BINOMIAL PROBABILITY

A **binomial experiment** consists of n independent trials whose outcomes are either successes or failures; the probability of success p is the same for each trial, and the probability of failure q is the same for each trial. Because there are only two outcomes, $p + q = 1$, or $q = 1 - p$. Below are some examples of binomial experiments:

Experiment	Success	Failure	P(success)	P(failure)
10 flips of a coin	Heads	Tails	$p = 0.5$	$q = 1 - p = 0.5$
100 rolls of a number cube	Roll a 3.	Roll any other number.	$p = \frac{1}{6}$	$q = \frac{5}{6}$

Binomial Probability

If a binomial experiment has n trials in which p is the probability of success and q is the probability of failure in any given trial, then the **binomial probability** that there will be exactly r successes is:

$$P(r) = {}_nC_r\, p^r q^{n-r}$$

success prob. failure prob.

A bowl contains 4 blue, 5 red and 1 white marble. (10 total) 3 marbles are picked. What is the probability of getting exactly 2 blue marbles?

${}_nC_r\, p^r q^{n-r}$

Without replacement

$$\frac{\underbrace{4C2}_{\text{Blue}}\ \underbrace{(6\ C1)}_{\text{other}}}{10\ C3}$$

$$\frac{6(6)}{120} = \frac{3}{10}$$

With replacement

success = getting a blue marble

P(success) = 4/10 = 0.4 P(failure) = 0.6

P (2 success out of 3 trials) =

$3C2\ (0.4)^2\ (0.6)^{1}$ ($3-2$)

$3\,(0.16)(0.6)$

$.288$

5 cards are drawn from a standard deck.
What is the probability of getting exactly 3 aces?

Without replacement

$$\frac{\underbrace{{}_4C_3}_{\text{aces}} \cdot \underbrace{{}_{48}C_2}_{\text{other}}}{{}_{52}C_5}$$

$$= \frac{4 \cdot 1128}{2598960}$$

$= .001736$

With replacement

P(failure) = 12/13

success = getting an ace

P(success) = 4/52 = 1/13

P(3 successes out of 5) =

$(5C3)\ (1/13)^3\ (12/13)^2$ ($5-3$)

$10\,(.000455166)(.852071006)$

$.003878338$

A number cube is rolled 20 times. What is the probability of getting a six exactly 12 times?

$P(\text{success}) = \frac{1}{6}$

$P(\text{failure}) = \frac{5}{6}$

$${}_{20}C_{12}\left(\frac{1}{6}\right)^{12}\left(\frac{5}{6}\right)^{20-12}$$

$125970\,(4.59\times10^{-10})(.232568039)$

$.000013459$

Jean usually makes three fourth of her free throws in basketball practice. Today, she tries 3 free throws. What is the probability that Jean will make exactly 1 of her free throws?

P(success = making a free throw) $= \frac{3}{4} = .75$

P(failure) $= \frac{1}{4} = .25$

$_3C_1 (.75)^1 (.25)^{3-1}$

3

.140625

Students are assigned randomly to 1 of 3 guidance counselors. What is the probability that Counselor Jenkins will get 2 of the next 3 students assigned?

The probability that the counselor will be assigned 1 of the 3 students is $\frac{1}{3}$ = P(success)

P(failure) $= \frac{2}{3}$

$P(r) = {}_nC_r p^r q^{n-r}$

$P(2) = {}_3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2}$ *Substitute 3 for n, 2 for r, $\frac{1}{3}$ for p, and $\frac{2}{3}$ for q.*

$= 3\left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) = \frac{2}{9} \approx 0.22$

The probability that Counselor Jenkins will get 2 of the next 3 students assigned is about 22%.

You make 4 trips to a drawbridge. There is a 1 in 5 chance that the drawbridge will be raised when you arrive. What is the probability that the bridge will be down for at least 3 of your trips?

P(success/bridge being down) $= \frac{4}{5} = .8$

P(failure) $= \frac{1}{5} = .2$

P(3) + P(4)

$_4C_3\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^1 + {}_4C_4\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)^0$

.4096 + .4096 = .8192

Jean usually makes three fourths of her free throws in basketball practice. Today, she tries 3 free throws. What is the probability that she will make at least 1 free throw?

P(1) + P(2) + P(3)

$x \geq 1$

$x < 1$

1 − P(0)

$_3C_0 (.75)^0 (.25)^{3-0}$

1 − .015625

.984375

Ellen takes a multiple-choice quiz that has 5 questions, with 4 answer choices for each question. What is the probability that she will get at least 2 answers correct by guessing?

P(correct/success) $= \frac{1}{4}$

P(failure) $= \frac{3}{4}$

2 or 3 or 4 or 5

$x \geq 2$

$x < 2$

0 or 1

1 − (P(0) + P(1))

$1 - \left({}_5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 + {}_5C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4\right)$

.237304688 + .395507813

1 − .632812501 = .3671875

Wendy takes a multiple-choice quiz that has 20 questions. There are 4 answer choices for each question. What is the probability that she will get at least 2 answers correct by guessing?

P(success) $= \frac{1}{4} = .25$

P(failure) $= \frac{3}{4} = .75$

1 − (P(0) + P(1))

$1 - \left({}_{20}C_0 (.25)^0 (.75)^{20} + {}_{20}C_1 (.25)^1 (.75)^{19}\right)$

1 − (.003171212 + .021141413)

.975687375

A machine has a 98% probability of producing a part within acceptable tolerance levels. The machine makes 25 parts an hour. What is the probability that there are 23 or fewer acceptable parts?

$P(\text{success}) = .98$

$P(\text{failure}) = .02$

$x \leq 23$

$x > 23$

24 or 25

$$1 - \left(P(24) + P(25) \right)$$

$$1 - \left({}_{25}C_{24} (.98)^{24} (.02)^{1} + {}_{25}C_{25} (.98)^{25} (.02)^{0} \right)$$

$$1 - \left(.307890168 + .60346473 \right)$$

$$= .08864512$$