2-1 Using Inductive Reasoning to Make Conjectures

Objectives

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.

Vocabulary

inductive reasoning

conjecture

counterexample

Example 1A: Identifying a Pattern

Find the next item in the pattern.

January, March, May, …

Alternating months of the year make up the pattern.

The next month is July.

Example 1B: Identifying a Pattern

Find the next item in the pattern.

7, 14, 21, 28, …

The next multiple is 35.

Multiples of 7 make up the pattern.

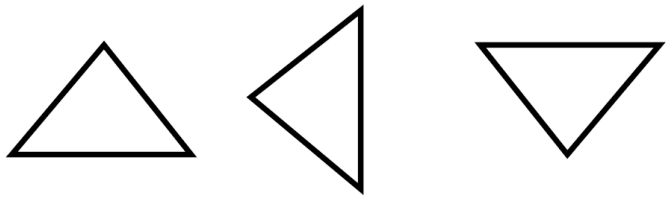
Find the next item in the pattern
0.4, 0.04, 0.004, …

When reading the pattern from left to right, the next item in the pattern has one more zero after the decimal point.

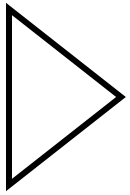
The next item would have 3 zeros after the decimal point, or 0.0004.

Example 1C: Identifying a Pattern

Find the next item in the pattern.



In this pattern, the figure rotates 90° counter-clockwise each time.

The next figure is  .

2-1 Using Inductive Reasoning to Make Conjectures

When several examples form a pattern and you assume the pattern will continue, you are applying *inductive reasoning*. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.

2-1 Using Inductive Reasoning to Make Conjectures

Example 2A: Making a Conjecture

Complete the conjecture.

The sum of two positive numbers is _?_.

List some examples and look for a pattern.

1 + 1 = 2 3.14 + 0.01 = 3.15

3,900 + 1,000,017 = 1,003,917

The sum of two positive numbers is **positive**.

Check It Out! Example 2

Complete the conjecture.

The product of two odd numbers is _?_.

List some examples and look for a pattern.

1 × 1 = 1 *3 × 3 = 9* *5 × 7 = 35*

The product of two odd numbers is odd.

2-1 Using Inductive Reasoning to Make Conjectures

To show that a conjecture is always true, you must prove it.

To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a **counterexample**.

A counterexample can be a drawing, a statement, or a number.

2-1 Using Inductive Reasoning to Make Conjectures

Inductive Reasoning
1. Look for a pattern.
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

Example 4A: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.
For every integer n, n^3 is positive.

Pick integers and substitute them into the expression to see if the conjecture holds.

Let $n = 1$. Since $n^3 = 1$ and $1 > 0$, the conjecture holds.

Let $n = -3$. Since $n^3 = -27$ and $-27 \leq 0$, the conjecture is false.

$n = -3$ is a counterexample.

Example 4B: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

Two complementary angles are not congruent.

45° + 45° = 90°

If the two congruent angles both measure 45°, the conjecture is false.

Example 4C: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

The monthly high temperature in Abilene is never below 90°F for two months in a row.

Monthly High Temperatures (ºF) in Abilene, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

The monthly high temperatures in January and February were 88°F and 89°F, so the conjecture is false.

Check It Out! Example 4a

Show that the conjecture is false by finding a counterexample.

For any real number x, $x^2 \geq x$.

Let $x = \frac{1}{2}$.

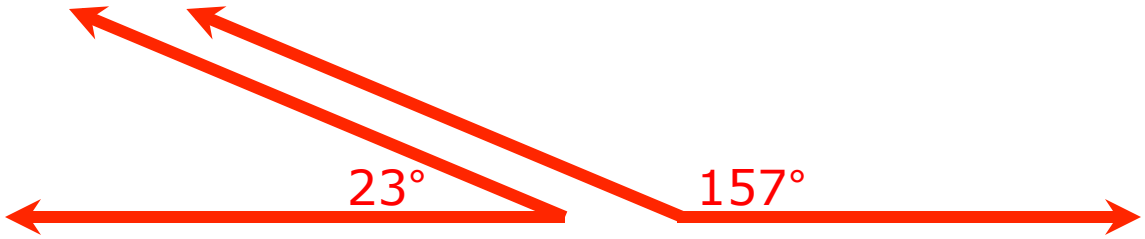
Since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\frac{1}{4} \not\geq \frac{1}{2}$.

The conjecture is false.

Check It Out! Example 4b

Show that the conjecture is false by finding a counterexample.

Supplementary angles are adjacent.



The supplementary angles are not adjacent, so the conjecture is false.

Check It Out! Example 4c

Show that the conjecture is false by finding a counterexample.

The radius of every planet in the solar system is less than 50,000 km.

Planets' Diameters (km)							
Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500

Since the radius is half the diameter, the radius of Jupiter is 71,500 km and the radius of Saturn is 60,500 km. The conjecture is false.