

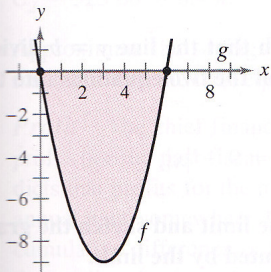
## REVIEW WORKSHEET OF SECTIONS 6.1, 6.2, AND 6.5

### SECTION 6.1

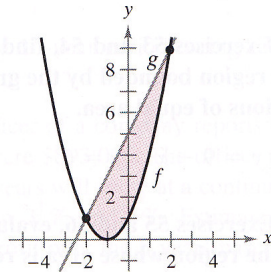
Complete #1-9 odd, 13-25 odd, 37, 49

In Exercises 1–6, set up the definite integral that gives the area of the region.

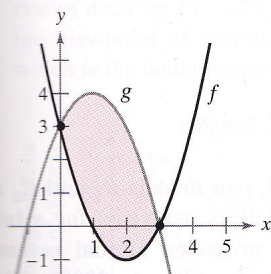
1.  $f(x) = x^2 - 6x$   
 $g(x) = 0$



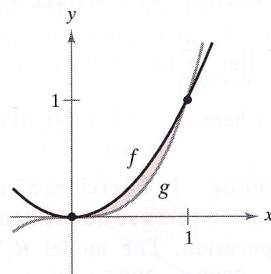
2.  $f(x) = x^2 + 2x + 1$   
 $g(x) = 2x + 5$



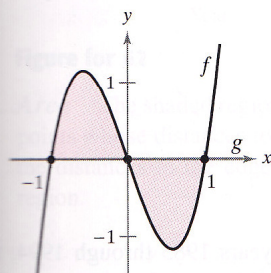
3.  $f(x) = x^2 - 4x + 3$   
 $g(x) = -x^2 + 2x + 3$



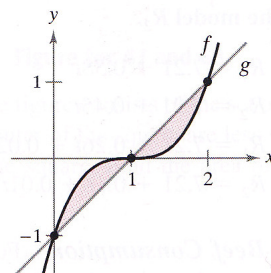
4.  $f(x) = x^2$   
 $g(x) = x^3$



5.  $f(x) = 3(x^3 - x)$   
 $g(x) = 0$



6.  $f(x) = (x - 1)^3$   
 $g(x) = x - 1$



In Exercises 7–10, the integrand of the definite integral is a difference of two functions. Sketch the graph of each function and shade the region whose area is represented by the integral.

7.  $\int_0^4 \left[ (x + 1) - \frac{x}{2} \right] dx$

8.  $\int_{-1}^1 [(1 - x^2) - (x^2 - 1)] dx$

9.  $\int_0^5 \left[ 4(2^{-x/3}) - \frac{x}{6} \right] dx$

10.  $\int_{-\pi/3}^{\pi/3} (2 - \sec x) dx$

In Exercises 13–26, sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

13.  $f(x) = x^2 - 4x$ ,  $g(x) = 0$

14.  $f(x) = 3 - 2x - x^2$ ,  $g(x) = 0$

15.  $f(x) = x^2 + 2x + 1$ ,  $g(x) = 3x + 3$

16.  $f(x) = -x^2 + 4x + 2$ ,  $g(x) = x + 2$

17.  $y = x$ ,  $y = 2 - x$ ,  $y = 0$

18.  $y = \frac{1}{x^2}$ ,  $y = 0$ ,  $x = 1$ ,  $x = 5$

19.  $f(x) = \sqrt{3x} + 1$ ,  $g(x) = x + 1$

20.  $f(x) = \sqrt[3]{x}$ ,  $g(x) = x$

21.  $f(y) = y^2$ ,  $g(y) = y + 2$

22.  $f(y) = y(2 - y)$ ,  $g(y) = -y$

23.  $f(y) = y^2 + 1$ ,  $g(y) = 0$ ,  $y = -1$ ,  $y = 2$

24.  $f(y) = \frac{y}{\sqrt{16 - y^2}}$ ,  $g(y) = 0$ ,  $y = 3$

25.  $f(x) = \frac{4}{x}$ ,  $x = 0$ ,  $y = 1$ ,  $y = 4$

26.  $g(x) = \frac{4}{2 - x}$ ,  $y = 4$ ,  $x = 0$

In Exercises 37–40, sketch the region bounded by the graphs of the transcendental functions and find the area of the region.

37.  $f(x) = 2 \sin x$ ,  $g(x) = \tan x$ ,  $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

In Exercises 49 and 50, set up and evaluate the definite integral that gives the area of the region bounded by the graph of the function and the tangent line to the graph at the indicated point.

49.  $f(x) = x^3$ ,  $(1, 1)$

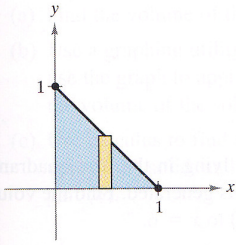
50.  $f(x) = \frac{1}{x^2 + 1}$ ,  $\left(1, \frac{1}{2}\right)$

## SECTION 6.2

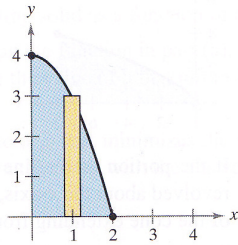
Complete #1-21 odd, 23, 25, 29, 55

In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the  $x$ -axis.

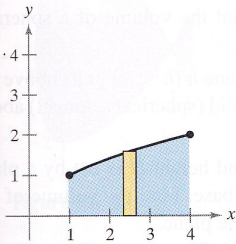
1.  $y = -x + 1$



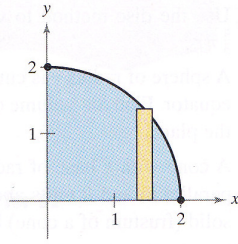
2.  $y = 4 - x^2$



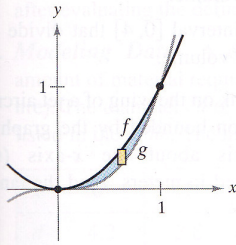
3.  $y = \sqrt{x}$



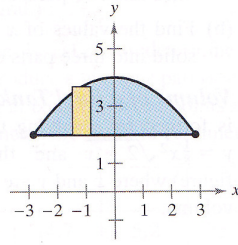
4.  $y = \sqrt{4 - x^2}$



5.  $y = x^2, y = x^3$

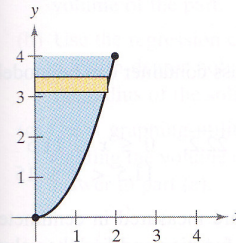


6.  $y = 2, y = 4 - \frac{x^2}{4}$

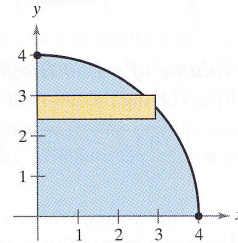


In Exercises 7–10, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the  $y$ -axis.

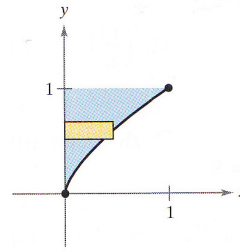
7.  $y = x^2$



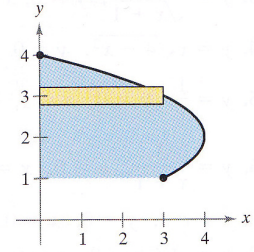
8.  $y = \sqrt{16 - x^2}$



9.  $y = x^{2/3}$



10.  $x = -y^2 + 4y$



In Exercises 11–14, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines.

11.  $y = \sqrt{x}, y = 0, x = 4$

- (a) the  $x$ -axis (b) the  $y$ -axis  
(c) the line  $x = 4$  (d) the line  $x = 6$

12.  $y = 2x^2, y = 0, x = 2$

- (a) the  $y$ -axis (b) the  $x$ -axis  
(c) the line  $y = 8$  (d) the line  $x = 2$

13.  $y = x^2, y = 4x - x^2$

- (a) the  $x$ -axis (b) the line  $y = 6$

14.  $y = 6 - 2x - x^2, y = x + 6$

- (a) the  $x$ -axis (b) the line  $y = 3$

In Exercises 15–18, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line  $y = 4$ .

15.  $y = x, y = 3, x = 0$

16.  $y = x^2, y = 4$

17.  $y = \frac{1}{x}, y = 0, x = 1, x = 4$

18.  $y = \sec x, y = 0, 0 \leq x \leq \frac{\pi}{3}$

In Exercises 19–22, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line  $x = 6$ .

19.  $y = x, y = 0, y = 4, x = 6$

20.  $y = 6 - x, y = 0, y = 4, x = 6$

21.  $x = y^2, x = 4$

22.  $xy = 6, y = 2, y = 6, x = 6$

In Exercises 23–28, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the  $x$ -axis.

23.  $y = \frac{1}{\sqrt{x+1}}, y = 0, x = 0, x = 3$

24.  $y = x\sqrt{4-x^2}, y = 0$

25.  $y = \frac{1}{x}, y = 0, x = 1, x = 4$

26.  $y = \frac{3}{x+1}, y = 0, x = 0, x = 8$

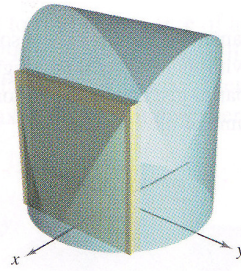
**In Exercises 29 and 30, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the  $y$ -axis.**

29.  $y = 3(2 - x)$ ,  $y = 0$ ,  $x = 0$

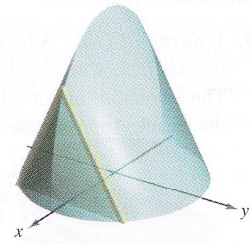
30.  $y = 9 - x^2$ ,  $y = 0$ ,  $x = 2$ ,  $x = 3$

55. Find the volume of the solid whose base is bounded by the circle  $x^2 + y^2 = 4$ , with the indicated cross sections taken perpendicular to the  $x$ -axis.

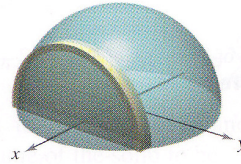
(a) Squares



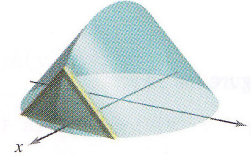
(b) Equilateral triangles



(c) Semicircles



(d) Isosceles right triangles



## SECTION 6.5

Complete #51, 57, 61, 63-68

In Exercises 51–56, find the value of  $c$  guaranteed by the *Mean Value Theorem for Integrals* for the function over the indicated interval.

Function	Interval
51. $f(x) = x - 2\sqrt{x}$	$[0, 2]$
52. $f(x) = \frac{9}{x^3}$	$[1, 3]$
53. $f(x) = 2 \sec^2 x$	$[-\pi/4, \pi/4]$
54. $f(x) = \cos x$	$[-\pi/3, \pi/3]$
55. $f(x) = 5 - \frac{1}{x}$	$[1, 4]$
56. $f(x) = 10 - 2^x$	$[0, 3]$

In Exercises 57–62, use a graphing utility to graph the function over the indicated interval. Find the average value of the function over the interval and all values of  $x$  in the interval for which the function equals its average value.

Function	Interval
57. $f(x) = 4 - x^2$	$[-2, 2]$
58. $f(x) = \frac{x^2 + 1}{x^2}$	$[\frac{1}{2}, 2]$
59. $f(x) = 2e^x$	$[-1, 1]$
60. $f(x) = \frac{1}{2x}$	$[1, 4]$
61. $f(x) = \sin x$	$[0, \pi]$
62. $f(x) = \cos x$	$[0, \pi/2]$

**Think About It** In Exercises 63–68, use the graph of  $f$  shown in the figure. The shaded region  $A$  has an area of 1.5, and  $\int_0^6 f(x) dx = 3.5$ . Use this information to fill in the blanks.

63.  $\int_0^2 f(x) dx =$
64.  $\int_2^6 f(x) dx =$
65.  $\int_0^6 |f(x)| dx =$
66.  $\int_0^2 -2f(x) dx =$
67.  $\int_0^6 [2 + f(x)] dx =$
68. The average value of  $f$  over the interval  $[0, 6]$  is .

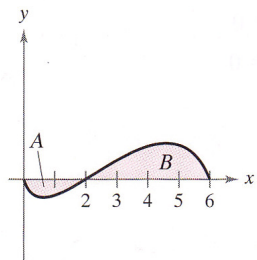


Figure for 63–68

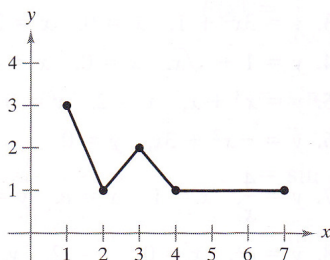


Figure for 69