12-1 Counting and Permutations

Fundamental Counting Principle

# of ways to complete task	= # of ways to choose 1st part	x # of ways to choose 2nd part	x # of ways to choose 3rd part	...

1) CA license plates have 1 digit, 3 letters, and 3 digits.

a) How many possible license plates?

$\frac{10}{\text{digit}} \times \frac{26}{3\text{ letters}} \times \frac{26}{} \times \frac{26}{} \times \frac{10}{3\text{ digits}} \times \frac{10}{} \times \frac{10}{}$

b) How many possible license plates that begin with a "3" if no letter can be repeated?

$\frac{1}{\text{"3"}} \times \frac{26}{3\text{ letters}} \times \frac{25}{} \times \frac{24}{} \times \frac{10}{\text{digits}} \times \frac{10}{} \times \frac{10}{}$

2) Five-question multiple choice quiz, with A, B, C, D answers. How many ways to complete the quiz?

$\frac{4}{1st} \times \frac{4}{} \times \frac{4}{} \times \frac{4}{} \times \frac{4}{5th} = 4^5$

A *permutation* is an ordering of objects.

3) You have homework in 5 classes tonight.

a) In how many different ways can you complete the homework?

$\frac{5}{1st} \times \frac{4}{2nd} \times \frac{3}{3rd} \times \frac{2}{4th} \times \frac{1}{5th} = 120$ (5!)

notation: ${}_5P_5$

permutation of n things taken n at a time — 5 factorial

b) In how many different ways can you do only 3 of the assignments?

$\frac{5}{1st} \times \frac{4}{2nd} \times \frac{3}{3rd} = 60$

notation: ${}_5P_3$

${}_5P_3$ ← at a time; 5 ← total

$= \frac{5!}{(5-3)!} \quad \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{5!}{2!}$

4) How many distinguishable permutations of the letters W, O, W?

WOW OWW WWO

$\frac{3!}{2!}$ ← W's

permutations of *n* objects with repetitions

$\frac{n!}{(r_1! \times r_2! \times r_3! \cdots)}$

n = total objects

$r_1, r_2, r_3 \ldots$ = repeated objects

5) Your dog has 8 puppies, 5 males and 3 females. One possible birth order is MMMMMFFF. How many different birth orders are possible?

$\frac{8!}{(5! \times 3!)} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56$

M F