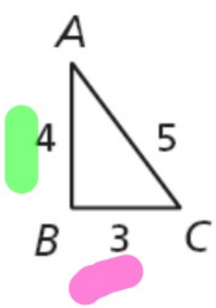
Section 9-7
Dilations

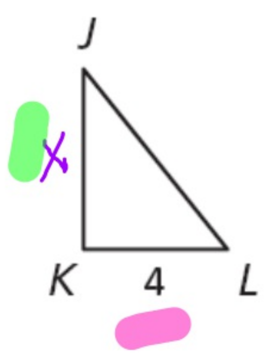
Warm Up

1. Translate the triangle with vertices *A*(2, –1), *B*(4, 3), and *C*(–5, 4) along the vector <2, 2>.

A'(4,1), *B'*(6, 5),*C*(–3, 6)

2. Δ*ABC* ~ Δ*JKL*. Find the value of *JK*.





$5\frac{1}{3}$

$\frac{4}{x} = \frac{3}{4}$ or $\frac{4}{3} = \frac{x}{4}$

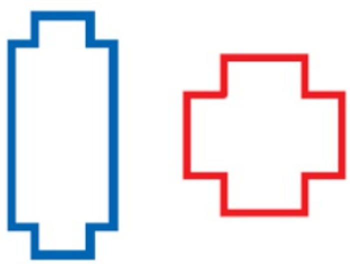
Recall that a dilation is a transformation that changes the size of a figure but not the shape. The image and the preimage of a figure under a dilation are similar. ~ (same ∠ and = ratio sides)

Because they change the size of the figure, dilations are NOT isometries

Example 1: Identifying Dilations

Tell whether each transformation appears to be a dilation. Explain.

A.



No; the figures are not similar.

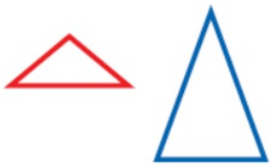
B.

Yes; the figures are similar and the image is not turned or flipped.

Check It Out! Example 1

Tell whether each transformation appears to be a dilation. Explain.

a.



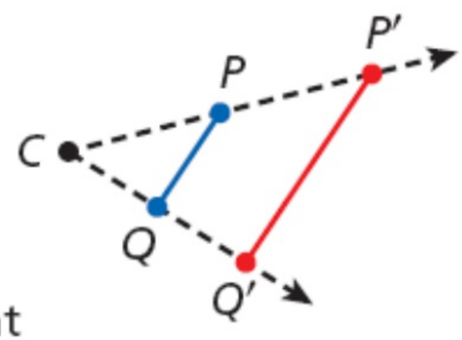
No, the figures are not similar.

b.

Yes, the figures are similar and the image is not turned or flipped.

Dilations

A dilation, or *similarity transformation*, is a transformation in which the lines connecting every point P with its image P' all intersect at a point C, called the **center of dilation**. $\frac{CP'}{CP}$ is the same for every point P.



The scale factor k of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.

A dilation enlarges or reduces all dimensions proportionally. A dilation with a scale factor greater than 1 is an **enlargement**, or *expansion*. A dilation with a scale factor greater than 0 but less than 1 is a **reduction**, or *contraction*.

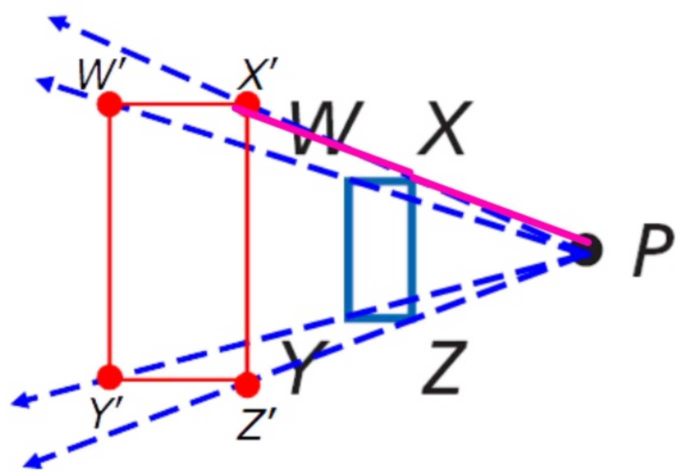
Example 2: Drawing Dilations

Copy the figure and the center of dilation *P*. Draw the image of $\Delta WXYZ$ under a dilation with a scale factor of 2.

Step 1 Draw a line through *P* and each vertex.

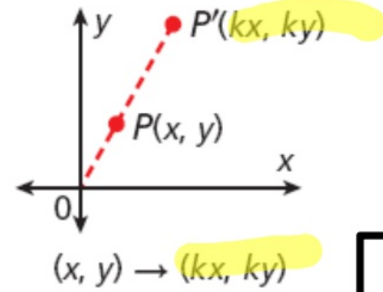
Step 2 On each line, mark twice the distance from *P* to the vertex.

Step 3 Connect the vertices of the image.

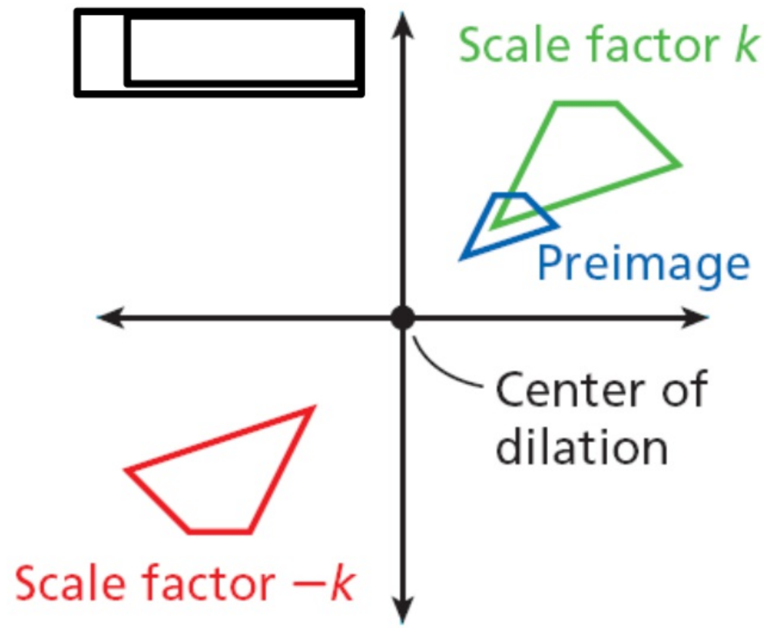


Dilations in the Coordinate Plane

If $P(x, y)$ is the preimage of a point under a dilation centered at the origin with scale factor k, then the image of the point is $P'(kx, ky)$.



If the scale factor of a dilation is negative, the preimage is rotated by 180°. For $k > 0$, a dilation with a scale factor of $-k$ is equivalent to the composition of a dilation with a scale factor of k that is rotated 180° about the center of dilation.



Example 4: Drawing Dilations in the Coordinate Plane

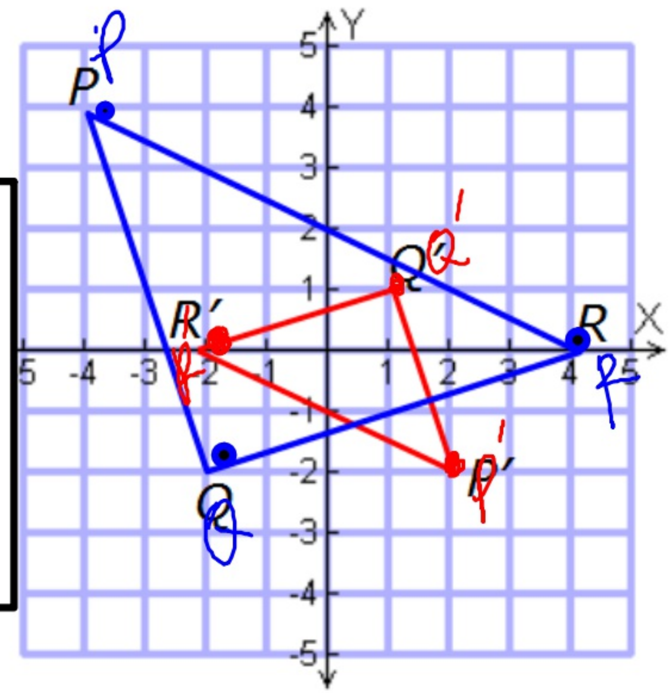
Draw the image of the triangle with vertices *P*(–4, 4), *Q*(–2, –2), and *R*(4, 0) under a dilation with a scale factor of $-\frac{1}{2}$ centered at the origin.

The dilation of (x, y) is $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.

$$P(-4,4) \rightarrow P'\left(-\frac{1}{2}(-4), -\frac{1}{2}(4)\right) = P'(2,-2)$$

$$Q(-2,-2) \rightarrow Q'\left(-\frac{1}{2}(-2), -\frac{1}{2}(-2)\right) = Q'(1,1)$$

$$R(4,0) \rightarrow R'\left(-\frac{1}{2}(4), -\frac{1}{2}(0)\right) = R'(-2,0)$$



Check It Out! Example 4

Draw the image of the triangle with vertices *R*(0, 0), *S*(4, 0), *T*(2, -2), and *U*(−2, −2) under a dilation centered at the origin with a scale factor of $-\frac{1}{2}$.

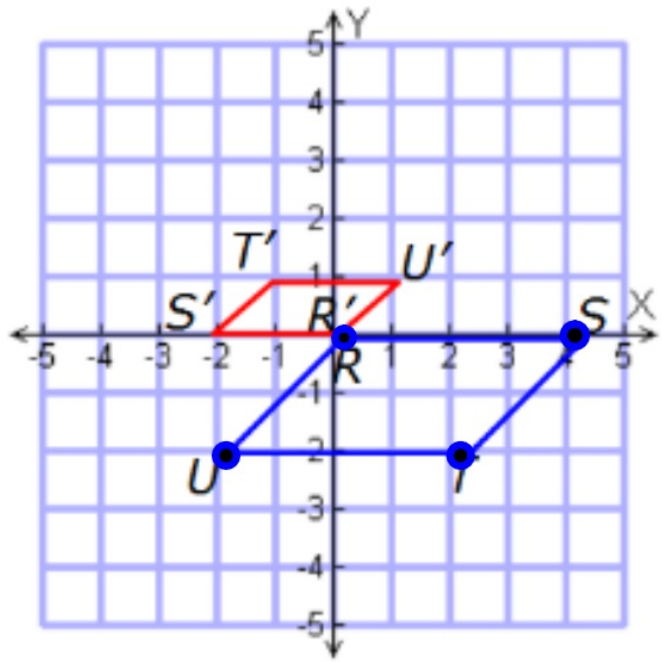
The dilation of (*x*, *y*) is $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.

$$R(0,0) \rightarrow R'\left(-\frac{1}{2}(0), -\frac{1}{2}(0)\right) = R'(0,0)$$

$$S(4,0) \rightarrow S'\left(-\frac{1}{2}(4), -\frac{1}{2}(0)\right) = S'(-2,0)$$

$$T(2,-2) \rightarrow T'\left(-\frac{1}{2}(2), -\frac{1}{2}(-2)\right) = T'(-1,1)$$

$$U(-2,-2) \rightarrow U'\left(-\frac{1}{2}(-2), -\frac{1}{2}(-2)\right) = U'(1,1)$$

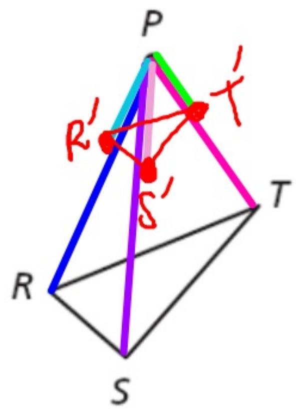


Lesson Quiz: Part I

1. Tell whether the transformation appears to be a dilation.

yes

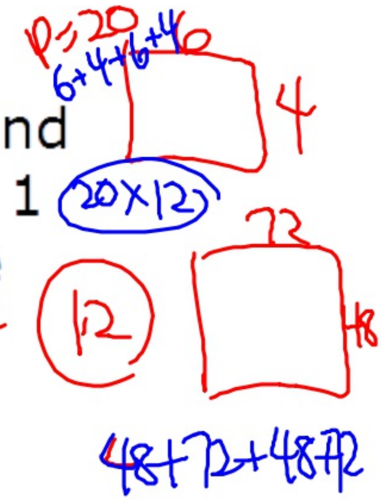
2. Copy ΔRST and the center of dilation. Draw the image of ΔRST under a dilation with a scale of $\frac{1}{3}$.



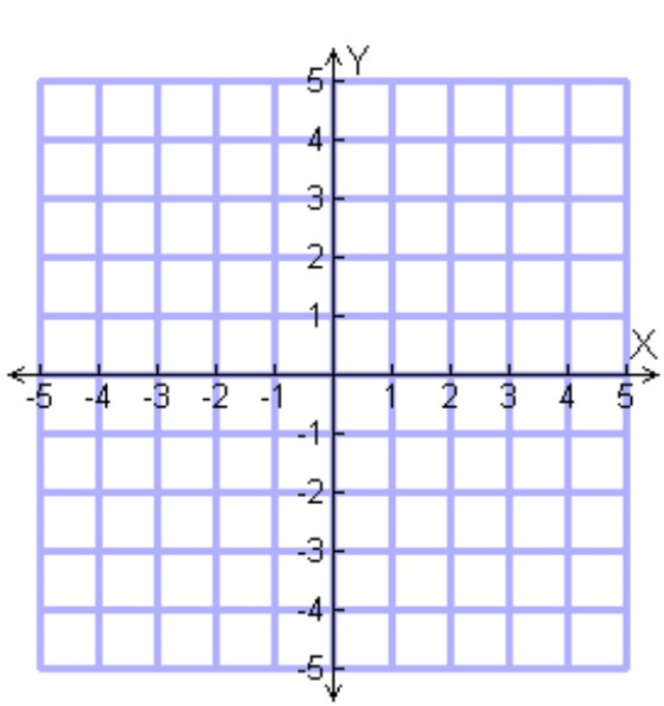
Lesson Quiz: Part II

3. A rectangle on a transparency has length 6cm and width 4 cm and with 4 cm. On the transparency 1 cm represents 12 cm on the projection. Find the perimeter of the rectangle in the projection.

240 cm



4. Draw the image of the triangle with vertices *E*(2, 1), *F*(1, 2), and *G*(−2, 2) under a dilation with a scale factor of −2 centered at the origin.



HW 9.7a Online

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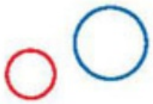
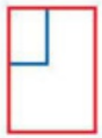
GUIDED PRACTICE

1. **Vocabulary** What are the *center of dilation* and scale factor for the transformation $(x, y) \rightarrow (3x, 3y)$?

Tell whether each transformation appears to be a dilation.

2. 

3. 

4. 

5. 

Copy each triangle and center of dilation *P*. Draw the image of the triangle under a dilation with the given scale factor.

6. Scale factor: 2

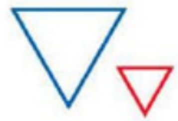
7. Scale factor: $\frac{1}{2}$

Draw the image of the figure with the given vertices under a dilation with the given scale factor centered at the origin.

9. $A(1, 0)$, $B(2, 2)$, $C(4, 0)$; scale factor: 2
10. $J(-2, 2)$, $K(4, 2)$, $L(4, -2)$, $M(-2, -2)$; scale factor: $\frac{1}{2}$
11. $D(-3, 3)$, $E(3, 6)$, $F(3, 0)$; scale factor: $-\frac{1}{3}$
12. $P(-2, 0)$, $Q(-1, 0)$, $R(0, -1)$, $S(-3, -1)$; scale factor: -2

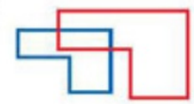
PRACTICE AND PROBLEM SOLVING

Tell whether each transformation appears to be a dilation.

13. 

14. 

15. 

16. 

Draw the image of the figure with the given vertices under a dilation with the given scale factor centered at the origin.

20. $M(0, 3)$, $N(6, 0)$, $P(0, -3)$; scale factor: $-\frac{1}{3}$
21. $A(-1, 3)$, $B(1, 1)$, $C(-4, 1)$; scale factor: -1
22. $R(1, 0)$, $S(2, 0)$, $T(2, -2)$, $U(-1, -2)$; scale factor: -2
23. $D(4, 0)$, $E(2, -4)$, $F(-2, -4)$, $G(-4, 0)$, $H(-2, 4)$, $J(2, 4)$; scale factor: $-\frac{1}{2}$

Homework:
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