#25

$h(w) = \sqrt{w}\, e^w$

$\frac{d}{dw} w^{1/2} = \frac{1}{2} w^{-1/2}$

$h'(w) = \underbrace{w^{1/2}}_{f} \underbrace{e^w}_{g}$ ← product rule $\frac{d}{dw} e^w = e^w$

$\frac{1}{2} w^{-1/2} e^w + w^{1/2} e^w$ ← product rule twice.

$h''(w) = -\frac{1}{4} w^{-3/2} e^w + \frac{1}{2} w^{-1/2} e^w + \frac{1}{2} w^{-1/2} e^w + w^{1/2} e^w$

when $w = 1$

$-\frac{1}{4} e^w + \frac{1}{2} e^w + \frac{1}{2} e^w + e^w = \frac{7}{4} e^1$

$w=1$

$g''(0)$, $g(s) = \frac{e^s}{s+1}$ $\qquad \frac{(s+1)e^s - e^s(1)}{(s+1)^2}$

#26
Higher order derivatives

$$f'(s) = \frac{se^s}{s^2+2s+1} = f'(x) = \frac{xe^x}{x^2+2x+1}$$

switch s to x for ease of use

$\frac{d}{dx}(xe^x)$ $\qquad \frac{d}{dx}(s^2+2s+1)$ $\qquad$ since $x=0$ makes whole part $=0$

$$g''(0) = \frac{(x^2+2x+1)(xe^x+e^x) - (2x+2)(xe^x)}{(x^2+2x+1)^2}$$

$$\frac{(1)(1)-(0)}{1} = 1$$

Proof of Derivative of sinx

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\lim_{h\to 0} \frac{\sin x \cosh + \cos x \sin h - \sin x}{h}$$

$$\lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

$$\lim_{h\to 0} \sin x \underbrace{\frac{(\cos h - 1)}{h}}_{0} + \cos x \underbrace{\frac{\sin h}{h}}_{1}$$

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

After taking the limit

$$\sin(x) \cdot 0 + \cos x \cdot 1 = \boxed{\cos x}$$