6-5: Factoring Polynomials and Finding Zeros

ex. 1 Use synthetic division: $\left(x^3+2x^2-6x-15\right)\div(x+3)$

$x-k$

$k=-3$ | 1 2 -6 -15

-3 3 9

1 -1 -3 | -6 = R

$x^2-x-3-\frac{6}{x+3}$

If $f(x)=x^3+2x^2-6x-15$, find $f(-3)$.

$f(-3) = -27+2(9)+18-15 = -6$

Remainder Theorem: If a polynomial *f(x)* is divided by *x - k*, then the remainder *R = f(k).*

Factor Theorem: A polynomial *f(x)* has a factor *x - k* if and only if *f(k)* = 0.

ex. 2 Factor $f(x)=3x^3+13x^2+2x-8$ given that $f(-4)=0$

R

factor $x+4$

k $3x^3$

-4 | 3 13 2 -8

-12 -4 8

3 1 -2 | 0

$(x+4)(3x^2+x-2) = \boxed{(x+4)(3x-2)(x+1)}$

ex. 3 Is -2 a zero of $f(x)=x^4+5x^2-6$?

zero: value of x that makes $f(x)=0$

$f(-2) = 16+5(4)-6 = 30$ NO

$f(-2)\neq 0$

ex. 4 **Given one zero of *f(x)*, find the other zeros.**

$f(x) = x^3 - 3x^2 + x + 1$; 1 ← zero

x^3

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 1 & 1 \\ & & 1 & -2 & -1 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$f(x) = x^2 - 2x - 1$

$0 = x^2 - 2x - 1$

$1 + 1 = x^2 - 2x + 1$

$\sqrt{2} = \sqrt{(x-1)^2}$

$x = 1 \pm \sqrt{2}$

$x = \dfrac{2 \pm \sqrt{4 - 4(-1)}}{2}$

$\dfrac{2 \pm 2\sqrt{2}}{2}$