Worksheet 10.3

1. $\sin 15^\circ$; 15° in QI so $\sin 15^\circ$ is $\oplus$

$\theta = 30^\circ \rightarrow 15^\circ = \frac{\theta}{2}$

$\sin\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{2}}$

$= \oplus\sqrt{\frac{1-\cos 30^\circ}{2}} = \oplus\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \oplus\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}}$

$= \oplus\sqrt{\frac{2-\sqrt{3}}{2} \div \frac{2}{1}} = \oplus\sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}} = \oplus\frac{\sqrt{2-\sqrt{3}}}{2}$

2. $\cos 22.5^\circ$; 22.5° is in QI so $\cos 22.5^\circ$ is $\oplus$

$\theta = 45^\circ \rightarrow 22.5^\circ = \frac{\theta}{2}$

$\cos\frac{\theta}{2} = \oplus\sqrt{\frac{1+\cos\theta}{2}} = \oplus\sqrt{\frac{1+\cos 45^\circ}{2}}$

$= \oplus\sqrt{\frac{1+\sqrt{2}/2}{2}} = \oplus\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \oplus\sqrt{\frac{2+\sqrt{2}}{2} \div$

$= \oplus\sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}} = \oplus\frac{\sqrt{2+\sqrt{2}}}{2}$

3. $\tan 67.5^\circ$; 67.5° is in QI so $\tan 67.5^\circ$ is $\oplus$

$\theta = 135^\circ \rightarrow 67.5^\circ = \frac{\theta}{2}$

$\tan\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \oplus\sqrt{\frac{1-\cos 135^\circ}{1+\cos 135^\circ}} = +\sqrt{\frac{1-(-\frac{\sqrt{2}}{2})}{1+(-\frac{\sqrt{2}}{2})}}$

$\oplus\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}} = \oplus\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2}{2}-\frac{\sqrt{2}}{2}}} = \oplus\sqrt{\frac{2+\sqrt{2}}{2} \div \frac{2-\sqrt{2}}{2}}$

$\oplus\sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{2}{2-\sqrt{2}}} = +\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}} \cdot \sqrt{\frac{2+\sqrt{2}}{2+\sqrt{2}}}$

$\oplus\sqrt{\frac{4+2\sqrt{2}+2\sqrt{2}+2}{4+2\sqrt{2}-2\sqrt{2}-2}} = +\sqrt{\frac{6+4\sqrt{2}}{2}} = +\sqrt{3+2\sqrt{2}}$

≈ 2.414213562

Or $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 135^\circ}{1+\cos 135^\circ} = \frac{\frac{\sqrt{2}}{2}}{1+\left(-\frac{\sqrt{2}}{2}\right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{2}{2}-\frac{\sqrt{2}}{2}}$

$= \frac{\sqrt{2}}{2} \div \frac{2-\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\cdot\frac{2}{2-\sqrt{2}} = \frac{\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \frac{2\sqrt{2}+2}{4+2\sqrt{2}-2\sqrt{2}-2} = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1 \approx 2.41421$

4. $\cos\frac{7\pi}{12}$; $\frac{7\pi}{12}$ is in Q II → cos ⊖

$\frac{\pi}{2} = \frac{6\pi}{12}$, $\pi - \frac{12\pi}{12}$

$\theta = \frac{7\pi}{6} \rightarrow \frac{7\pi}{12} = \frac{\theta}{2} \rightarrow \cos\frac{7\pi}{12} = \cos\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \ominus\sqrt{\frac{1+\cos\theta}{2}} = \ominus\sqrt{\frac{1+\cos 7\pi/6}{2}} = \ominus\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \ominus\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \ominus\sqrt{\frac{2-\sqrt{3}}{2}\div\frac{2}{1}} = \ominus\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}$

$= \ominus\frac{\sqrt{2-\sqrt{3}}}{2}$

5. $\tan\frac{\pi}{12}$; $\frac{\pi}{12}$ is in Q I so $\tan\frac{\pi}{12}$ is ⊕

$\theta = \frac{\pi}{6}$ so $\frac{\pi}{12} = \frac{\theta}{2} \rightarrow \tan\frac{\pi}{12} = \tan\frac{\theta}{2}$

$\tan\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \oplus\sqrt{\frac{1-\cos\pi/6}{1+\cos\pi/6}} = \oplus\sqrt{\frac{1-\sqrt{3}/2}{1+\sqrt{3}/2}}$

$= \oplus\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{\frac{2}{2}+\frac{\sqrt{3}}{2}}} = \oplus\sqrt{\frac{2-\sqrt{3}}{2}\div\frac{2+\sqrt{3}}{2}} =$

$\oplus\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{2}{2+\sqrt{3}}} = \oplus\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}}$

$$+\sqrt{\frac{4-2\sqrt{3}-2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3}} = +\sqrt{\frac{7-4\sqrt{3}}{1}} = +\sqrt{7-4\sqrt{3}} \approx .267949\ldots$$

or $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin\frac{\pi}{6}}{1+\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{2}{2}+\frac{\sqrt{3}}{2}}$

$$= \frac{1}{2} \div \frac{2+\sqrt{3}}{2} = \frac{1}{2}\cdot\frac{2}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3} \approx .2679\ldots$$

6. $\sin\frac{5\pi}{8} \Rightarrow \frac{5\pi}{8}$ is in QII (sin +) $\quad \frac{\pi}{2} = \frac{4\pi}{8}$, $\pi = \frac{8\pi}{8}$

$\theta = \frac{5\pi}{4} \rightarrow \frac{5\pi}{8} = \frac{\theta}{2} \rightarrow \sin\frac{5\pi}{8} = \sin\frac{\theta}{2}$

$$\sin\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{2}} = \oplus\sqrt{\frac{1-\cos\frac{5\pi}{4}}{2}} = \oplus\sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \oplus\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \oplus\sqrt{\frac{2+\sqrt{2}}{2}\div\frac{2}{1}} = \oplus\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= \oplus\frac{\sqrt{2+\sqrt{2}}}{2}$$

7. $\cos\frac{3\pi}{8} \rightarrow \frac{3\pi}{8}$ is in QI so $\sin\frac{3\pi}{8}$ is +

$\theta = \frac{3\pi}{4} \rightarrow \frac{3\pi}{8} = \frac{\theta}{2} \rightarrow \cos\frac{3\pi}{8} = \cos\frac{\theta}{2}$

$$\cos\frac{\theta}{2} = +\sqrt{\frac{1+\cos\theta}{2}} = +\sqrt{\frac{1+\cos 3\pi/4}{2}} = +\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= +\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{2-\sqrt{2}}{2}\div\frac{2}{1}} = +\sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= +\frac{\sqrt{2-\sqrt{2}}}{2}$$

8. $\sin 165^\circ$; 165° is in Q II so sin is +

$\theta = 330^\circ \rightarrow 165^\circ = \frac{\theta}{2} \rightarrow \sin 165^\circ = \sin\frac{\theta}{2}$

$$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\cos 330^\circ}{2}} = +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= +\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = +\sqrt{\frac{2-\sqrt{3}}{2} \div \frac{2}{1}} = +\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}$$

$$= +\frac{\sqrt{2-\sqrt{3}}}{2}$$

9. $\sin\frac{5\pi}{12}$; $5\pi/12$ is in Q I so sin is +

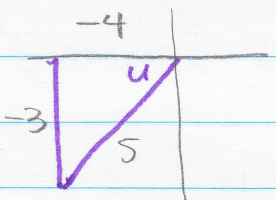
$\theta = \frac{5\pi}{6} \rightarrow \frac{5\pi}{12} = \frac{\theta}{2} \rightarrow \sin\frac{5\pi}{12} = \sin\frac{\theta}{2}$

$$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\cos 5\pi/6}{2}} = +\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= +\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = +\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{2}{1}} = +\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$$

$$= +\frac{\sqrt{2+\sqrt{3}}}{2}$$

10. $\tan u = \frac{3}{4}$



$\cos u = -4/5$

$\sin u = -3/5$

u is in Q III ; must find where $\frac{u}{2}$ is

$\pi < u < \frac{3\pi}{2} \rightarrow \frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4} \rightarrow$ Q II

$\sin\frac{u}{2}$ is (+) ; $\cos\frac{u}{2}$ is (−) ; $\tan\frac{u}{2}$ is (−)

* *

$$\sin\frac{u}{2} = +\sqrt{\frac{1-\cos u}{2}} =$$

$$+\sqrt{\frac{1-(-4/5)}{2}} =$$

$$+\sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}}$$

$$+\sqrt{\frac{9}{5}\div\frac{2}{1}}$$

$$+\sqrt{\frac{9}{5}\cdot\frac{1}{2}} = +\sqrt{\frac{9}{10}} = +\frac{3\sqrt{10}}{10}$$

see p.4

$$\cos\frac{u}{2} = \overset{*}{\ominus}\sqrt{\frac{1+\cos u}{2}} = \ominus\sqrt{\frac{1+(-4/5)}{2}} \ominus\sqrt{\frac{1-\frac{4}{5}}{2}} \ominus\sqrt{\frac{\frac{1}{5}}{2}}$$

$$\ominus\sqrt{\frac{1}{5}\cdot\frac{1}{2}} = \ominus\sqrt{\frac{1}{10}\frac{10}{10}} = \ominus\frac{\sqrt{10}}{10}$$

$$\tan\frac{u}{2} = \overset{*}{\ominus}\sqrt{\frac{1-\cos u}{1+\cos u}} = \ominus\sqrt{\frac{1-(-\frac{4}{5})}{1+(-\frac{4}{5})}} = \ominus\sqrt{\frac{\frac{9}{5}}{\frac{1}{5}}}$$

$$= \ominus\sqrt{\frac{9}{5}\cdot\frac{5}{1}} = \ominus\sqrt{9} = \ominus 3$$

OR $\tan\frac{u}{2} = \frac{\sin u}{1+\cos u} = \frac{-\frac{3}{5}}{1+(-\frac{4}{5})} = \frac{-\frac{3}{5}}{\frac{1}{5}} = -\frac{3}{5}\cdot\frac{5}{1} = -3$

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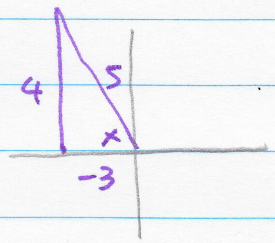


$\sin x = \frac{4}{5}$

$\cos x = -\frac{3}{5}$

$\tan x = -\frac{4}{3}$

$$\sin 2x = 2\sin x\cos x = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

$$\cos 2x = 2\cos^2 x - 1 = 2\left(-\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = -\frac{7}{25}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-8/3}{1-\frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}}$$

$$= -\frac{8}{3}\cdot\frac{-9^3}{7} = \frac{24}{7}$$

12 $$\frac{\sin 2x}{2\sin x} = \frac{2\sin x\cos x}{2\sin x} = \cos x$$

13 $$(\sin x+\cos x)^2 = \sin^2 x + \sin x\cos x + \sin x\cos x + \cos^2 x$$

$$= \sin^2 x + \cos^2 x + 2\sin x\cos x$$

$$= 1 + \sin 2x$$

WLS 3 # 6

$\tan 2\pi = \frac{0}{1} = 0$

14 $\tan(x-2\pi) = \frac{\tan x - \tan 2\pi}{1+\tan x \tan 2\pi} = \frac{\tan x - 0}{1+\tan x(0)}$

$= \frac{\tan x}{1} = \tan x$

15. $\sin(x-\frac{\pi}{2}) = \sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2}$

$= \sin x(0) - \cos x(1) = -\cos x$

16 $\frac{\cos 2x + \cos x}{2\cos^2 x - 1 + \cos x}$ $= 2\cos^2 x + \cos x - 1$

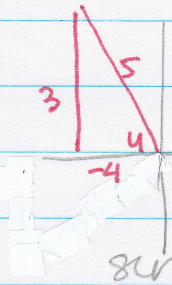
$2\cos^2 x + \cos x - 1$

17 $\frac{\cos 2x}{\cos x}$ $= 2\cos x - \sec x$

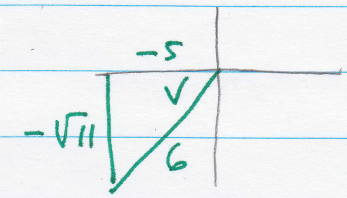
$\frac{2\cos^2 x - 1}{\cos x}$

$\frac{2\cos^2 x}{\cos x} - \frac{1}{\cos x}$

$2\cos x - \sec x$



$\sin u = 3/5$
$\cos u = -4/5$
$\tan u = -\frac{3}{4}$



$\sin v = -\sqrt{11}/6$
$\cos v = -5/6$
$\tan v = \sqrt{11}/5$

18 $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\frac{3}{5}\left(-\frac{5}{6}\right) + \left(-\frac{4}{5}\right)\left(-\sqrt{11}/6\right) = \frac{-15+4\sqrt{11}}{30}$

19 $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$(-4/5)(-5/6) + (3/5)(-\sqrt{11}/6) = \frac{20-3\sqrt{11}}{30}$