

# DIFFERENTIAL EQUATIONS FRQ SET # 1

- ① At time  $t, t \geq 0$ , the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius.

At  $t = 0$ , the radius of the sphere is 1 and at  $t = 15$ , the radius is 2. (The volume  $V$  of a sphere with radius  $r$  is  $V = (4/3)\pi r^3$ .)

- (a) Find the radius of the sphere as a function of  $t$ .
- (b) At what time  $t$  will the volume of the sphere be 27 times its volume at  $t = 0$ ?

- ② A certain rumor spreads through a community at the rate  $dy/dt = 2y(1 - y)$ , where  $y$  is the proportion of the population that has heard the rumor at time  $t$ .

- (a) What proportion of the population has heard the rumor when it is spreading the fastest?
- (b) If at time  $t = 0$  ten percent of the people have heard the rumor, find  $y$  as a function of  $t$ .
- (c) At what time  $t$  is the rumor spreading the fastest?

- ③ Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is,  $dy/dt = ky$ , where  $y$  is the amount of oil left in the well at any time  $t$ . Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

- (a) Write an equation for  $y$ , the amount of oil remaining in the well at any time  $t$ .
- (b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
- (c) In order not to lose money, at what time  $t$  should oil no longer be pumped from the well?

④ At any time  $t \geq 0$ , in days, the rate of growth of a bacteria population is given by  $y' = ky$ , where  $k$  is a constant and  $y$  is the number of bacteria present. The initial population is 1,000 and the population triples during the first 5 days.

- (a) Write an expression for  $y$  at any time  $t \geq 0$ .
- (b) By what factor will the population have increased in the first 10 days?
- (c) At what time  $t$ , in days, will the population have increased by a factor of 6?

⑤ Given the differential equation  $dy/dx = -xy/\ln y$ ,  $y > 0$ .

- (a) Find the general solution of the differential equation.
- (b) Find the solution that satisfies the condition that  $y = e^2$  when  $x = 0$ . Express your answer in the form  $y = f(x)$ .
- (c) Explain why  $x = 2$  is not in the domain of the solution found in part (b).

⑥ At time  $t = 0$ , a jogger is running at a velocity of 300 meters per minute. The jogger is slowing down with a negative acceleration that is directly proportional to time  $t$ . This brings the jogger to a stop in 10 minutes.

- (a) Write an expression for the velocity of the jogger at time  $t$ .
- (b) What is the total distance traveled by the jogger in that 10-minute interval?