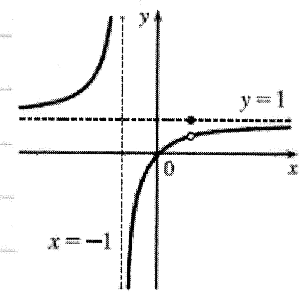
18.

$$f(x) = \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a=1$$



$f(1) = 1$

$$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{x^2-x}{x^2-1}$$

$$= \lim_{x\to1} \frac{x(x-1)}{(x+1)(x-1)}$$

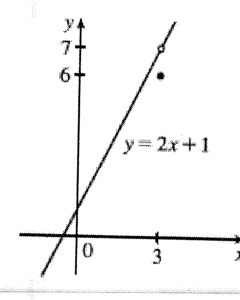
$$= \lim_{x\to1} \frac{x}{x+1}$$

$$= \frac{1}{2}$$

f is discontinuous at $x=1$ since
$\lim_{x\to1} f(x) \neq f(1)$

20.

$$f(x) = \begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases} \qquad a=3$$



$f(3) = 6$

$$\lim_{x\to3} f(x) = \lim_{x\to3} \frac{2x^2-5x-3}{x-3}$$

$$= \lim_{x\to3} \frac{(2x+1)(x-3)}{x-3}$$

$$= \lim_{x\to3} (2x+1)$$

$$= 7$$

f is discontinuous at $x=3$ since
$\lim_{x\to3} f(x) \neq f(3)$

31. $\lim_{x \to 4} \frac{5+\sqrt{x}}{\sqrt{5+x}}$ Let $f(x) = \frac{5+\sqrt{x}}{\sqrt{5+x}}$

$h(x) = 5$ is a polynomial so h is cont on $(-\infty, \infty)$
$g(x) = \sqrt{x}$ is a root function so g is cont on its domain which is $[0, \infty)$
$K(x) = \sqrt{5+x}$ is a root function so k is cont on its domain which is $[-5, \infty)$
Then $f(x) = \frac{h(x) \cdot g(x)}{K(x)}$ is cont on $[0, \infty)$
(Note: don't need to worry about dividing by zero because domain of f is $[0, \infty)$)

Since f is cont at $x = 4$
$$\lim_{x \to 4} f(x) = f(4) = \frac{5+\sqrt{4}}{\sqrt{5+4}} = \frac{7}{3}$$

33 $\lim_{x \to \frac{\pi}{4}} x\cos^2(x)$ Let $F(x) = x\cos^2(x)$

$g(x) = x$ is a polynomial so g is cont on $(-\infty, \infty)$
$h(x) = \cos(x)$ is a trig function so h is cont on its domain which is $(-\infty, \infty)$
then $F(x) = g(x) \cdot f(x) \cdot f(x)$ so F is cont on $(-\infty, \infty)$

Since F is cont at $x = \frac{\pi}{4}$ then
$$\lim_{x \to \frac{\pi}{4}} F(x) = F(\tfrac{\pi}{4}) = \tfrac{\pi}{4}\cos^2(\tfrac{\pi}{4}) = \tfrac{\pi}{8}$$

35. $f(x) = \begin{cases} x^2 & \text{if } x<1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$

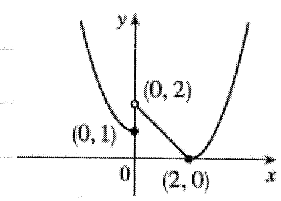
let $g(x) = x^2$ since g is a polynomial
g is cont on $(-\infty, 1)$
let $h(x) = \sqrt{x}$ since h is a root function
h is cont on $(1, \infty)$
Now we need to check that f is cont at $x=1$
$f(1) = \sqrt{1} = 1$

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} x^2 = 1 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} \sqrt{x} = 1 \end{aligned}\right\} \lim_{x\to 1} f(x) = 1$$

f is cont at $x=1$ since $\lim_{x\to 1} f(x) = f(1)$

Thus f is cont on $(-\infty, \infty)$

37. $f(x) = \begin{cases} 1+x^2 & \text{if } x \leq 0 \\ 2-x & \text{if } 0<x\leq 2 \\ (x-2)^2 & \text{if } x>2 \end{cases}$



⊛ to start we know...
$f(0) = 1$

$$\left.\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} (1+x^2) = 1 \\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} (2-x) = 2 \end{aligned}\right\} \begin{aligned} &\lim_{x\to 0} f(x) \text{ DNE} \\ &\text{since } \lim_{x\to 0} f(x) \neq \lim_{x\to 0^-} f(x \end{aligned}$$

f is discontinuous at $x=0$ since $\lim_{x\to 0} f(x)$ DNE

f is continuous at $x=0$ from the left
since $\lim_{x\to 0^-} f(x) = f(0)$

⊛ f is cont on $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$ since each piece is a polynomial

37 cont'd

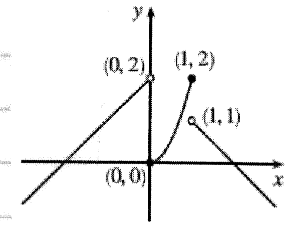
$f(2) = 0$

$$\left.\begin{array}{l} \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (2-x) = 0 \\ \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (x-2)^2 = 0 \end{array}\right\} \lim\limits_{x \to 2} f(x) = 0$$

f is continuous at $x=2$ since $\lim\limits_{x \to 2} f(x) = f(2)$

Now we can say f is cont on $(-\infty, 0) \cup (0, \infty)$

39. $f(x) = \begin{cases} x+2 & \text{if } x<0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x>1 \end{cases}$



to start we know f is cont on $(-\infty, 0)$, $(0,1)$, and $(1,\infty)$ sinch each piece of f is a polynomial.

Now look into $x=0$ and $x=1$

$f(0) = 0$

$\left.\begin{array}{l} \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x+2) = 2 \\ \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (2x^2) = 0 \end{array}\right\}$ $\lim\limits_{x\to 0} f(x)$ DNE since $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x$

f is discontinuous at $x=0$ since $\lim\limits_{x\to 0} f(x)$ DNE

f is continuous at $x=0$ from the right since $\lim\limits_{x\to 0^+} f(x) = f(0)$

$f(1) = 2$

$\left.\begin{array}{l} \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 2x^2 = 2 \\ \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (2-x) = 1 \end{array}\right\}$ $\lim\limits_{x\to 1} f(x)$ DNE since $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x$

f is discontinuous at $x=1$ since $\lim\limits_{x\to 1} f(x)$ DNE

f is continuous at $x=1$ from the left since $\lim\limits_{x\to 1^+} f(x) = f(1)$

Now we know f is cont on $(-\infty, 0) \cup [0,1] \cup (1,\infty)$