

Lesson 2 - 4

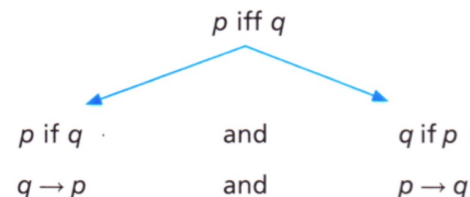
Biconditional Statements and Definitions Going Deeper

Essential question: How can you analyze the truth of a biconditional statement?

When you combine a conditional statement and its converse, you create a statement.

A statement is a statement that can be written in the form " p q ." This means and .

You have seen *If p , then q* and the notation $p \rightarrow q$. The notation is shorthand for and means a pair of statements.



The conjunction of these two statements is a biconditional statement. It is true only if both parts are true. It is false if either part is false.

$$p \leftrightarrow q \text{ means } p \rightarrow q \text{ and } q \rightarrow p$$

Writing Math

The biconditional " p if and only if q " can also be written as " p iff q " or $p \leftrightarrow q$.

CC.MP.3

EXAMPLE Analyzing Biconditionals and Definitions

A student defined *square* this way:

A polygon is a square if and only if it has four right angles.

A Write the biconditional as a pair of conditional statements.

If a polygon _____, then the polygon has four right angles.

If a polygon _____, then the polygon is a square.

Write the conditional statement and converse within the biconditional.

A solution is neutral \leftrightarrow its pH is 7.

For a biconditional statement to be true, the conditional statement and its must be true. If either the conditional or the converse is , then the biconditional statement is .

REFLECT

1a. Why is it necessary to prove only one of the two parts of a biconditional is false to conclude that the biconditional is false?

Determine if the biconditional is true. If false, give a counterexample.

A rectangle has side lengths of 12 cm and 25 cm if and only if its area is 300 cm².

If a rectangle has side lengths of 12 cm and 25 cm, then its area is 300 cm². *The conditional is*

If a rectangle's area is 300 cm², then it has side lengths of 12 cm and 25 cm. *The converse is*

Determine if the biconditional is true. If false, give a counterexample.

$$y = -5 \leftrightarrow y^2 = 25$$

Conditional: If then *The conditional is*

Converse: If then *The converse is*

An angle is a right angle iff its measure is 90°.

Conditional: If an angle is a right angle, then its measure is 90°. *The conditional is*

Converse: If the measure of an angle is 90°, then it is a right angle. *The converse is*

Example 4: Writing Definitions as Biconditional Statements

Write each definition as a biconditional.

A. A pentagon is a five-sided polygon.

B. A right angle measures 90°.

Lesson Quiz

1. For the conditional "If an angle is right, then its measure is 90°," write the converse and a biconditional statement.

2. Determine if the biconditional "Two angles are complementary if and only if they are both acute" is true. If false, give a counterexample.

3. Write the definition "An acute triangle is a triangle with three acute angles" as a biconditional.