GeoGebra

in
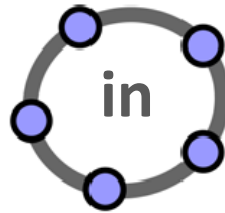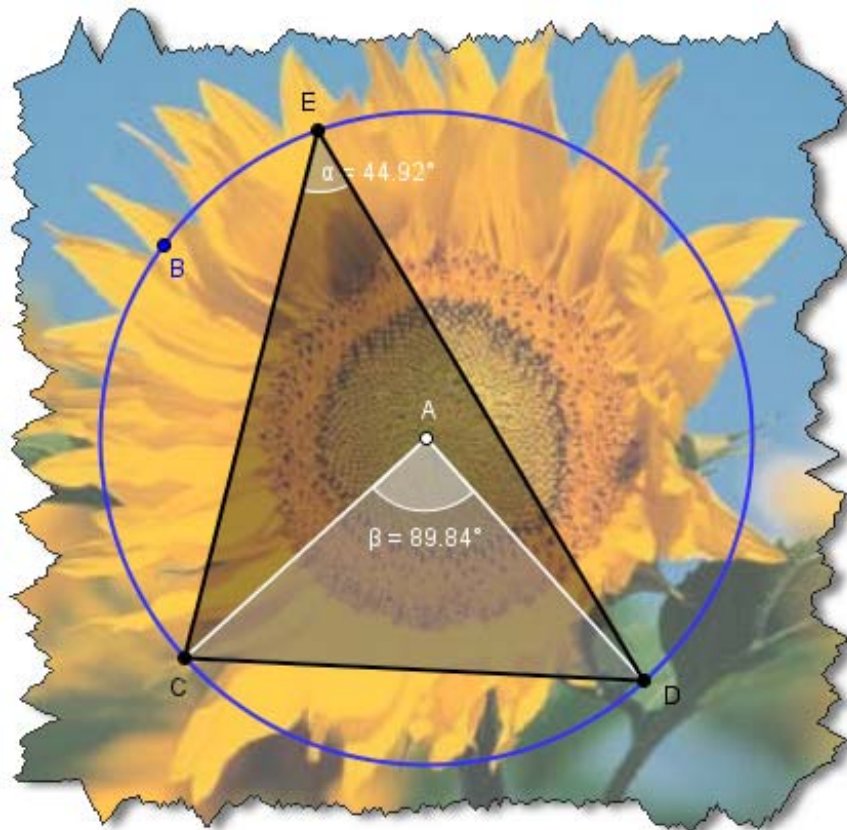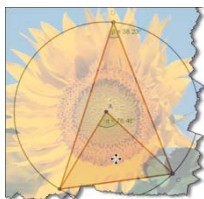
10 lessons

Gerrit Stols

Acknowledgements

GeoGebra is dynamic mathematics open source (free) software for learning and teaching mathematics in schools. It was developed by Markus Hohenwarter and an international team of programmers. They did a brilliant job and we as mathematics teachers and lecturers must salute them. GeoGebra combines geometry, algebra, statistics and calculus. You can download it for free.

Download GeoGebra from http://www.geogebra.org

For more information contact the author:

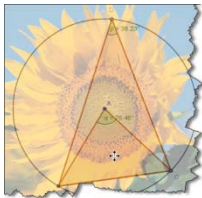**Gerrit Stols**
University of Pretoria
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gerrit.stols@up.ac.za
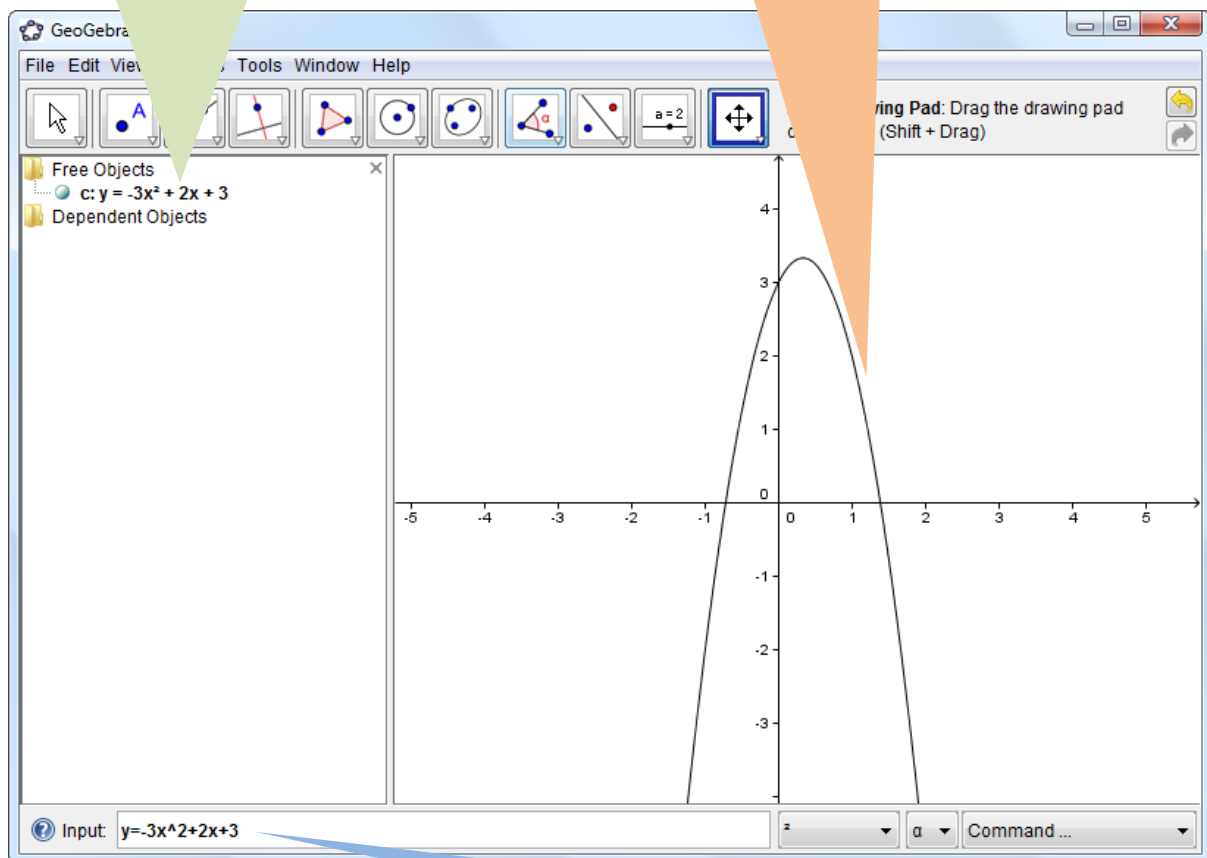+27 12 82 415 7583

Last modified: October 05, 2009

Contents

GeoGebra Interface

The GeoGebra basic interface is divided into three sections:
Input bar, Algebra View, and Graphic View.

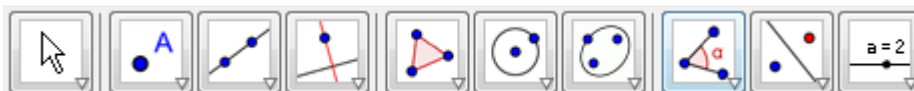ALGEBRA VIEW: Show and edit all the created objects and functions.
Just double click on equation to edit it.

GRAPHIC VIEW: Show and construct objects and the graphs of functions.

INPUT BAR: Create new objects, equations and functions
E.g. Construct the graph of $y = 3x^2 + 4x + 6$
Type: "y = 3x^2+4x+6" or "y = 3*x^2+4*x+6" and

Construction tools:

Menu:

File Edit View Options Tools Window Help

Menus

File Edit View Options Tools Window Help

File	Edit View Options Tools Window Help	
	New Window	Ctrl+N
	New	
	Open ...	Ctrl+O
	Save	Ctrl+S
	Save As ...	
	Export	
	Print Preview ...	Ctrl+P
	assignment 1.ggb	
	Circumference of earth.ggb	
	Trigonometric graphs transformations.ggb	
	Parabola transformations y = ax^2+bx+c.ggb	
	Close	Alt+F4

View	Options Tools Window Help	
	Axes	
	Grid	
	Algebra View	Ctrl+Shift+A
	Spreadsheet View	Ctrl+Shift+S
	Auxiliary Objects	
	Horizontal Splitting	
	Input Bar	
	Command List	
	Construction Protocol ...	
	Navigation Bar for Construction Steps	
	Refresh Views	Ctrl+F
	Recompute All Objects	Ctrl+R

Edit	View Options Tools Window Help	
	Undo	Ctrl+Z
	Redo	Ctrl+Y
	Delete	Delete
	Select All	Ctrl+A
	Select Current Layer	Ctrl+L
	Select Descendants	Ctrl+Shift+J
	Select Ancestors	Ctrl+J
	Graphics View to Clipboard	Ctrl+Shift+C
	Properties ...	Ctrl+E

Options	Tools Window Help	
	Point Capturing	▶
	Angle Unit	▶
	Rounding	▶
	Continuity	▶
	Point Style	▶
	Checkbox Size	▶
	Right Angle Style	▶
	Coordinates	▶
	Labelling	▶
	Font Size	▶
	Language	▶
	Drawing Pad ...	
	Save Settings	
	Restore Default Settings	

Tools	Window Help
	Create New Tool ...
	Manage Tools ...
	Customise Toolbar ...

2

Construction Tools

| ⓪ | ❶ | ❷ | ❸ | ❹ | ❺ | ❻ | ❼ | ❽ | ❾ | ❿ |

⓪

Icon	Label
▹	Move
▹	Rotate around Point
36 24 12	Record to Spreadsheet

❶

Icon	Label
•A	New Point
✕	Intersect Two Objects
• •	Midpoint or Centre

❷

Icon	Label
╱	Line through Two Points
╱	Segment between Two Points
╱	Segment with Given Length from Point
╱	Ray through Two Points
➚	Vector between Two Points
➚	Vector from Point

❸

Icon	Label
✛	Perpendicular Line
╱	Parallel Line
✕	Perpendicular Bisector
✦	Angle Bisector
◯	Tangents
◯	Polar or Diameter Line
∴	Best Fit Line
✗	Locus

❹

Icon	Label
△	Polygon
⬠	Regular Polygon

3

5

⊙	Circle with Centre through Point
⊙	Circle with Centre and Radius
⊙	Compasses
◯	Circle through Three Points
◠	Semicircle through Two Points
⌒	Circular Arc with Centre between Two Points
⌒	Circumcircular Arc through Three Points
△	Circular Sector with Centre between Two Point
⌒	Circumcircular Sector through Three Points

6

◯	Ellipse
⧖	Hyperbola
⌒	Parabola
◯	Conic through Five Points

7

∠α	Angle
∠α	Angle with Given Size
cm	Distance or Length
cm²	Area
◺	Slope

8

╱	Reflect Object in Line
∴	Reflect Object in Point
⌐	Reflect Point in Circle
⟳	Rotate Object around Point by Angle
➶	Translate Object by Vector
k	Enlarge Object from Point by Factor

9

a = 2	Slider
☑	Check Box to Show / Hide Objects
ABC	Insert Text
✿	Insert Image
a ? b / a = b	Relation between Two Objects

10

✛	Move Drawing Pad
🔍	Zoom In
🔍	Zoom Out
○	Show / Hide Object
A A	Show / Hide Label
🖌	Copy Visual Style
⌫	Delete Object

4

Polygons and Angles

① Let's hide the axes because we do not need them now:
In the **View** menu, click the **Axes** button.

View	Options	Tools	Window	Help
⊥ Axes				
▦ Grid				
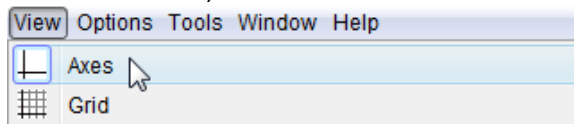
② Go to the Construction Tools: select the **Polygon** tool

| Polygon |
| Regular Polygon |

③ In the **Graphic View** area: create a triangle by selecting three points which will be the vertices of the polygon. (Remember to click the first point again in order to close the polygon)

④ Measure the interior angles:
Go to the **Construction Tools** & select the **Angle** tool
Select the three vertices counter clockwise (the measured angle second)

| Angle |
| Angle with Given Size |
| cm Distance or Length |
| cm² Area |
| Slope |

⑤ Calculate the sum of the interior angles

Input: α+β+γ

Go to the **Input bar** and type: $\alpha + \beta + \gamma$
Because there is no α, β and γ on the keyboard you have to select them from the dropdown list at the bottom:

α ▼
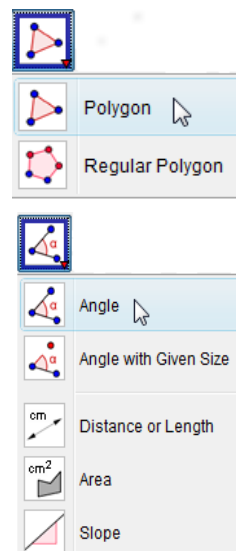
⑥ The sum of the angles (which is 180°) will appear in the **Algebra window**

GeoGebra

File Edit View Options Tools Window Help

Move Drawing Pad

Free Objects
- A = (2, 3.54)
- B = (-1.38, 0.58)
- C = (3.9, -0.14)

Dependent Objects
- a = 5.33
- b = 4.14
- c = 4.49
- poly1 = 9.03
- α = 76.1°
- β = 54.93°
- γ = 48.98°
- δ = 180°

A
α = 76.1°
γ = 48.98°
B
β = 54.93°
C

Input:

² ▼   γ ▼   Command ... ▼
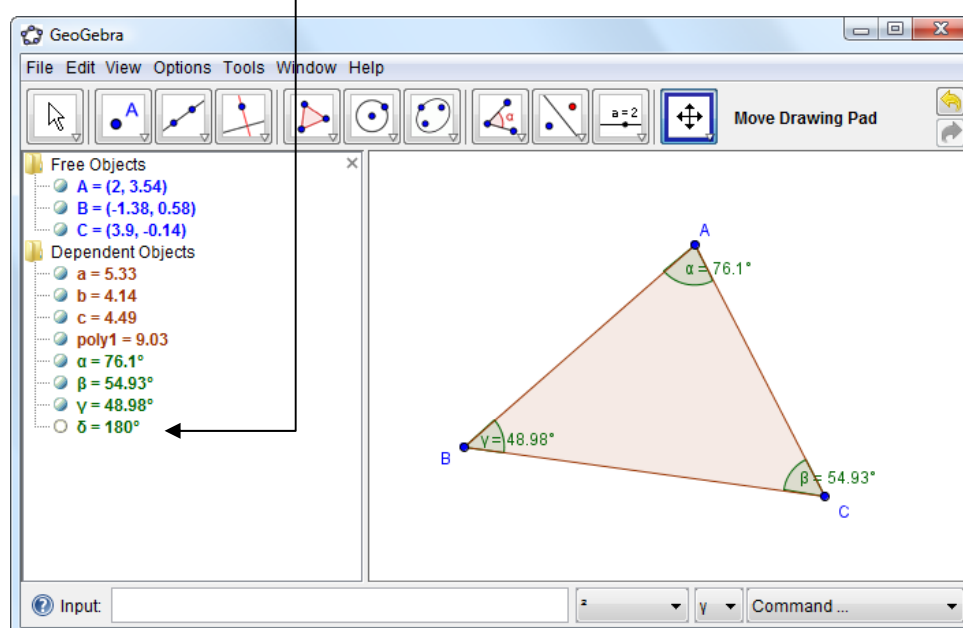
⑦ The question that any mathematician will ask is if this is a special case or is it always true?
Go to the Construction Tools and select the Arrow

Drag the vertices (A, B and C) of the triangle. GeoGebra will measure the angles imediately and also update the sum of the interior angles.

⑧ To save the construction: select the File tab and click the Save button

Construction of a regular polygon

① Hide the axes: select View / Click Axes button

② Select the Regular Polygon (click the small arrow in the bottom righthand corner of the icon to see all the options)

③ Create an equilateral triangle by selecting the two base points. A window will open: type the number of vertices (in the case of a triangle 3, square 4, regular pentagon 5) and hit the enter key.
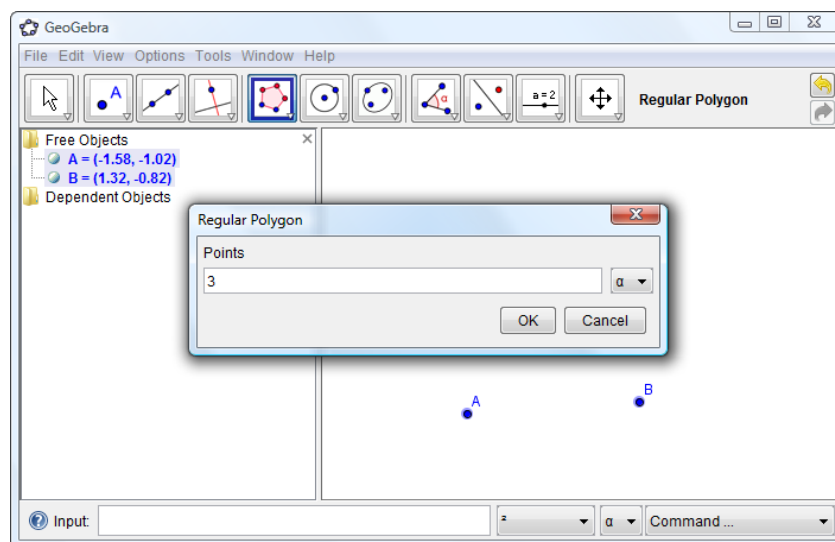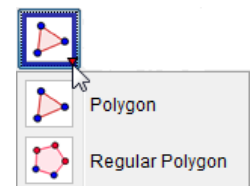
④ Measure an interior angle: select the Angle tool / Select the three vertices counter clockwise (the measured angle second).

⑤ Repeat the steps 1 to 5 to construct a square, regular polygon, etc.
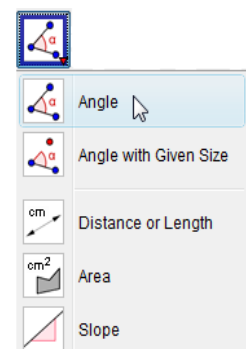
2 Perpendicular and Parallel lines

To construct a median, altitude, perdendicular bisector, and angle bisector of a triangle you need to know the following:

Construct the midpoint of a line segment

① Construct a line segment
Use the Segment between Two Points tool

② Construct the midpoint of the line segment:
Use the Midpoint or Centre tool

Line through Two Points
Segment between Two Points
Segment with Given Length from Point
Ray through Two Points
Vector between Two Points
Vector from Point

New Point
Intersect Two Objects
Midpoint or Centre
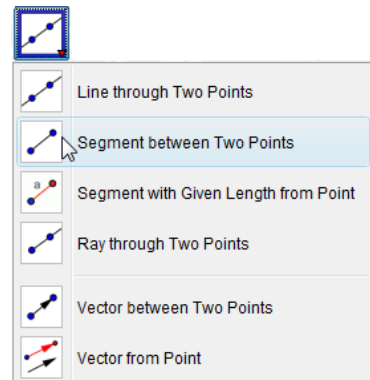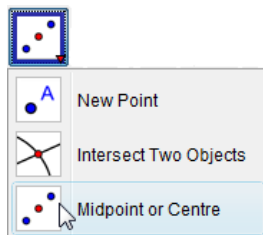
Construct a line perpendicular to a given line and through a given point

GeoGebra

File Edit View Options Tools Window Help

Move Drawing Pad

Free Objects
A = (-2.02, 2)
B = (2.36, 0.64)
Dependent Objects
C = (-0.46, 1.51)
a = 4.59
b: -4.38x + 1.36y = 4.05

Input: Command ...

① Construct a line segment using the
Segment between Two Points tool
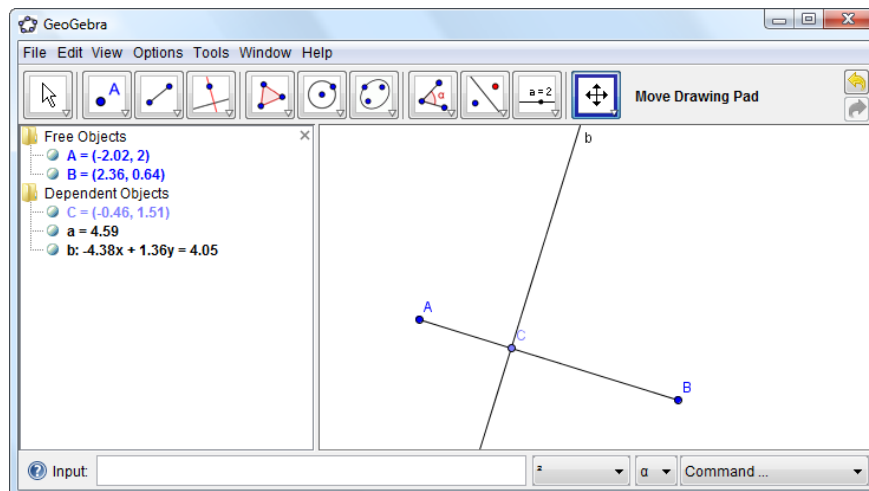
Segment between Two Points

② Construct a point on the line segment:
select the New Point tool and click on the line segment

New Point

③ Construct a perpendicular line: select the Perpendicular Line tool
and click on the point and the line.

Perpendicular Line

Perpendicular Line
Parallel Line
Perpendicular Bisector
Angle Bisector
Tangents
Polar or Diameter Line
Best Fit Line
Locus

7
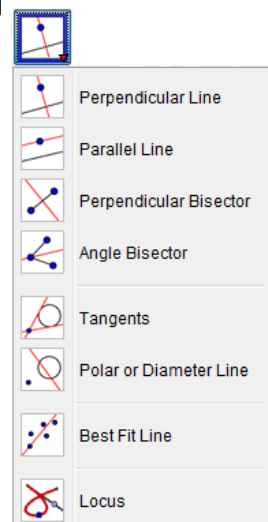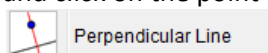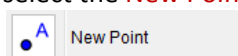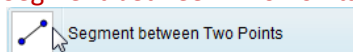
Construct a line parallel to a given line and through a given point:

① Construct a line segment using the
Segment between Two Points tool

② Construct any other point (not on the line segment) using the
New Point tool.

③ Construct a parallel line:
select the Parallel Line tool and click on the point and the line.
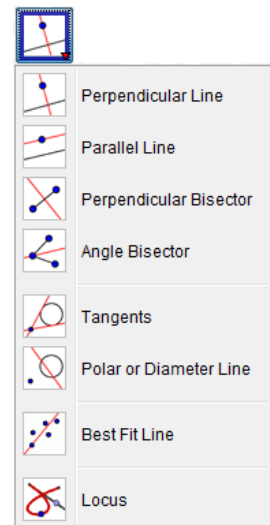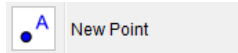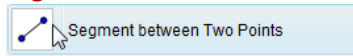
Construct the perpendicular bisector of a line segment:
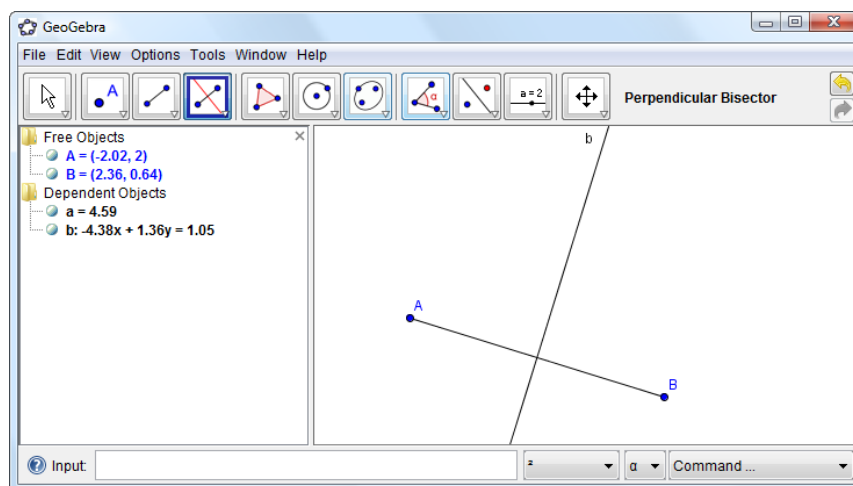
① Construct a line segment using the
Segment between Two Points tool

② Construct the perpendicular bisector of the line segment:
select the Perpendicular Bisector tool and click on the two end points of the line segment.
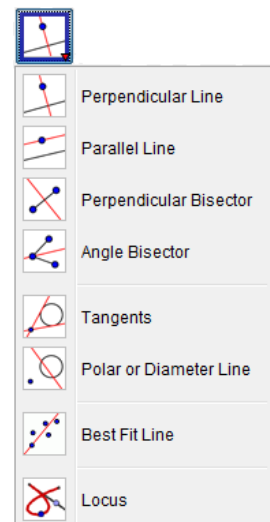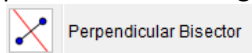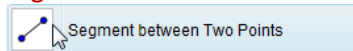
Construct the angle bisector of an angle:
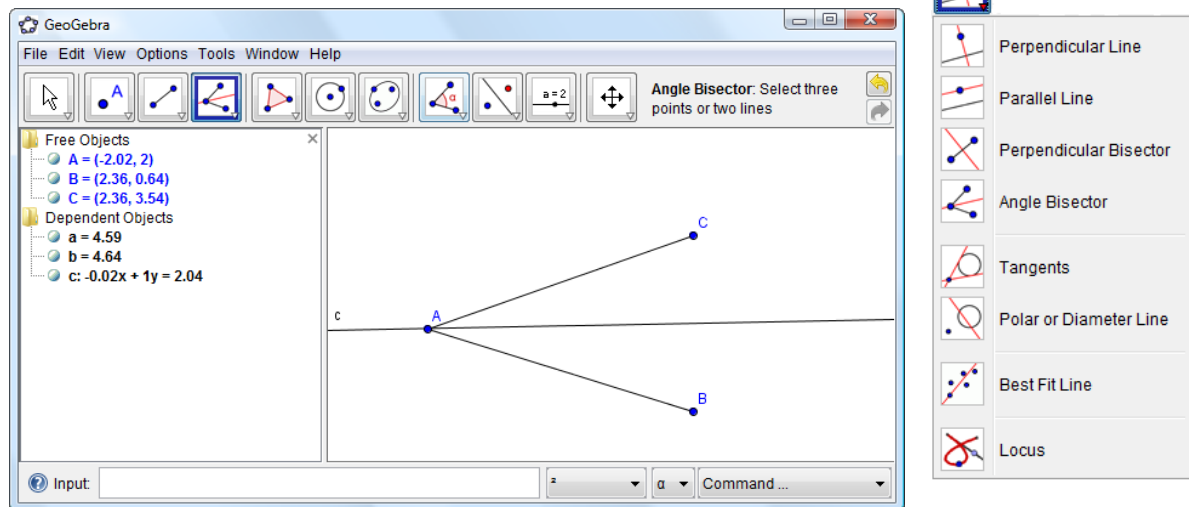
① Construct an angle by clicking the Segment between Two Points tool twice in succession.

② Construct the angle bisector:
select the Angle Bisector tool and click on the three points of the angle.

3 Drawing graphs

You can create and modify algebraic coordinates and equations by using the Input Bar at the bottom of the GeoGebra window.

Construction of the graphs of

a) $3x + 2y = 6$

b) $y = 3x^2 - 4x - 6$

c) $x^2 + 3x - 2y^2 - 3y = 25$

d) $y = \frac{3}{x-2} - 3$

e) $y = 2.3^{x+2} - 1$

① Click on the Input Bar on the bottom of the GeoGebra window.

| ? Input: | | ⌐ | ▾ | α | ▾ | Command ... | ▾ |

② Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation:

? Input: y=3x-4

$3x + 2y = 6$

? Input: 3x+2y=6

$y = 3x^2 - 4x - 6$

? Input: y=3x^2-4x-6

$x^2 + 3x - 2y^2 - 3y = 25$

? Input: x^2+3x-2y^2-3y=25

$y = \frac{3}{x-2} - 3$

? Input: y=3/(x-2)-3

$y = 2.3^{x+2} - 1$

? Input: y=2*3^(x+2)-1

③ Press the enter key on the keyboard after typing each equation.

You can create and modify trigonometric equations by using the Input Bar at the bottom of the GeoGebra window. You can use radian measure or degrees. The default mode is radian measure.

Construction of a trigonometric graphs (in radian measure)

① Click on the Input Bar on the bottom of the GeoGebra window.

Input: [_____] ² ▼ α ▼ Command ... ▼

② Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation:
a) $y = \sin x$

Input: y=sin(x)|

③ Press the enter key on the keyboard.

① Move the cursor to the *x*-axis. Press the right button on the mouse (right click).

② The following screen will appear:

Drawing Pad

- ▦ Axes
- ▦ Grid
- 🔍 Zoom ▸
- xAxis : yAxis ▸
- Show All Objects
- Standard View
- 🔲 Properties

Drawing Pad

Background Color: ▭

Axes | Grid

☑ Axes   Colour: ■   Line Style: → ▾

xAxis | yAxis

☑ Show   Ticks: | | | ▾

☑ Numbers   ☐ Distance: 1 ▾

Unit: ▾

Label: ▾

min: -5.5 ▾   max: 8.26 ▾

xAxis : yAxis = 1 : 1

Close

From the dropdown list select degrees:

Unit: ° ▾

Adjust the minimum and maximum *x*-values:

min: -360 ▾   max: 360 ▾
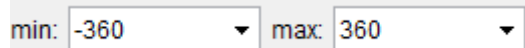
Change the distance between the *x*-values:

☑ Distance: 60 ▾
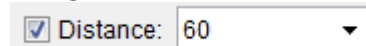
③ Close the window and click on the Input Bar on the bottom of the GeoGebra window.

④ Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation:

Input: y=sin(x°)

Use the dropdown list for the degree sign:

```
a
3
°
π
e
∞
sqrt(x)
cbrt(x)
°
```

12
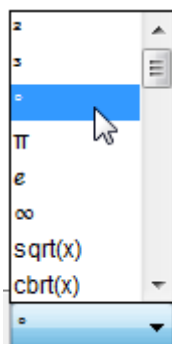
⑤ Press the enter key on the keyboard.
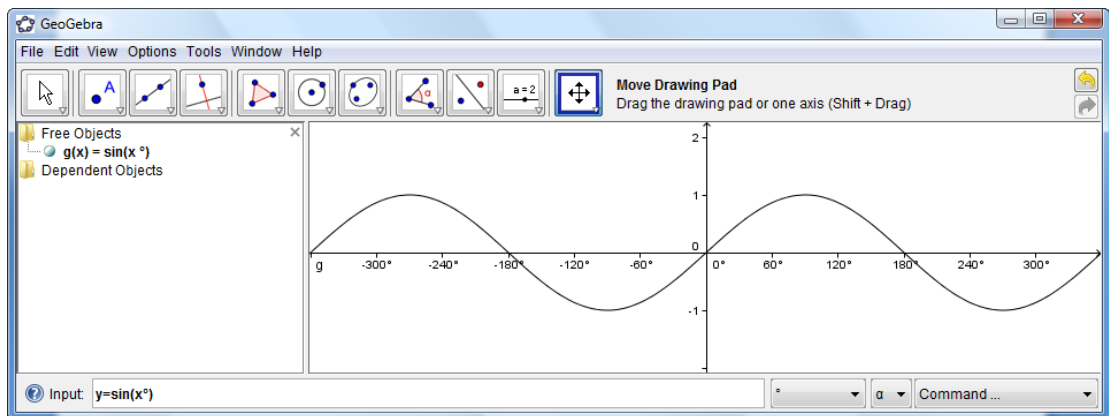
⑥ If you want to you can change the appearance of the graph:
Right click on the graph and select properties.
Click the Colour tab and select any colour.
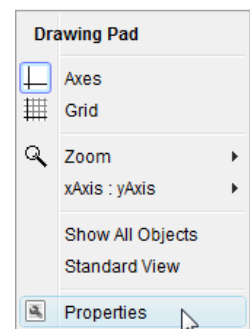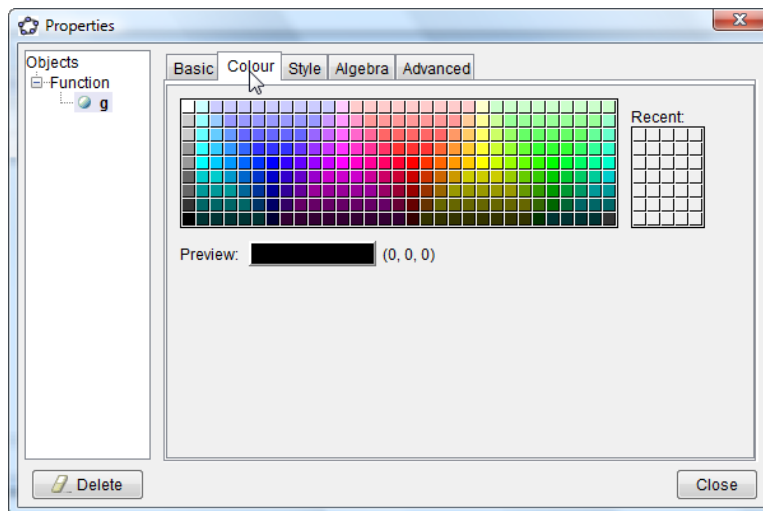
Click the style tab and select the line thickness and style.

Follow steps 1 to 4 in the previous section, but type:

❓ Input: f(x)=2*cos(x°)+1

❓ Input: g(x)=-tan(x°-30°)
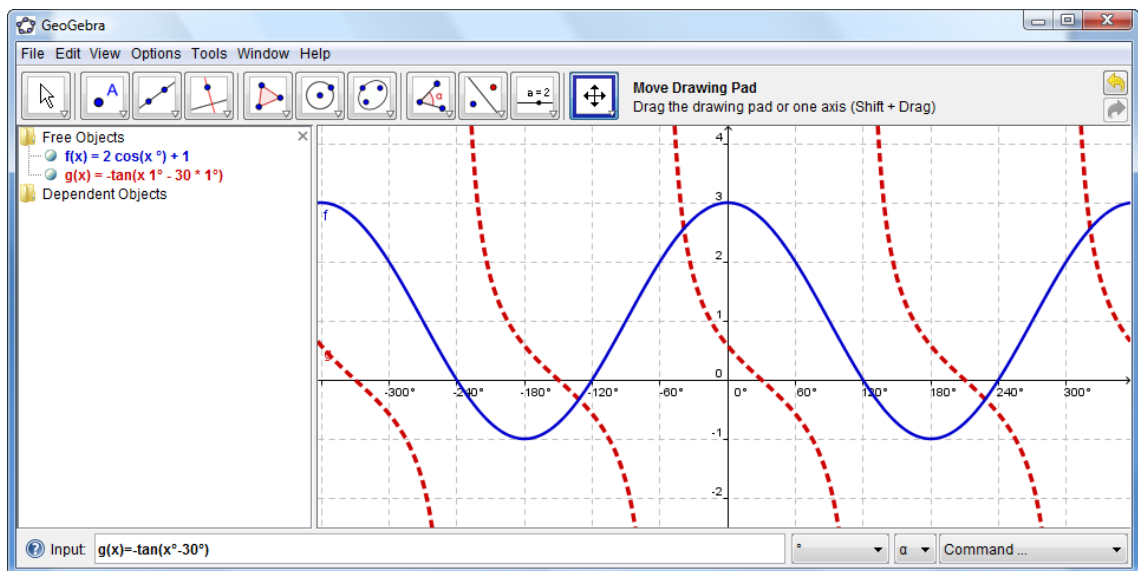
To add a grid as you noticed in the background of the previous sketch right click the *x*-axis and make the following selections:
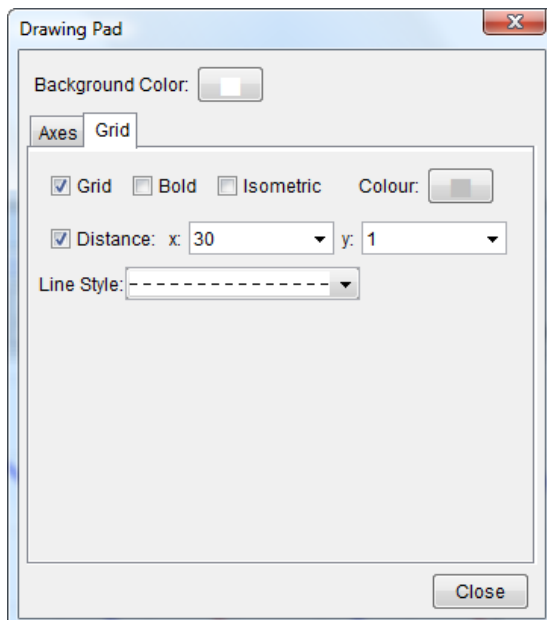
4 Use sliders to transform graphs

You can create and use sliders to change the coefficients of the equations of graphs.

① Select the Slider tool from the Construction Tools:

② Click where you want to locate the slider. The following window will appear:
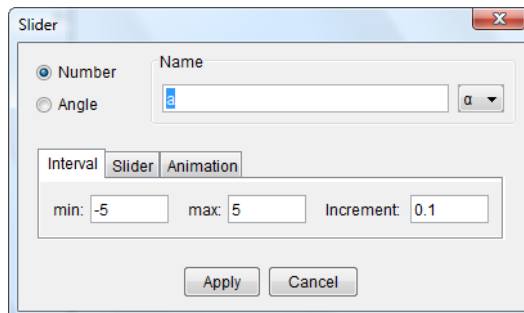
③ Click the Apply button and a slider will appear.

$a = 1$

④ Go to the Construction Tools and select the Arrow

Use the arrow to drag the point a on the slider. You will notice the value of point a on the slider will change.

⑤ Repeat steps 1 to 4 to create more sliders but rename them k, p and q.
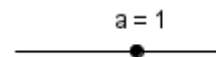
Using sliders in equations

⑥ Click on the Input Bar on the bottom of the GeoGebra window.

Input:

⑦ Use the keyboard and the dropdown menus (next to the Input Bar) to type the equation (on separate pages) and press enter:
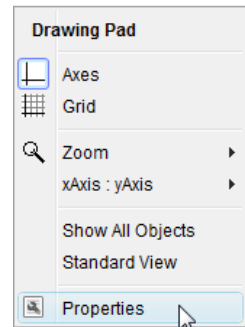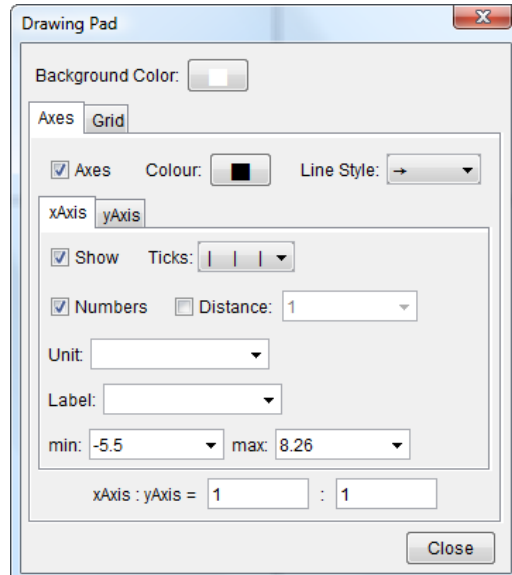
$y = a(x + p)^2 + q$:

Input: y=a*(x+p)^2+q

$y = a . 2^{x+p} + q$

Input: y=a*2^(x+p)+q

$y = \dfrac{a}{x+p} + q$

15

$$y = a. \sin k(x + p) + q$$

Input: y=a*sin(x+p)+q

⑧ Go to the Construction Tools and select the Arrow. Use the arrow to drag the points on the sliders. You will notice what the effect of the changing coefficient is on the graph.
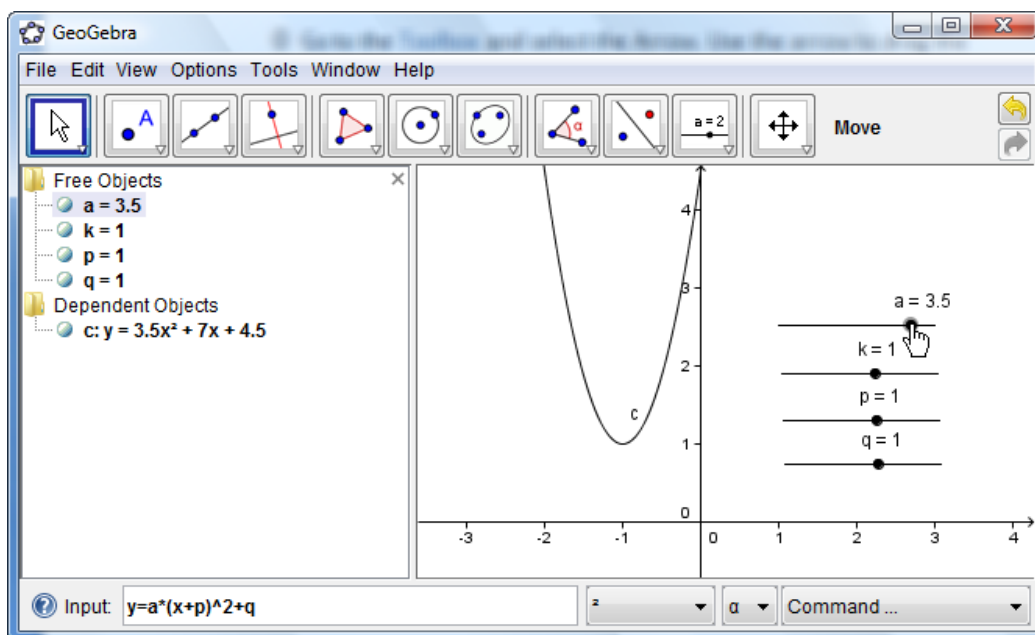
5 Transformation Geometry

You can do all the basic transformation geometry in GeoGebra. You will find all the transformation functions if you click the third icon from the left on the Toolbox:

Reflect Object in Line

Reflect Object in Point

Reflect Point in Circle

Rotate Object around Point by Angle

Translate Object by Vector

Enlarge Object from Point by Factor
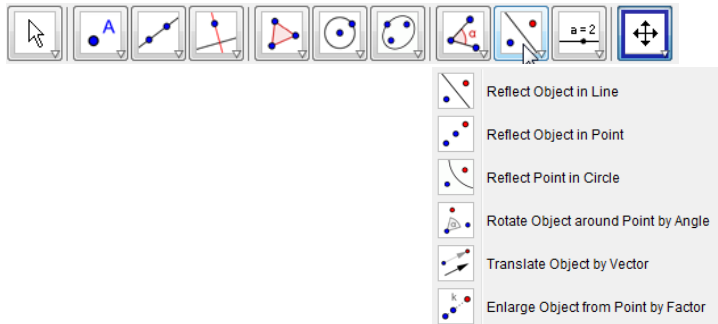
Constructing a reflection in the *x*-axis

Example: Determine the coordinates of the image of P(3; 2) if P is reflected across the *x*-axis.

① Select View / Grid in order to show the grid.

② Type: (3,2) in the Input bar

Input: (3,2)

③ Select the Reflect Object in Line option

Reflect Object in Line

④ Click the point (3, 2) and the *x*-axis. GeoGebra will in fact give you instructions needed on the right hand side of the Toolbox.

Reflect Object in Line
Select object to reflect, then line of reflection

Example: Determine the coordinates of the image of P(3; 2) if P is reflected across the line *y = x*.

① Type: (3,2) in the Input bar

⑦ Input:	(3,2)

② Type: *y = x* in the Input bar

⑦ Input:	y=x

③ Select the Reflect Object in Line option

 Reflect Object in Line

④ Click the point (3, 2) and the line *y* = x.

Example: Determine the coordinates of the image of P(3; 2) if P has been rotated about the origin through 90° in an anti clockwise direction.

① Type: (3,2) in the Input bar and press the enter key

⑦ Input:	(3,2)

② Type: (0,0) in the Input bar and press the enter key

⑦ Input:	(0,0)

③ Select the Rotate Object around Point by Angle option

 Rotate Object around Point by Angle

④ The following message will appear next to the Construction Tools:

Rotate Object around Point by Angle
Select object to rotate, then centre point and enter angle

Follow the instructions: Select the point (3, 2), then the centre (0, 0). The following screen will appear:
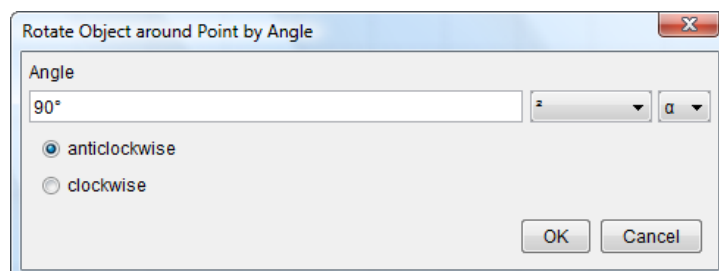
⑤ Type 90 and select the degree sign from the dropdown menu. Select "anticlockwise" and press the enter key.

18
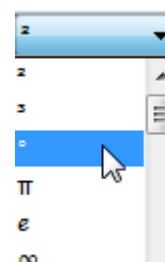
Translation of a point

Example: Determine the coordinates of the image of P(3; 2) if P has been translated 4 units horizontally to the left.

① Type: (3,2) in the Input bar and press the enter key

> ⑦ Input: (3,2)

② Select Vector between two points from the Toolbox.

Construct any vector of 4 units horizontally to the left.

- Line through Two Points
- Segment between Two Points
- Segment with Given Length from Point
- Ray through Two Points
- Vector between Two Points
- Vector from Point

③ Select the Translate Object by Vector option from the Toolbox.

> Translate Object by Vector
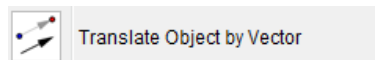
④ The following message will appear next to the Construction Tools:

> **Translate Object by Vector**
> Select object to translate, then vector

Follow the instructions: Select the point (3, 2), then the vector.

GeoGebra

File Edit View Options Tools Window Help

Free Objects
- A = (3, 2)
- B = (5, -1)
- C = (1, -1)
Dependent Objects
- A' = (-1, 2)
- u = (-4, 0)

Input: (0,0)   Command ...
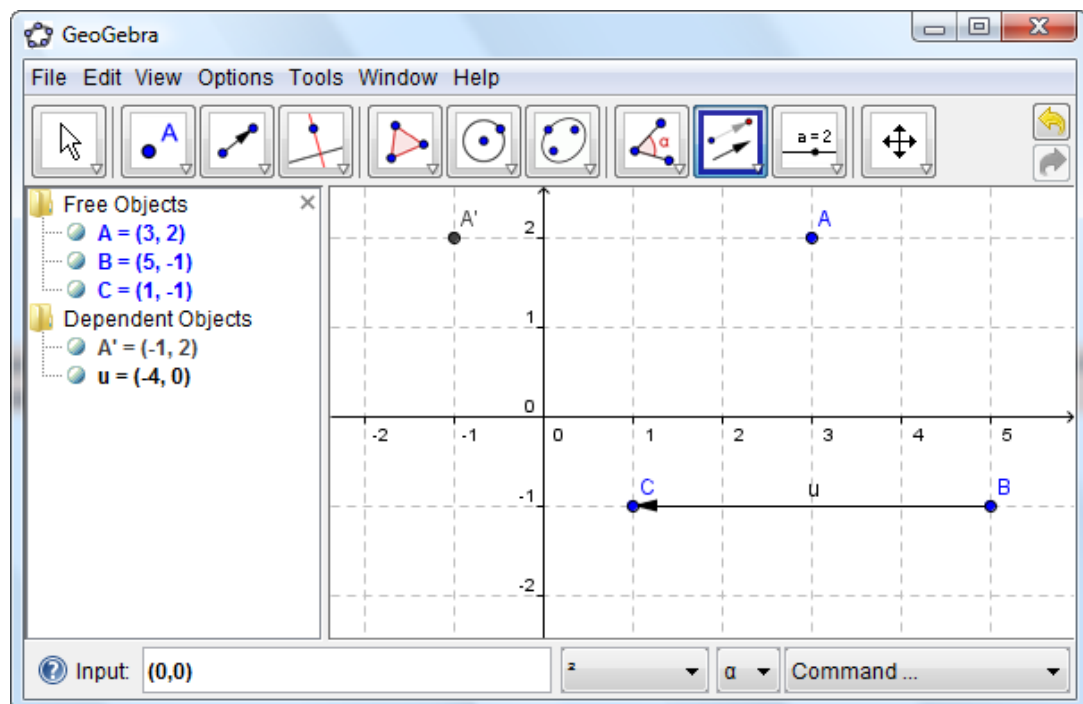
⑤ You will notice that the vector detemines the translation. You can changes the translation by dragging the vector.

Example: Determine the coordinates of the image of P(3; 2) if P has been enlarged by a factor 3 with the origin as centre of enlargement.
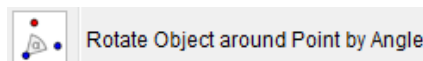
① Type: (3,2) in the Input bar and press the enter key

> Input: (3,2)

② Type: (0,0) in the Input bar and press the enter key

> Input: (0,0)

③ Select the Rotate Object around Point by Angle option
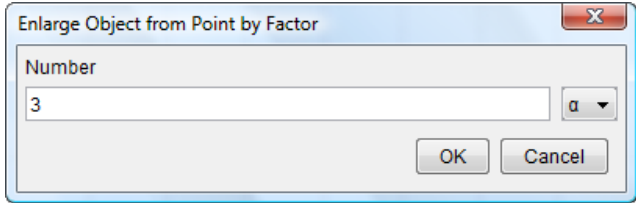
> Enlarge Object from Point by Factor
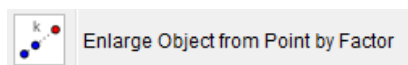
④ The following message will appear next to the Construction Tools:

> **Enlarge Object from Point by Factor**
> Select object to enlarge, then centre point and enter factor

Follow the instructions: Select the point (3, 2), then the centre (0, 0). The following screen will appear:

> Enlarge Object from Point by Factor
> Number
> 3   α ▼
> OK   Cancel

⑤ Type 3 and Click OK.

20

6 User define tools (centroid)
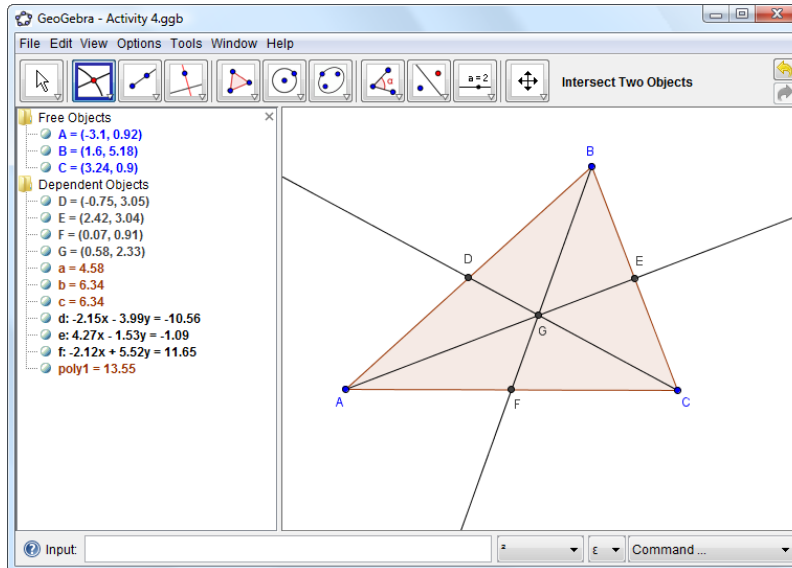
① Construct a triangle

② Construct the midpoint of each of the sides of the triangle:
use the Midpoint or Centre Tool

③ Construct the medians: select the Ray through Two Points.
Connect the vertex with the midpoint of the opposite side.

④ Construct a point on the intersection of the medians.
Use Intersect Two Objects tool
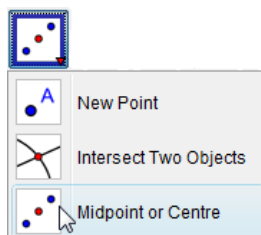
⑤ Hide all the unwanted information and constructions.
Go to the Algebra View: click all the dots except for A, B, C
and the centroid G.

21
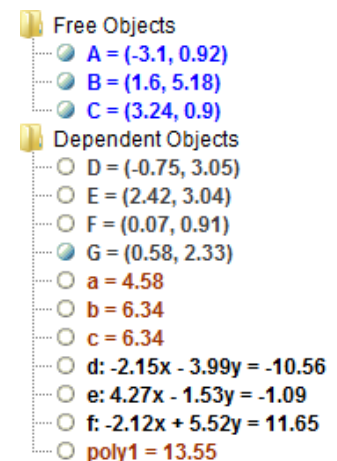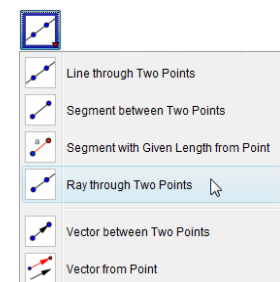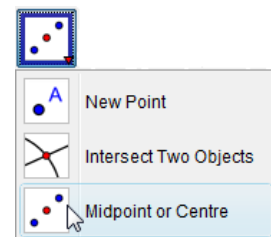
GeoGebra allows you to create your own construction tools based on your existing constructions. All these tools created by you are saved in your GeoGebra file.

⑥ Create your own "Centroid" tool for future use.
Use the Arrow tool and select the points A, B, C and G by dragging. Select the Tools menu and Create New Tool.
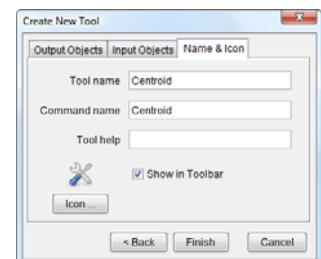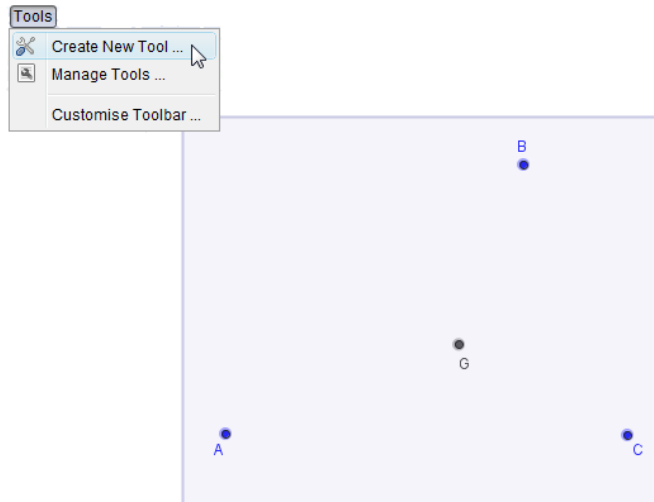
Click "Next" twice and enter the name for the new tool.
If you want to construct a centroid in future, use this tool.

To construct a centroid: select the "Centroid" tool and select the three vertices of any triangle.

⑦ If you open GeoGebra again your custom tools will not appear in the Construction Tools unless you save the settings. Select the Option menu and click Save Settings.
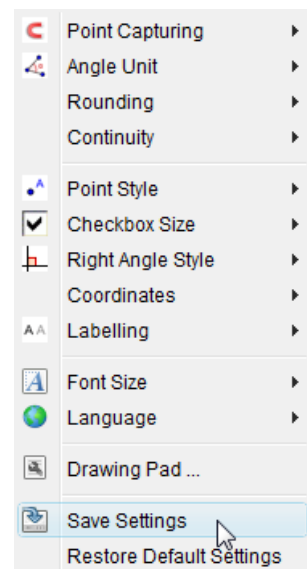
7 Kites and parallelograms

Construction of a parallelogram

A parallelogram is a quadrilateral with 2 pairs of opposite sides parallel.
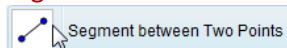
① Construct a line segment using the
Segment between Two Points tool

Segment between Two Points

② Construct any other point (not on the line segment) using the
New Point tool.

A New Point

③ Connect point A and C using the
Segment between Two Points tool

Segment between Two Points
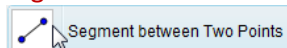
④ Construct a parallel line by selecting the Parallel Line tool and click
on the point C and the line segment AB.

Parallel Line

Perpendicular Line
Parallel Line
Perpendicular Bisector
Angle Bisector
Tangents
Polar or Diameter Line
Best Fit Line
Locus

GeoGebra

File Edit View Options Tools Window Help

Move
Drag or select objects (Esc)

Free Objects
A = (-2.7, 1.94)
B = (1.72, 1.94)
C = (-1.42, 4)
Dependent Objects
a = 4.42
b = 2.43
c: y = 4
d: -2.06x + 1.28y = -1.06

C

A   B

Input: ²   α   Command ...
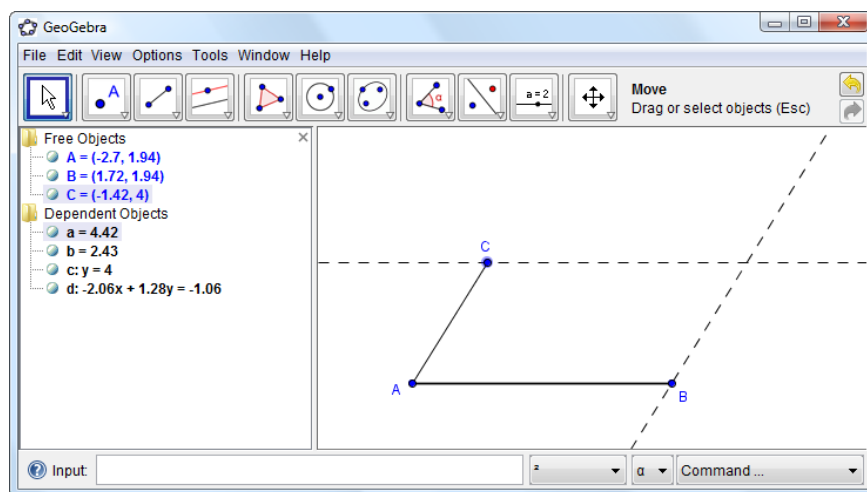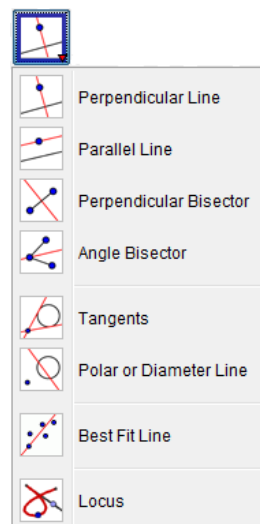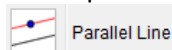
⑤ Construct another parallel line by selecting the Parallel Line tool
and click on the point B and the line segment AC.
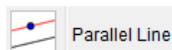
Parallel Line

⑥ Construct a point on the intersection of the two newly
constructed parallel lines using the Intersect Two Objects tool

Intersect Two Objects

A New Point
Intersect Two Objects
Midpoint or Centre

23
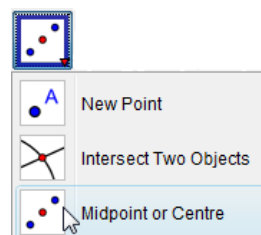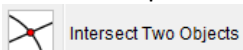
⑦ Select the Polygon Tool and select the four vertices of the parrallelogram.

Polygon
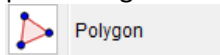
⑧ Hide all the unwanted information and constructions.
Right click on the line or object and select either Show Object or Show Label.
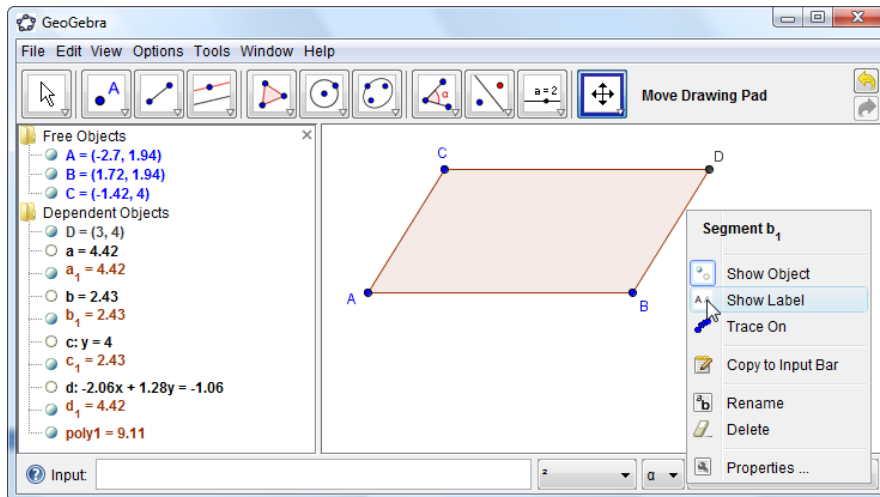
Construction of a kite

A kite is a quadrilateral with two pairs of adjacent sides equal.
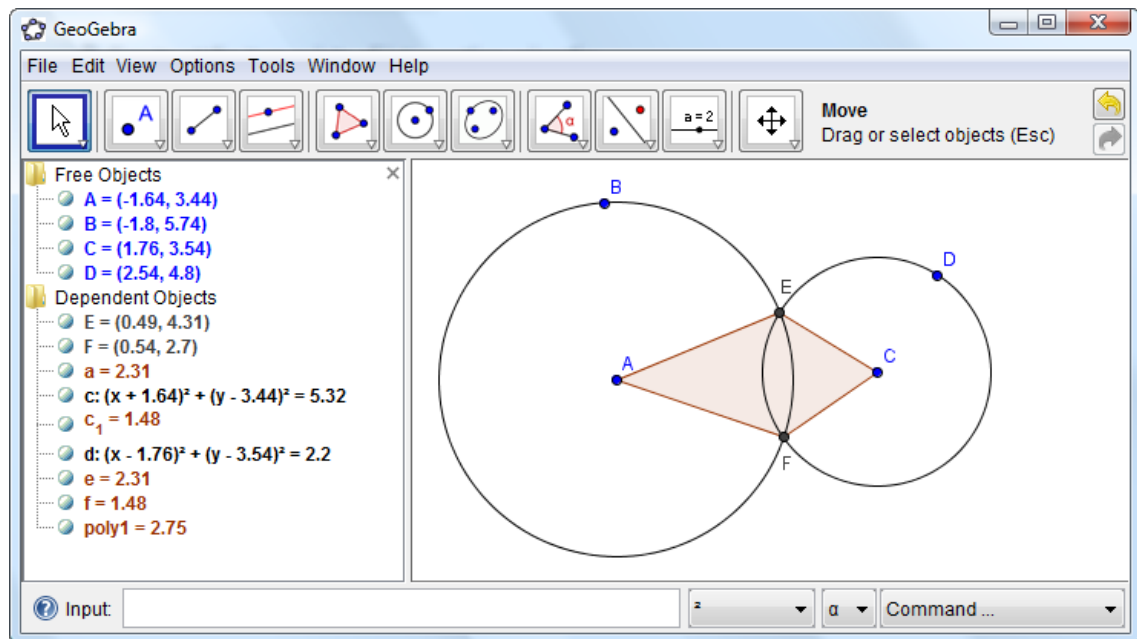
① Construct two intersecting circles using the
   Circle with Centre through Point tool

② Construct the two points of intersection using the
   Intersect Two Objects tool
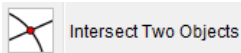
③ Select the Polygon tool and select the four vertices of the kite (the
   centre of the two circles and the two intersections of the circles).
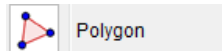
④ Hide all the unwanted information and constructions.
   Right click on the line or object and select either Show
   Object or Show label.

8 Statistics

GeoGebra has a range of statistical uses which can be used with or without the Spreadsheet View. It has a number of statistical functions and graphing tools.

Finding the mean, median and mode

The marks for a math test, out of 60, are given below. Use GeoGebra to find the mean, median and mode: 48 38 42 54 40 34 58 44 52 36 26 46 60 20 26

Method 1: Use the Input bar:

① Type the following in the Input bar

> Input: mean[48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26]
>
> Input: median[48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26]
>
> Input: mode[48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26]

② GeoGebra will list the results in the Algebra Window:

Mean = 42.67

Median = 42

Mode = 26
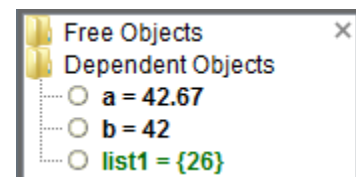
> Free Objects
> Dependent Objects
> ○ a = 42.67
> ○ b = 42
> ○ list1 = {26}

Method 2: Use the Input bar and Spreadsheet View:

① Open the Spreadsheet View:
click the View menu / Spreadsheet View

> Spreadsheet View   Ctrl+Shift+S

② Type the data in the first column of the spreadsheet.

③ Select the data by dragging the mouse.

④ Right-click on the selected block and select Create List.

Create List

The following list will appear in the Algebra window:

> Free Objects
> Dependent Objects
> ○ L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}

⑤ Type the following in the Input bar

> Input: mean[L_1]
>
> Input: median[L_1]
>
> Input: mode[L_1]

Example: The marks for a math test, out of 60, are given below. Use GeoGebra to draw a histogram for the marks: 48 38 42 54 40 34 58 44 52 36 26 46 60 20 26

Method 1: Use the Input bar:

① Type the following in the Input bar to creates a histogram using the raw data: Histogram[{List of Class Boundaries}, {List of Raw Data}]

Input: Histogram[{10, 20, 30, 40, 50, 60, 70}, {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}]

The class boundaries determine the width and position of each bar of the histogram.

② GeoGebra will construct the following histogram:

Method 2: Use the Input bar and Spreadsheet View:

① Open the Spreadsheet View:
click the View menu / Spreadsheet View

Spreadsheet View   Ctrl+Shift+S

② Type the data in the first column of the spreadsheet.

③ Select the data by dragging the mouse.

④ Right-click on the selected block and select Create List.

Create List

The following list will appear in the Algebra window:

Free Objects
Dependent Objects
L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}

⑤ Type the following in the Input bar

Input: Histogram[{10, 20, 30, 40, 50, 60, 70}, L_1]

27

Example: The marks for a math test, out of 60, are given below. Use GeoGebra to draw a box and whisker diagram for the marks: 48 38 42 54 40 34 58 44 52 36 26 46 60 20 26

① Open the Spreadsheet View:
   click the View menu / Spreadsheet View
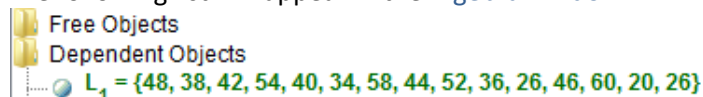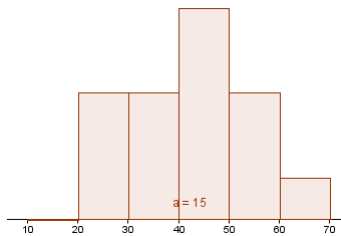
Spreadsheet View	⌖	Ctrl+Shift+S

② Type the data in the first column of the spreadsheet.

③ Select the data by dragging the mouse.

④ Right-click on the selected block and select Create List.

   Create List

   The following list will appear in the Algebra window:

   Free Objects
   Dependent Objects
      L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}

⑤ Type the following in the Input bar to creates a Box and whisker plot: BoxPlot[a, b, L_1] where
   • a is the vertical position on the coordinate system
   • b is the relative height of the box and whisker diagram

   Input: BoxPlot[3, 1, L_1]

⑥ GeoGebra will construct the following Box and whisker plot:

⑦ Changing the first two values to a = 1 and b = 0.2 will result in:

   Input: BoxPlot[1, .2, L_1]
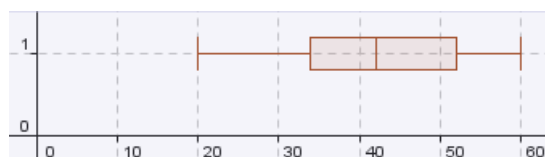
The marks for a math test, out of 60, are given below. Use GeoGebra to find the quartiles, standard deviation and variance: 48 38 42 54 40 34 58 44 52 36 26 46 60 20 26

① Open the Spreadsheet View:
click the View menu / Spreadsheet View

Spreadsheet View		Ctrl+Shift+S

② Type the data in the first column of the spreadsheet.

③ Select the data by dragging the mouse.

④ Right-click on the selected block and select Create List.

Create List

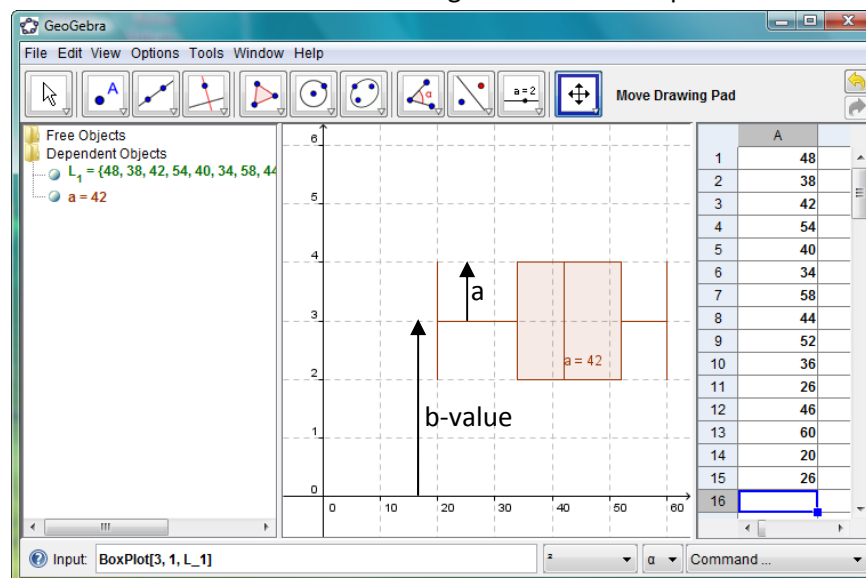The following list will appear in the Algebra window:

Free Objects
Dependent Objects
L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}

⑤ Type the following in the Input bar:

For calculating the **Lower Quartile (Q1):**

Input: Q1[L_1]

For calculating the **Upper Quartile (Q3):**

Input: Q3[L_1]

For calculating the **Standard Deviation (SD):**

Input: SD[L_1]

For calculating the **Variance (σ):**

Input: Variance[L_1]

⑥ GeoGebra will list the results in the same order as it was created in the Algebra Window:

Free Objects
Dependent Objects
L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}
a = 34
b = 52
c = 11.53
d = 132.91
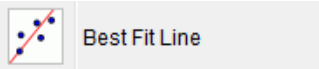
Example: The table below represents the number of new businesses that were started in Pretoria between 2003 and 2008.

Year	Number of businesses
2002	754
2003	881
2004	943
2005	1 083
2006	1 182
2007	1 304
2008	1 402

Sketch a scatter plot which represents the information above and draw the line of best fit.

① Open the Spreadsheet View:
   click the View menu / Spreadsheet View

   | Spreadsheet View | ⊾ | Ctrl+Shift+S |

② Type the data in the first and second columns of the spreadsheet.

③ Select the Best Fit Line from the Construction Tools

   Best Fit Line

④ The equation of the line will appear in the Algebra window

Perpendicular Line

Parallel Line

Perpendicular Bisector

Angle Bisector

Tangents

Polar or Diameter Line

Best Fit Line

Locus
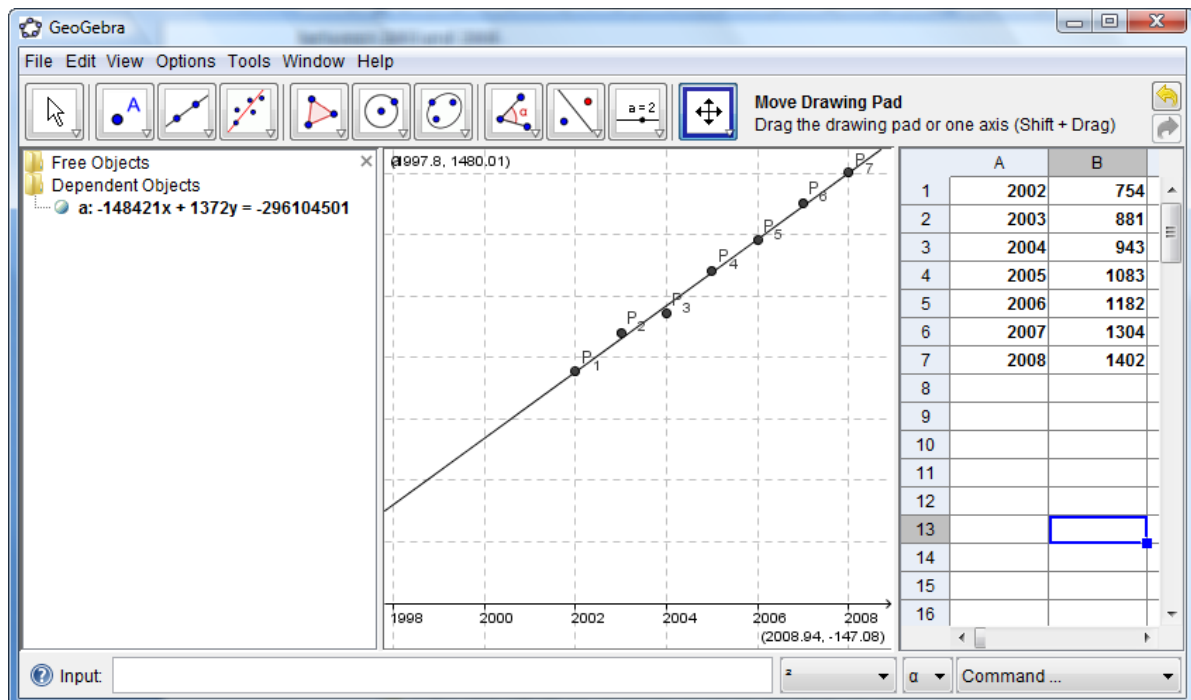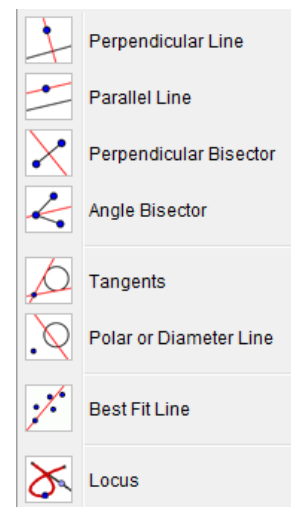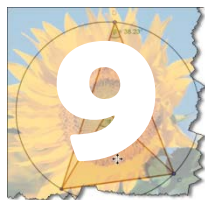
9 Calculus

Construct a tangent at a point (must be able to drag the point of contact) to any curve of a function f

Example: Construct a tangent to the graph of $f(x) = 2x^3 + 3x - 2$

① Type the equation in the Input Bar and press enter.

> Input: f(x)=2x^3+3x-2

② Select the New Point from the Construction Tools and click on the graph.

> A New Point

A New Point	
Intersect Two Objects	
Midpoint or Centre	
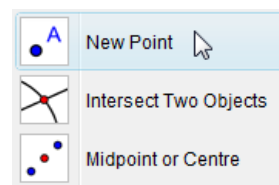
③ Select the Tangents from the Toolbox and click on the point and on the graph.

> Tangents

Perpendicular Line	
Parallel Line	
Perpendicular Bisector	
Angle Bisector	
Tangents	
Polar or Diameter Line	
Best Fit Line	
Locus	
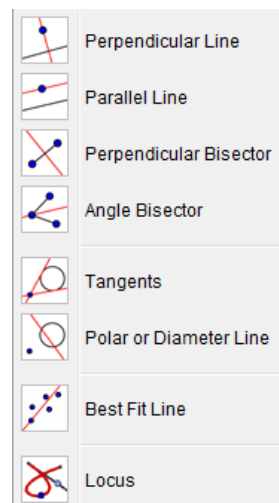
④ Drag the point on the graph

GeoGebra

File Edit View Options Tools Window Help

Free Objects
  f(x) = 2 x³ + 8 x² + 4 x - 2
Dependent Objects
  A = (-2.96, 4.4)
  a: y = 9.18x + 31.56

Input: f(x)=2x^3+3x-2
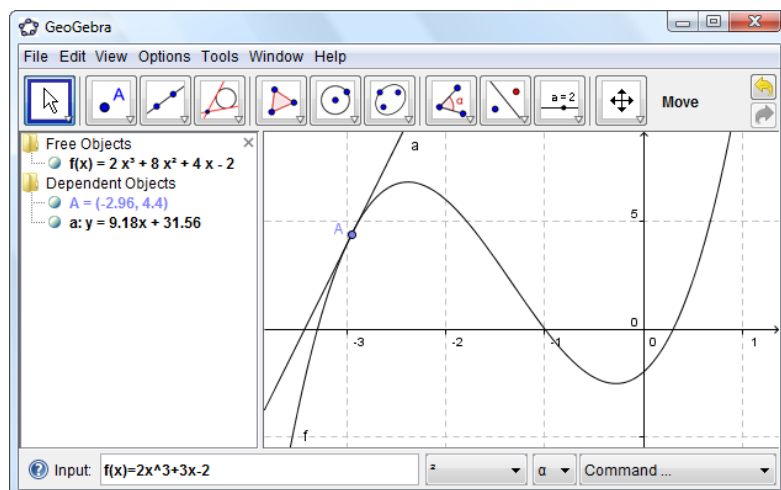
Example: Find the derivate of $f(x) = 3x^3 + 2x^2 - 6x + 5$

① Type the equation in the Input Bar and press enter.

② Input: f(x)=3x^3+2x^2-6x+5

② Type the following command (or select it from the drop down list) in the Input Bar and press enter.

② Input: f'(x)

or

② Input: derivative[f]

GeoGebra will calculate the derivative in the algebra view and construct the curve of f'(x).

Free Objects
f(x) = 6 x³ - 3 x² - 8 x - 3
Dependent Objects
f'(x) = 18 x² - 6 x - 8

Example: Evaluate and demonstrate the Riemann sum for $f(x) = 2x^3 + 8x^2 + 4x - 2$ taking the sample points to be under the curve and $a = -3$, $b = -1$, and $n = 8$. Sketch a graph of the function and the Riemann rectangles and use the GeoGebra to determine these areas.

① Type the equation in the Input Bar and press enter.

> Input: f(x)=2x^3+8x^2+4x-2

② Type the following command (or select it from the drop down list) in the Input Bar and press enter.

> Input: LowerSum[f, -3, -1, 8]

This command will yields the lower sum of the function f on the interval $[-3, -1]$ with 8 rectangles.

A similar command is available for the upper sum. If you want to increase the number of rectangles you can also create a slider.

Example: Calculate the area under the graph of $f(x) = 2x^3 + 3x - 2$
between $x = -3$ and $x = -2$.

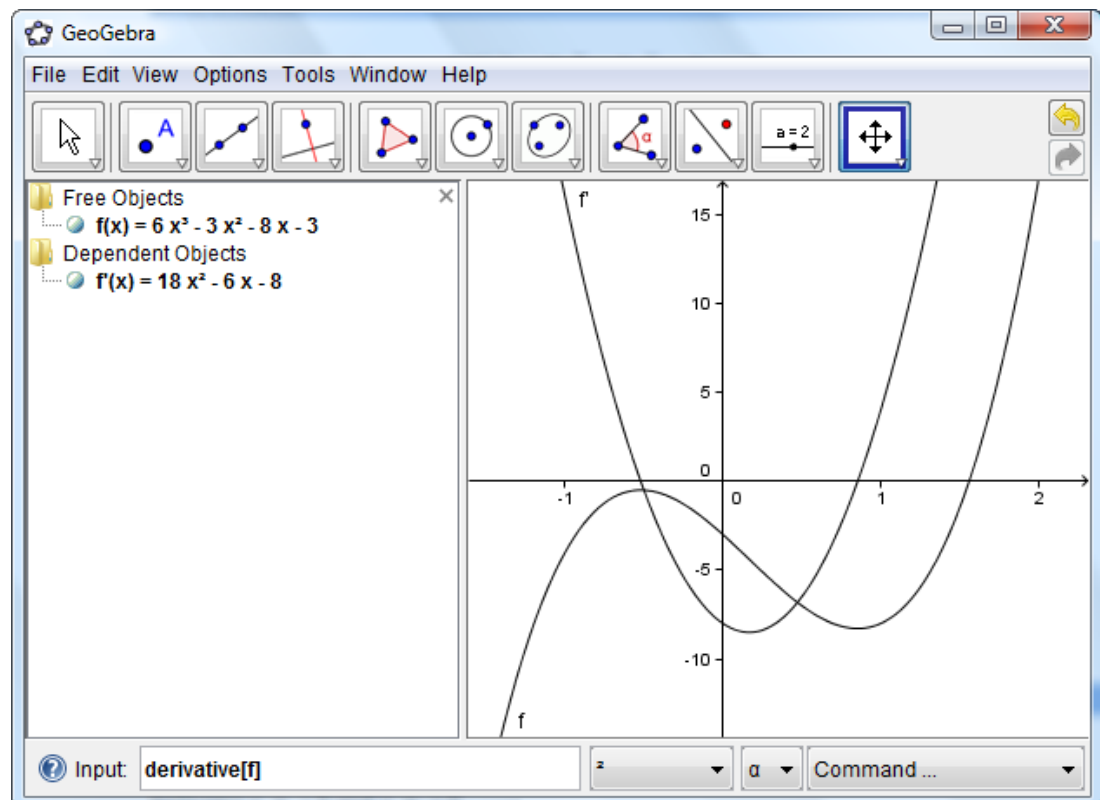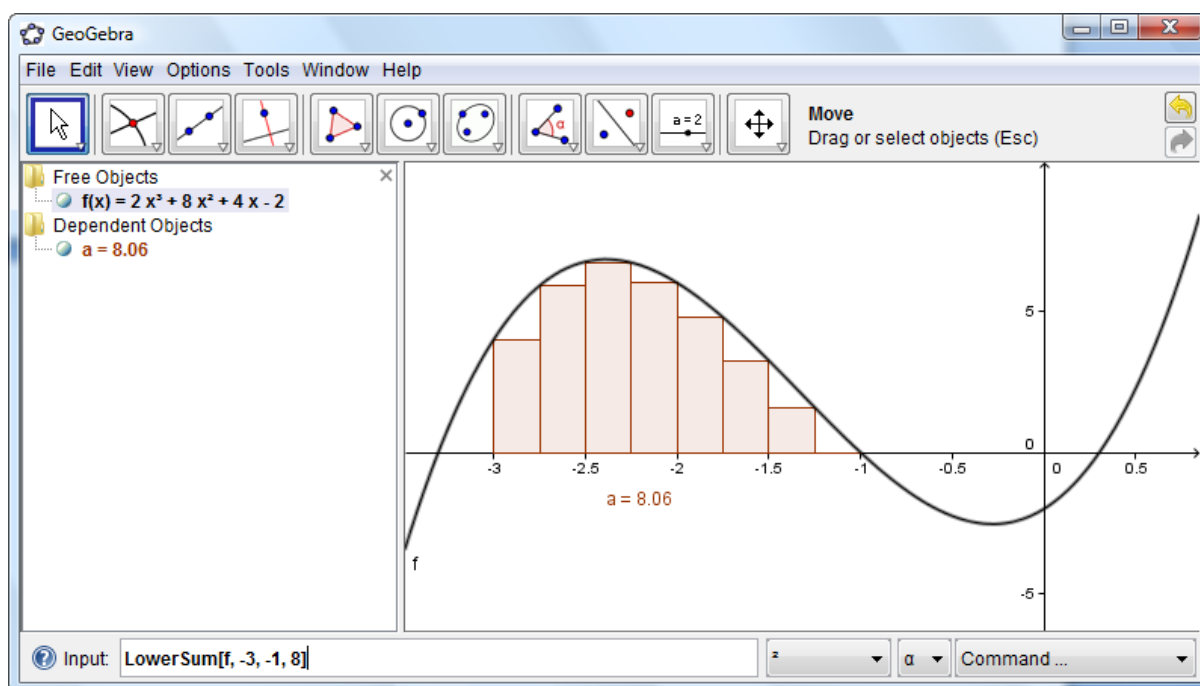   ① Type the equation in the Input Bar and press enter.

       Input: f(x)=2x^3+3x-2

    ② Type the following command (or select it from the drop down list)
       in the Input Bar and press enter.

       Input: Integral[f,-3,-2]
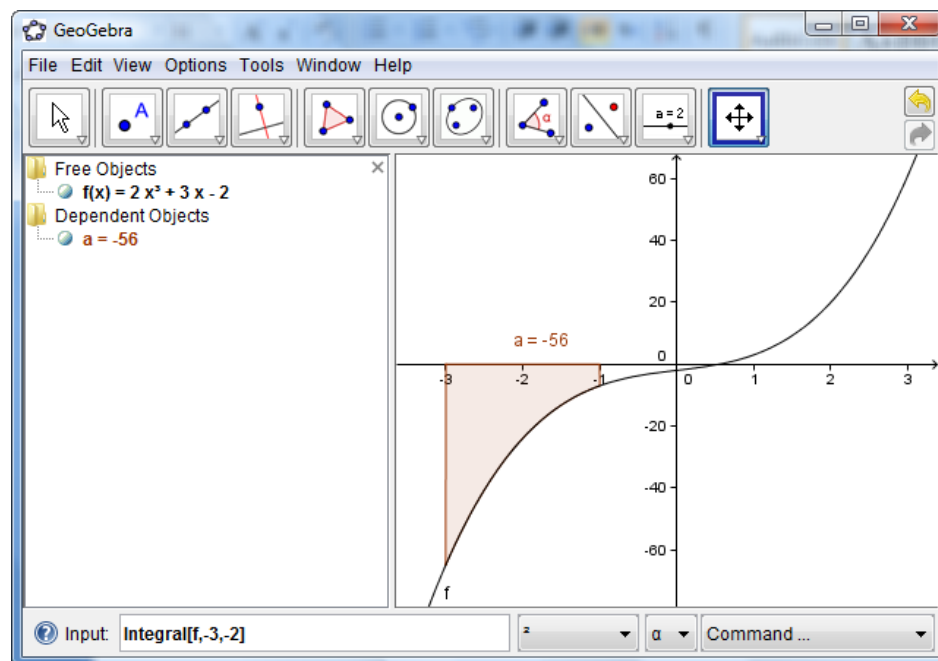
    ③ In the algebra view GeoGebra will returns the definite integral of
       the function f in the interval $[-3, -1]$.

The following command will yields the definite integral of the difference f(x) – g(x) in the interval [a, b]: Integral[f, g, a, b]

Example find the area between the curves of $f(x) = 3x^3 + 2x^2 - 6x + 5$ and $g(x) = 18x^2 - 6x - 8$ between their points of intersection.
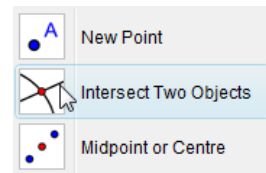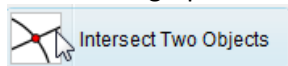
① Type the equation in the Input Bar and press enter.

> ❓ Input: **f(x)=3x^3+2x^2-6x+5**

> ❓ Input: **g(x)=18x^2-6x-8**

② From the Construction Tools select Intersect Two Objects and click on the two graphs.
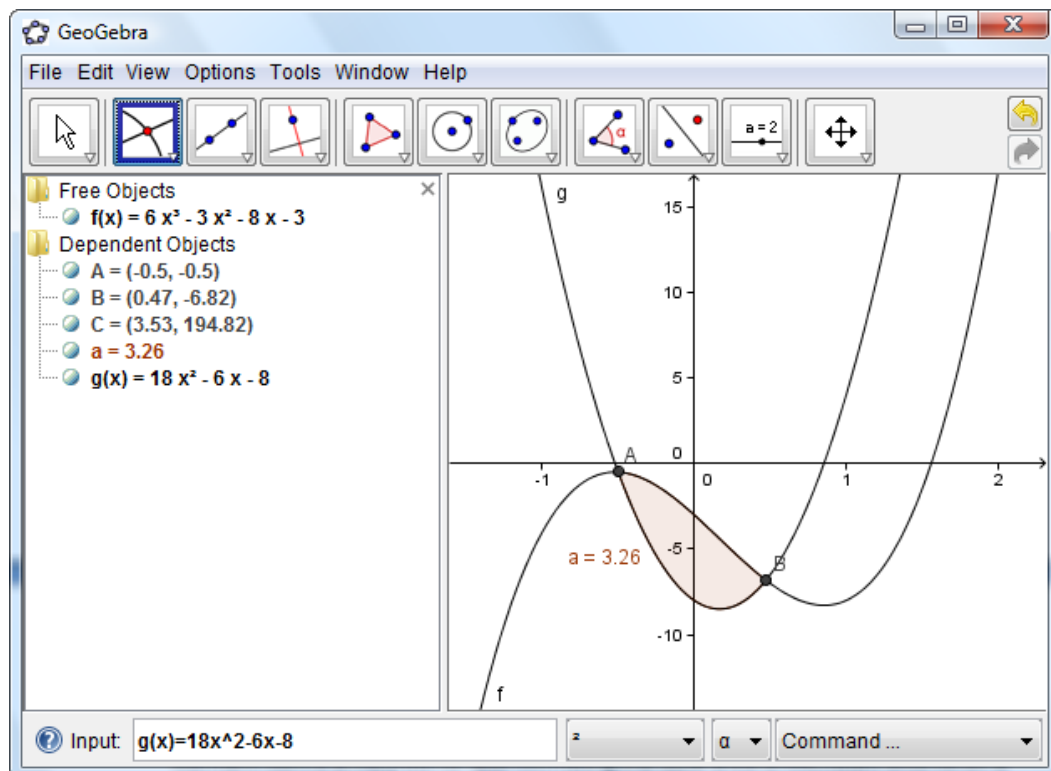
> Intersect Two Objects

GeoGebra will constructs the points of intersection A and B.

③ Type the following command (or select it from the drop down list) in the Input Bar and press enter.

> ❓ Input: **integral[f, g, x(A), x(B)]**

GeoGebra will construct and measure the area between the curves of f and g between the *x*-value of point A and the *x*-value of point B.
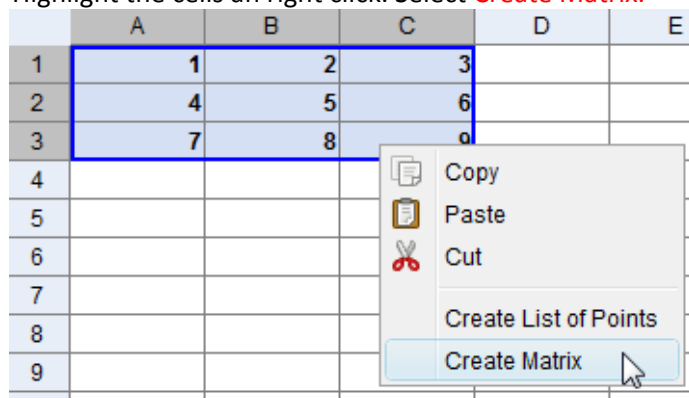
Matrices

You can use GeoGebra to do matrix operations. For example: calculate:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 5 \\ 1 & 9 & 5 \\ 4 & 7 & 9 \end{bmatrix}$$

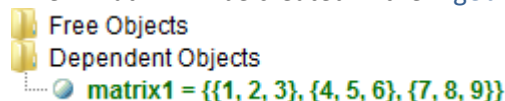① Open the Spreadsheet View:
   click the View menu / Spreadsheet View

   | Spreadsheet View | | Ctrl+Shift+S |

② Type the data in the same order as the matrix in the spreadsheet.

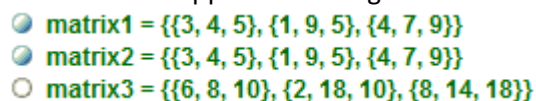③ Highlight the cells an right click. Select Create Matrix.

	A	B	C	D	E
1	1	2	3		
2	4	5	6		
3	7	8	9		
4			Copy		
5			Paste		
6			Cut		
7					
8			Create List of Points		
9			Create Matrix		

A new matrix will be created in the Algebra View area

Free Objects
Dependent Objects
    matrix1 = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}

④ Repeat steps 1 to 4 and create another matrix.

⑤ Type the following command (or select it from the drop down list) in the Input Bar and press enter.

Input: matrix1+matrix2

The result will appear in the Algebra View under matrix 3:

matrix1 = {{3, 4, 5}, {1, 9, 5}, {4, 7, 9}}
matrix2 = {{3, 4, 5}, {1, 9, 5}, {4, 7, 9}}
matrix3 = {{6, 8, 10}, {2, 18, 10}, {8, 14, 18}}

It is also possible to calculate the determinant, or to invert or transpose matrices using the following commands:

Input: Invert[matrix1]

Input: Transpose[matrix1]

Input: Determinant[matrix1]