows heads.
$P(\text{quarter shows heads}) = \frac{4}{8} = \frac{1}{2}$

3. There are 8 possible outcomes, and 2 outcomes where the penny and the nickel both show heads.
$P(\text{penny and nickel show heads}) = \frac{2}{8} = \frac{1}{4}$

4. There are 8 possible outcomes, and 3 outcomes where 1 coin shows heads.
$P(\text{one coin shows heads}) = \frac{3}{8}$

5. There are 8 possible outcomes, and 2 outcomes where all coins land the same way.
$P(\text{all coins land the same way}) = \frac{2}{8} = \frac{1}{4}$

6. $P(\text{does not end in 5}) = 1 - P(\text{ends in 5})$
$= 1 - \frac{10}{100} = \frac{9}{10}$
The probability that a random 2-digit number does not end in 5 is $\frac{9}{10}$.

7. $P(\text{not in Dec or Jan}) = 1 - P(\text{in Dec or Jan})$
$= 1 - \frac{31 + 31}{365} = \frac{303}{365}$
The probability that a date is not in December or January is $\frac{303}{365}$.

8. Order is important. It is a permutation.
The number of possible outcomes is
${}_4P_4 = \frac{4!}{0!} = 24.$
There is only 1 way to place all the letters in the correct envelope.
$P(\text{all letters are in correct envelopes}) = \frac{1}{24}$

9. Order is not important. It is a combination.
The number of possible outcomes is
${}_{12}C_3 = \frac{12!}{3!9!} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$
There is only 1 way to choose all 3 green balloons.
$P(\text{3 green balloons}) = \frac{1}{220}$

10. Area of large circle is $A_t = \pi(6)^2 = 36\pi$
Area of middle circle is $A_m = \pi(4)^2 = 16\pi$
Area of small circle is $A_u = \pi(2)^2 = 4\pi$
Shaded area is $A_s = 16\pi - 4\pi = 12\pi$
$\frac{A_s}{A_t} = \frac{12\pi}{36\pi} = \frac{1}{3}$
The probability that a point is in the shaded area is $\frac{1}{3}$.

11. $\frac{A_u}{A_t} = \frac{4\pi}{36\pi} = \frac{1}{9}$
The probability that a point is in the smallest circle is $\frac{1}{9}$.

12. $\text{P(red)} = \frac{5}{20} = \frac{1}{4}$

13. $P(\text{red or blue}) = \frac{5+7}{20} = \frac{3}{5}$

PRACTICE AND PROBLEM SOLVING

14. There are 15 possible outcomes, and 5 outcomes of a white marble.
$P(\text{white marble}) = \frac{5}{15} = \frac{1}{3}$

15. There are 15 possible outcomes, and 12 outcomes of a red or white marble.
$P(\text{red or white marble}) = \frac{12}{15} = \frac{4}{5}$

16. There are 64 possible outcomes.
The number of possible outcomes where both numbers are greater than 2 is
$(\text{first number} > 2)(\text{second number} > 2) = 6 \times 6 = 36$
$P(\text{both numbers greater than 2}) = \frac{36}{64} = \frac{9}{16}$.
The probability that both numbers are greater than 2 is $\frac{9}{16}$.

17. Order is not important. It is a combination.
${}_8C_3 = \frac{8!}{3!5!} = \frac{8 \bullet 7 \bullet 6}{3 \bullet 2 \bullet 1} = 56$
There is only 1 way to choose the 3 strongest swimmers.
$P(\text{3 strongest swimmers}) = \frac{1}{56}$

18. Order is not important. It is a combination.
${}_7C_2 = \frac{7!}{2!5!} = \frac{7 \bullet 6}{2 \bullet 1} = 21$
There is only 1 way to choose books 1 and 2.
$P(\text{books 1 and 2}) = \frac{1}{21}$

19. 2 ft = 24 in., and 4 ft = 48 in.
Area of platform is $A_t = 24(48) = 1152$
Area of hole is $A_s = \pi(3)^2 = 9\pi$
$\frac{A_s}{A_t} = \frac{9\pi}{1152} = \frac{\pi}{128} \approx \frac{1}{42}$
The probability of the bag landing in the hole is $\frac{1}{42}$.

20. $P(\text{red card}) = \frac{16}{28} = \frac{4}{7}$
The experimental probability that a card is red is $\frac{4}{7}$.

21. never; the theoretical probability of tossing tails on a fair coin is $\frac{1}{2}$. Tossing the coin 25 times, implies that tails will appear 12.5 times in order to be equal to the theoretical probability.

22. $P(\text{state that does not border Mississippi})$
$= 1 - P(\text{state that borders Mississippi})$
$= 1 - \frac{4}{49} = \frac{45}{49}$
The probability that the winner will be from a state that does not border Mississippi is $\frac{45}{49}$.

23a. Area of circle is $A_c = \pi r^2$
Area of square is $A_s = (2r)^2 = 4r^2$
$\frac{A_c}{A_s} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$

b. Possible answer: $\frac{7852}{10{,}000} \approx \frac{\pi}{4}$,
and so, $\pi \approx 4 \times \frac{7852}{10{,}000} \approx 3.141$.

24. Possible answer: A roll of a die shows less than 20.

25a. $P(\text{made throws 1–25}) = \frac{17}{25} = 0.68$
$P(\text{made throws 26–50}) = \frac{21}{25} = 0.84$
$P(\text{made throws 51–75}) = \frac{19}{25} = 0.76$
$P(\text{made throws 76–100}) = \frac{16}{25} = 0.64$

b. $P(\text{throws made}) = \frac{73}{100} = 0.73$

c. The greater the number of experiments, the closer the experimental probability will be to the theoretical probability.

26a. $P(\text{two 4s}) = P(\text{4 and 4}) = \frac{1}{36}$
The probability that Mei will have 5 of a kind is $\frac{1}{36}$.

b. $P(\text{one 4}) = P(\text{4 and not 4}) + P(\text{not 4 and 4})$
$= \frac{5+5}{36} = \frac{5}{18}$
The probability that Mei will have 4 of a kind is $\frac{5}{18}$.

c. $P(\text{zero 4s}) = P(\text{not 4 and not 4}) = \frac{25}{36}$
The probability that Mei will have three 4s is $\frac{25}{36}$.

d. $\frac{1}{36} + \frac{10}{36} + \frac{25}{36} = 1$

27. *Length* $\overline{BD} = 4 - 2 = 2$; *Length* $\overline{AF} = 6 - 1 = 5$
$\frac{\text{Length } \overline{BD}}{\text{Length } \overline{AF}} = \frac{2}{5}$
Probability that a point lies on $\overline{BD}$ is $\frac{2}{5}$.

28. $P(\text{temperature} > 90°\text{F in April})$
$= \frac{5}{30} = \frac{1}{6} \approx 0.167$

29. June;
$P(\text{temperature} \leq 90°\text{F})$
$= 1 - P(\text{temperature} > 90°\text{F})$
$= 1 - \frac{26}{30} = \frac{4}{30} \approx 0.13$

30. it will be slightly greater:
$P(\text{temperature} \leq 90°\text{F})$
$= 1 - P(\text{temperature} > 90°\text{F})$
$= 1 - \frac{11}{31} = \frac{20}{31} \approx 0.645$ vs. $= 1 - \frac{11}{30} = \frac{19}{30} \approx 0.633$

31. No; yes; a theoretical probability of 1 means all possible outcomes are favorable outcomes, but a theoretical probability of 0.99 means there is at least 1 unfavorable outcome.