10 Review C

1. $10\sin 5x\cos 5x = y$ Let $\theta = 5x$

$10\sin\theta\cos\theta = y$ Divide by 5

$2\sin\theta\cos\theta = y/5$

$\sin 2\theta = y/5 \rightarrow y = 5\sin 2\theta$

$= 5\sin 2(5x)$

$= 5\sin 10x$

2. $\dfrac{\tan 195^\circ + \tan 15^\circ}{1 - \tan 195^\circ \tan 15^\circ} =$ → tan formula; keep top

$\tan(195^\circ + 15^\circ) = \tan 210^\circ = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

3. $\sin 165^\circ\cos 15^\circ + \cos 165^\circ \sin 15^\circ$ → sin formula; keep sign

$\sin(165^\circ + 15^\circ) = \sin 180^\circ$ (−1, 0) C, S

$= 0$

4. $\cos^2 75^\circ - \sin^2 75^\circ$ $\theta = 75^\circ$

$\cos^2\theta - \sin^2\theta = \cos 2\theta = \cos 2(75^\circ)$

$= \cos 150^\circ = -\dfrac{\sqrt{3}}{2}$

5. $\cos(45^\circ - x) - \cos(45^\circ + x)$

$\cos 45^\circ\cos x + \sin 45^\circ \sin x - (\cos 45^\circ\cos x - \sin 45^\circ \sin x)$

$\cos 45^\circ\cos x + \sin 45^\circ \sin x - \cos 45^\circ\cos x + \sin 45^\circ \sin x$

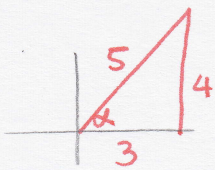
$\dfrac{\sqrt{2}}{2}\sin x \qquad + \dfrac{\sqrt{2}}{2}\sin x$

$= \dfrac{2\sqrt{2}}{2}\sin x = \sqrt{2}\sin x$

6. $\csc(90^\circ - x) = \dfrac{1}{\sin(90^\circ - x)} = \dfrac{1}{\sin 90^\circ\cos x - \cos 90^\circ \sin x}$

(0, 1) C, S

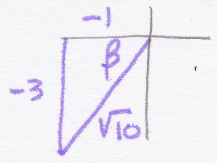
$= \dfrac{1}{1(\cos x) - 0(\sin x)} = \dfrac{1}{\cos x} = \sec x$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$\tan\alpha = \frac{4}{3}$



$\sin\beta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\cos\beta = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$

$\tan\beta = 1$

7 $\cos(\alpha+\beta) = \cos\alpha\cos\beta \ominus \sin\alpha\sin\beta$

$\frac{3}{5}\left(\frac{-\sqrt{10}}{10}\right) - \frac{4}{5}\left(\frac{-3\sqrt{10}}{10}\right) = \frac{-3\sqrt{10}+12\sqrt{10}}{50} = \frac{9\sqrt{10}}{50}$

8 $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} \approx 0.9659258263$

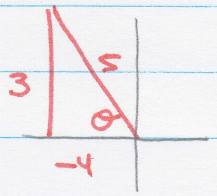
9 $\cos 15^\circ$ is + because 15° is in QI

$\theta = 30^\circ$

$\cos 15^\circ = \cos\frac{\theta}{2} = +\sqrt{\frac{1+\cos\theta}{2}} = +\sqrt{\frac{1+\cos 30^\circ}{2}}$

$\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2} \approx 0.9659258$

10



$\sin\theta = 3/5$

$\cos\theta = -4/5$

$90^\circ < \theta < 180^\circ$

$45^\circ < \frac{\theta}{2} < 90^\circ \rightarrow \frac{\theta}{2}$ is in QI

10. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$

11 $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(-\frac{4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \frac{7}{25}$

12 $\sin\frac{\theta}{2}$ is + because $\frac{\theta}{2}$ is in QI

$= +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = +\sqrt{\frac{9}{5}\cdot\frac{1}{2}} = \sqrt{\frac{9}{10}\cdot\frac{10}{10}}$

$= \frac{3\sqrt{10}}{10}$

13 $\tan\theta = -\frac{1}{3}$ $\quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-1/3)}{1-(-1/3)^2} = \frac{-2/3}{1-1/9}$

$$-\frac{2}{3} \div \frac{8}{9} = -\frac{2}{3}\cdot\frac{9}{8} = -\frac{3}{4}$$

14 $\tan\left(\frac{5\pi}{4} - \theta\right) = \frac{\tan\frac{5\pi}{4} - \tan\theta}{1+\tan\frac{5\pi}{4}\cdot\tan\theta} = \frac{-1-(-\frac{1}{3})}{1+(1)(-\frac{1}{3})} = \frac{\frac{4}{3}}{\frac{2}{3}}$

$$\frac{4}{3}\cdot\frac{3}{2} = 2$$

15. $\frac{\sin 2A}{\cos 2A+1} = \frac{2\sin A\cos A}{2\cos^2 A - 1 + 1} = \frac{2\sin A \cancel{\cos A}}{2\cos^2 A} = \frac{\sin A}{\cos A} = \tan A$

16 $\sin(\alpha+\beta)$ $\quad -\sin(\alpha-\beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta \quad -(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

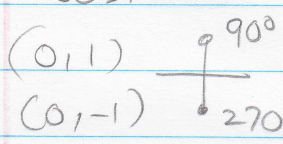
$\sin\alpha\cancel{\cos\beta} + \cos\alpha\sin\beta - \sin\alpha\cancel{\cos\beta} + \cos\alpha\sin\beta$

$2\cos\alpha\sin\beta$

17 $\sin 2x + \cos x = 0$

$2\sin x\cos x + \cos x = 0 \rightarrow \cos x(2\sin x + 1) = 0$

$\cos x = 0$



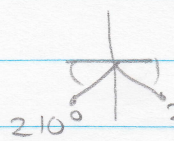
$x = 90° + n\cdot 180°$

$n=0$ $x = 90°$

$n=1$ $x = 270°$

$n=2$ $x = \cancel{450°}$

$2\sin x + 1 = 0 \rightarrow \sin x = -\frac{1}{2}$



$x = 210° + n\cdot 360°$

$n=0 \rightarrow x = 210°$

$n=1 \rightarrow$ too big

$x = 330° + n\cdot 360°$

$n=0 \rightarrow x = 330°$

$n=1 \rightarrow$ too big

18 $\cos 2x = 3\cos x + 1$

$2\cos^2 x - 1 - 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x - 2 = 0$

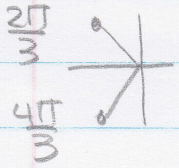
$2\cos x \quad +1$

$1\cos x \quad -2$

$(2\cos x + 1)(\cos x - 2) = 0$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$



$x = \frac{2\pi}{3} + 2n\pi$

$n=0 \rightarrow \frac{2\pi}{3}; \frac{4\pi}{3}$

$n=1$ too big

$\cos x - 2 = 0$

$\cos x = 2$

$x = \frac{4\pi}{3} + 2n\pi$ not poss.

19. $\cos 2x \cos x - \sin 2x \sin x = -1$ → cos formula; chg sign

$\cos(2x + x) = -1$

$\cos 3x = -1$ $\quad \theta = 3x$

$\cos\theta = -1$ → $(-1, 0)$

$\theta = 180° + n \cdot 360°$

$3x = 180° + n \cdot 360°$

$x = 60° + n \cdot 120°$

$n=0$ $x = 60°$

$n=1$ $x = 180°$

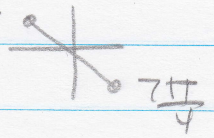
$n=2$ $x = 300°$

$n=3$ too big

20 $\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} = -1$ → tan formula keeptop

$\tan(2x + x) = -1 \rightarrow \tan 3x = -1 \quad \theta = 3x$

$\tan\theta = -1$



$\theta = \frac{3\pi}{4} + n\pi$

$3x = \frac{3\pi}{4} + n\pi$

$x = \frac{3\pi}{12} + \frac{n\pi}{3}\cdot\frac{4}{4} = \frac{3\pi}{12} + \frac{4n\pi}{12}$

$n=0$; $x = 3\pi/12 = \pi/4$

$n=1$; $x = 7\pi/12$

$n=2$; $x = 11\pi/12$

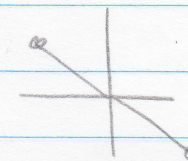
$n=3$; $x = 15\pi/12 = 5\pi/4$

$n=4$; $x = 19\pi/12$

$n=5$; $x = 23\pi/12$

$n=6$; $x = 27\pi/12$ → too big

OR: $\tan\theta = -1$



$\theta = \frac{3\pi}{4} + 2n\pi$

$3x = \frac{3\pi}{4} + 2n\pi$

$x = \frac{3\pi}{12} + \frac{2n\pi}{3}\cdot\frac{4}{4}$

$x = \frac{3\pi}{12} + \frac{8n\pi}{12}$

$\theta = \frac{7\pi}{4} + 2n\pi$

$3x = \frac{7\pi}{4} + 2n\pi$

$x = \frac{7\pi}{12} + \frac{2n\pi}{3}\cdot\frac{4}{4}$

$x = \frac{7\pi}{12} + \frac{8n\pi}{12}$

$n=0$	$3\pi/12$	$7\pi/12$
$n=1$	$11\pi/12$	$15\pi/12$
$n=2$	$19\pi/12$	$23\pi/12$
$n=3$	too big	