47. $f(x) = x^4 + x - 3$ is continuous on the interval $[1, 2]$, $f(1) = -1$, and $f(2) = 15$. Since $-1 < 0 < 15$, there is a number c in $(1, 2)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $x^4 + x - 3 = 0$ in the interval $(1, 2)$.

48. $f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.

49. $f(x) = \cos x - x$ is continuous on the interval $[0, 1]$, $f(0) = 1$, and $f(1) = \cos 1 - 1 \approx -0.46$. Since $-0.46 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\cos x - x = 0$, or $\cos x = x$, in the interval $(0, 1)$.

50. $f(x) = \tan x - 2x$ is continuous on the interval $[0, 1.4]$, $f(1) = \tan 1 - 2 \approx -0.44$, and $f(1.4) = \tan 1.4 - 2.8 \approx 3.00$. Since $-0.44 < 0 < 3.00$, there is a number c in $(0, 1.4)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\tan x - 2x = 0$, or $\tan x = 2x$, in the interval $(0, 1.4)$.

SECTION 3.1

1. $f(x) = x^2 + 2x$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{[(3+h)^2 + 2(3+h)] - 15}{h}$$

$$= \lim_{h\to 0} \frac{9 + 6h + h^2 + 6 + 2h - 15}{h}$$

$$= \lim_{h\to 0} \frac{h(8+h)}{h}$$

$$= \lim_{h\to 0} (8+h)$$

$$= 8$$

2. $f(x) = \frac{1}{x-1}$ at $a = 2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{(2+h)-1} - \frac{1}{1}}{h}$$

$$= \lim_{h\to 0} \left(\frac{1}{1+h} - 1\right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1-(1+h)}{1+h} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-h}{h(1+h)}$$

$$= \lim_{h\to 0} \frac{-1}{1+h}$$

$$= -1$$

3. $G(x)=4x+3$

ⓐ $G'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{[(4)(2+h)+3]-11}{h}$

$=\lim_{h\to 0}\frac{8+4h+3-11}{h}$

$=\lim_{h\to 0}\frac{4h}{h}$

$=\lim_{h\to 0} 4$

$=4$

ⓑ since $G(x)=4x+3$ is linear, the slope of any tangent line is 4

4. $f(t)=t^2+2t$

$V(3)=f'(3)=\lim_{h\to 0}\frac{f(3+t)-f(3)}{h}=8\frac{ft}{sec}$ (from #1)

(same equation)

$V(3)=8\ ft/sec$

5. ⓐ $f(t)=6t^2-4t+1$

interval	avg vel (ft/sec)
$[1,4]$	$\frac{f(4)-f(1)}{4-1}=26$
$[1,2]$	$\frac{f(2)-f(1)}{2-1}=14$
$[1,1.2]$	$\frac{f(1.2)-f(1)}{1.2-1}=9.2$
$[1,1.01]$	$\frac{f(1.01)-f(1)}{1.01-1}=8.06$

5 cont'd

$f(t) = 6t^2 - 4t + 1$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[6(1+h)^2 - 4(1+h) + 1] - 3}{h}$$

$$= \lim_{h \to 0} \frac{6(1+2h+h^2) - 4 - 4h + 1 - 3}{h}$$

$$= \lim_{h \to 0} \frac{h(8+6h)}{h}$$

$$= \lim_{h \to 0} (8+6h)$$

$= 8$ ft/sec

6. 3:00 → 120 miles

5:00 → 250 miles

(a) avg vel $= \frac{250-120}{2} = 65$ mi/hr

(b) instantaneous velocity at that moment

7. $f'(a) = \lim_{h \to 0} \frac{f(h) - f(a)}{h}$

FALSE!! should be $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

8. $f(x) = x^2 + 10x$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{[(3+h)^2 + 10(3+h)] - 39}{h}$$

$$= \lim_{h \to 0} \frac{9 + 6h + h^2 + 30 + 10h - 39}{h}$$

$$= \lim_{h \to 0} \frac{h(16+h)}{h}$$

$= 16$

9. $f(x) = 2x - x^2$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{[2(a+h) - (a+h)^2] - (2a - a^2)}{h}$$

$$= \lim_{h \to 0} \frac{2a + 2h - a^2 - 2ah - h^2 - 2a + a^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2 - 2a - h)}{h}$$

$$= \lim_{h \to 0} (2 - 2a - h)$$

$$= 2 - 2a$$

10.

interval	avg pop $\left(\frac{\text{people}}{\text{year}}\right)$
1998 to 1999	$\frac{P(1999) - P(1998)}{1999 - 1998} = -11$
1999 to 2000	$\frac{P(2000) - P(1999)}{2000 - 1999} = -9$

to estimate $P'(1999)$ we will average these

$$P'(1999) \approx \frac{-11 + -9}{2} = -10 \ \frac{\text{people}}{\text{year}}$$

11. $f'(a)$

12. FALSE
instantaneous rate of change

13. $f'(x) = 3x^2 + 2$

$f(x) = x^3 + 2x + 1$

(a) $f'(2) = 3(2)^2 + 2 = 14$

(b) $f(2) = (2)^3 + 2(2) + 1 = 13$

pt $(2, 13)$ $f'(2) = 14$

$y - 13 = 14(x - 2)$

$y = 14x - 15$

(c) $f'(3) = 3(3)^2 + 2 = 29$

(d) $v(4) = f'(4) = 3(4)^2 + 2 = 50$ ft/sec

15. $f'(b)$

SECTION 3.2

1. true

3. FALSE

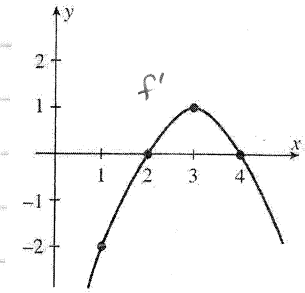
$$f'(x) = \frac{dy}{dx}$$

4. Yes
example: $f(x) = |x-3|$
f is cont at $x=3$ but f is not diff at $x=3$ since there is a corner at $x=3$

5. No
If a function is diff then the function is cont.

6. (a) true
(b) false (vert tangent)
(c) true
(d) false (corner)
(e) true
(f) true
(g) false ($\lim_{x\to 4} f(x)$ DNE since $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$)
(h) false (not cont so not diff)
(i) false (not at $x=2$)
(j) true

7. $f'(1) \approx -2$
$f'(2) \approx 0$
$f'(3) \approx 1$
$f'(4) \approx 0$



10. f is not diff at $x=0$ since f is discontinuous at $x=0$. f is discontinuous at $x=0$ since $f(0)$ is undefined.
f is not diff at $x=2$ since f is discontinuous at $x=2$. f is discontinuous at $x=2$ since $f(2)$ is undefined.
f is not diff at $x=4$ since f has a vertical tangent at $x=4$.

12. $f(t) = t^3 - 2t^2 + 3$

Note: can plug 3 into equations first

$$v(t) = f'(t) = \lim_{h\to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h\to 0} \frac{[(t+h)^3 - 2(t+h)^2 + 3] - (t^3 - 2t^2 + 3)}{h}$$

$$= \lim_{h\to 0} \frac{t^3 + 3t^2h + 3th^2 + h^3 - 2(t^2 + 2th + h^2) + 3 - t^3 + 2t^2 - 3}{h}$$

$$= \lim_{h\to 0} \frac{h(3t^2 + 3th + h^2 - 4t - 2h)}{h}$$

$$= \lim_{h\to 0} (3t^2 + 3th + h^2 - 4t - 2h)$$

$$= 3t^2 - 4t$$

12 cont'd

$a(t) = v'(t) = f''(t)$

$= \lim_{h \to 0} \dfrac{v(t+h) - v(t)}{h}$

$= \lim_{h \to 0} \dfrac{[3(t+h)^2 - 4(t+h)] - (3t^2 - 4t)}{h}$

$= \lim_{h \to 0} \dfrac{3(t^2 + 2th + h^2) - 4t - 4h - 3t^2 + 4t}{h}$

$= \lim_{h \to 0} \dfrac{h(6t + 3h - 4)}{h}$

$= \lim_{h \to 0} (6t + 3h - 4)$

$= 6t - 4$

$v(3) = 3(3)^2 - 4(3) = 15\ ^{m}/_{sec}$

$a(3) = 6(3) - 4 = 14\ ^{m}/_{sec^2}$