Date ____________________

Dear Family,

In Chapter 4, your child will learn about matrices and how to use them to solve linear systems.

A **matrix** is a rectangular array of numbers enclosed in brackets. A matrix is like a table without headings.

The numbers of rows and columns are called the **dimensions** of the matrix. A **square matrix** has the same numbers of rows and columns.

Each value, or **entry**, in the matrix is named by an **address** that gives its row and column.

> **matrix:** $A = \begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}$
>
> **dimensions:** Matrix A is a 3×2 matrix.
>
> **address:** The address of 3 is a_{21}.

Several operations can be performed with matrices:

Operation	Requirements	How To	Example
Addition/ Subtraction	The matrices must have the same dimensions.	Add or subtract corresponding entries.	$\begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -8 & 9 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2+3 & 5+6 \\ 3+(-8) & -4+9 \\ 1+2 & 0+5 \end{bmatrix}$
Scalar Multiplication	No requirements.	Multiply each entry by the same number, or **scalar**.	$4\begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4(2) & 4(5) \\ 4(3) & 4(-4) \\ 4(1) & 4(0) \end{bmatrix}$
Matrix Multiplication	The *inside* dimensions must be the same. (The dimensions of the product will be the *outside* dimensions.)	Working with row *i* of the first matrix and column *j* of the second matrix, add the products of consecutive entries. The result goes in row *i* column *j* of the matrix product.	The dimensions are 3×2 and 2×2. Same inside dimension. The product will be 3×2. $\begin{bmatrix} 2 & 5 \\ 3 & -4 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ -8 & 9 \end{bmatrix} =$ $\begin{bmatrix} 2(3)+5(-8) & 2(6)+5(9) \\ 3(3)+(-4)(-8) & 3(6)+(-4)(9) \\ 1(3)+0(-8) & 1(6)+0(9) \end{bmatrix}$ The entry at p_{32} is calculated from row 3 of the first matrix and column 2 of the second matrix.

A **multiplicative identity matrix** is a square matrix with 1's along the **main diagonal** (top left to bottom right) and 0's for every other entry. The product of square matrix *A* and the identity matrix is simply matrix *A*. The product of a matrix and its **multiplicative inverse** is the identity.

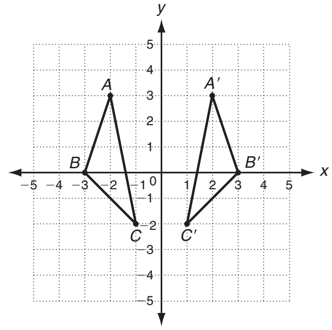
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A matrix can represent the *x*- and *y*-coordinates of the vertices of a polygon. You can then transform the polygon using matrix operations.

$$\Delta ABC: \begin{bmatrix} -2 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} \begin{matrix} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ A & B & C \end{matrix}$$

$$\Delta A'B'C': \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & -2 \end{bmatrix}$$



Matrices can also represent a linear system in two ways:

Linear System	**Matrix Equation**	**Augmented Matrix**
$\begin{cases} 2x + 4y = 20 \\ 3x - y = 9 \end{cases}$	$\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \end{bmatrix}$	$\left[\begin{array}{cc:c} 2 & 4 & 20 \\ 3 & -1 & 9 \end{array}\right]$

Cramer's rule allows you to solve a linear system using determinants. A **determinant** is a single number calculated from the entries in the matrix.

determinant of coefficient matrix: det $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = (2)(-1) - (3)(4) = -14$

replace *x*-coefficients with constants: det $\begin{bmatrix} 20 & 4 \\ 9 & -1 \end{bmatrix} = -56$, so $x = \frac{-56}{-14} = 4$

replace *y*-coefficients with constants: det $\begin{bmatrix} 2 & 20 \\ 3 & 9 \end{bmatrix} = -42$, so $y = \frac{-42}{-14} = 3$

You can also solve a linear system with a matrix equation and the multiplicative inverse of the coefficient matrix.

matrix equation: $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \end{bmatrix}$

solution: $\begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ \frac{3}{14} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 20 \\ 9 \end{bmatrix}$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, so $\begin{matrix} x = 4 \\ y = 3 \end{matrix}$

A third way to solve a linear system uses **row operations** to change the augmented matrix into **reduced row-echelon form**.

augmented matrix: $\left[\begin{array}{cc:c} 2 & 4 & 20 \\ 3 & -1 & 9 \end{array}\right]$

reduced row-echelon form: $\left[\begin{array}{cc:c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array}\right]$, so $\begin{matrix} x = 4 \\ y = 3 \end{matrix}$

All three methods can be extended to systems with more than two equations and two variables.

For additional resources, visit go.hrw.com and enter the keyword MB7 Parent.