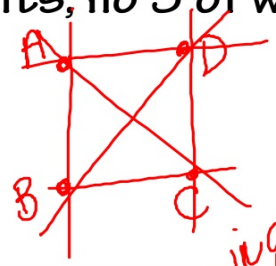
12-2 Combinations

March 18

A *combination* is a selection of objects in groups, without regard to order.

Given 4 points, no 3 of which are collinear. How many lines can b drawn?



6

$\overleftrightarrow{AC}$ $\overleftrightarrow{DC}$
$\overleftrightarrow{BD}$ $\overleftrightarrow{BC}$
$\overleftrightarrow{AD}$ $\overleftrightarrow{AB}$

$$\frac{4!}{2!(4-2)!} = \frac{4!}{2!2!}$$

r in group, n total

Combinations of r objects, taken from a group of n objects is:

$$\frac{n!}{r!(n-r)!} = {}_nC_r$$

n: total; r: in a group

2 standard 52-card deck, p. 708

Standard Deck of 52 Playing Cards:

Kinds

Suits													
Diamonds (Red):	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦	A♦
Hearts (Red):	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥	A♥
Clubs (Black):	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣	A♣
Spades (Black):	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠	A♠

a) How many different 7-card hands are possible? (groups)

$$_{52}C_7 = \frac{52!}{(7! \times 45!)} = 133,784,560$$

b) How many 7-card hands with 4 kings and any other 3 cards are possible?

17,296

4K and 3 others

$$\frac{4!}{4!\,0!} \; {}_4C_4 \times {}_{48}C_3 = 1 \times \frac{48!}{3!\,45!}$$

$$\frac{48 \cdot 47 \cdot 46 \cdot 45!}{3 \cdot 2 \cdot 1 \cdot 45!}$$

3 Twelve students apply for an ASB committee.
In how many ways can committees of 2 or 3 or 4 members be chosen?

2 or 3 or 4

$${}_{12}C_2 + {}_{12}C_3 + {}_{12}C_4$$

$$\frac{12!}{2!\,10!} + \frac{12!}{3!\,9!} + \frac{12!}{(4!\times 8!)}$$

$$\frac{12^{6}\cdot 11\cdot \cancel{10!}}{\cancel{2}\;\cancel{10!}} + \frac{\overset{2}{\cancel{12}}\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{3\cdot 2}\cdot \cancel{9!}} + \frac{\overset{5}{\cancel{12}}\cdot 11\cdot \cancel{10}\cdot 9\cdot \cancel{8!}}{\cancel{4\cdot 3\cdot 2}\cdot 1\,\cancel{8!}}$$

$$66 + 220 + 495$$

mutually exclusive events (no overlap)

1
66
220
495
181