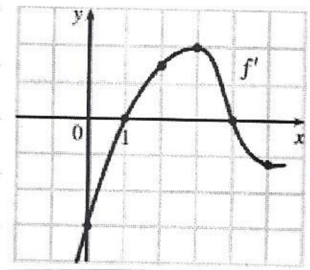
3.2b

2. $f'(0) \approx -3$
$f'(1) \approx 0$
$f'(2) \approx 1.5$
$f'(3) \approx 2$
$f'(4) \approx 0$
$f'(5) \approx -1.5$



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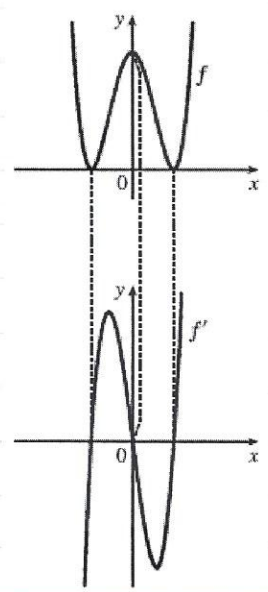

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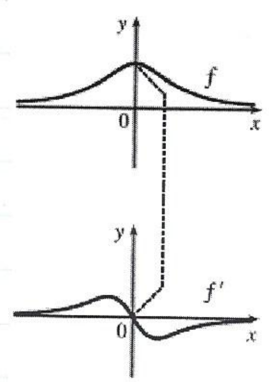

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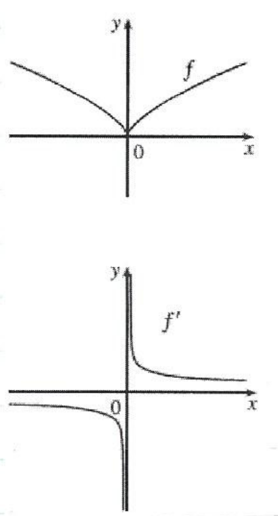

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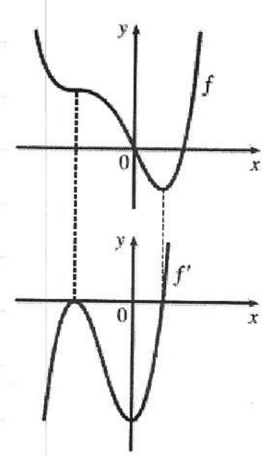

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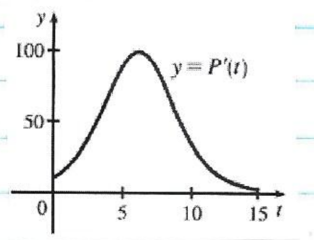
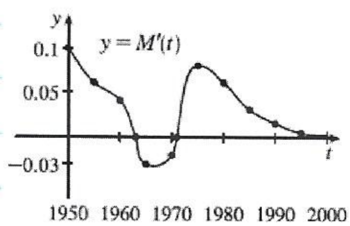


The slopes of the tangent lines on the graph of $y = P(t)$ are always positive, so the y-values of $y = P'(t)$ are always positive. These values start out relatively small and keep increasing, reaching a maximum at about $t = 6$. Then the y-values of $y = P'(t)$ decrease and get close to zero. The graph of P' tells us that the yeast culture grows most rapidly after 6 hours and then the growth rate declines.

13.



It appears that there are horizontal tangents on the graph of M for $t = 1963$ and $t = 1971$. Thus, there are zeros for those values of t on the graph of M'. The derivative is negative for the years 1963 to 1971.

18. $f(x) = mx + b$ (note: x is the variable, m and b represent numbers)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[m(x+h)+b] - (mx+b)}{h}$$

$$= \lim_{h \to 0} \frac{mx + mh + b - mx - b}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= \lim_{h \to 0} m$$

$$= m$$

$f(x) = mx + b$ D: $\mathbb{R}$

$f'(x) = m$ D: $\mathbb{R}$

Note: started with $y = mx + b$ which is a line with slope m. Then $y' = m$ which makes sense. ☺

20. $f(x) = 1.5x^2 - x + 3.7$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[1.5(x+h)^2 - (x+h) + 3.7] - (1.5x^2 - x + 3.7)}{h}$$

$$= \lim_{h \to 0} \frac{1.5(x^2 + 2hx + h^2) - x - h + 3.7 - 1.5x^2 + x - 3.7}{h}$$

$$= \lim_{h \to 0} \frac{3hx + 1.5h^2 - h}{h}$$

$$= \lim_{h \to 0} (3x + 1.5h - 1)$$

$$= 3x - 1$$

$f(x) = 1.5x^2 - x + 3.7$ D: $\mathbb{R}$
$f'(x) = 3x - 1$ D: $\mathbb{R}$

22. $f(x) = x + \sqrt{x}$ $\boxed{\text{SKIP}}$

24. $f(x) = \dfrac{3+x}{1-3x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{3+(x+h)}{1-3(x+h)} - \dfrac{3+x}{1-3x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{3+x+h}{1-3x-3h} - \frac{3+x}{1-3x} \right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(3+x+h)(1-3x) - (3+x)(1-3x-3h)}{(1-3x-3h)(1-3x)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{3-9x+x-3x^2+h-3hx-(3-9x-9h+x-3x^2-3hx)}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h\to 0} \frac{10h}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h\to 0} \frac{10}{(1-3x-3h)(1-3x)}$$

$$= \frac{10}{(1-3x)^2}$$

$f(x) = \dfrac{3+x}{1-3x}$ D: $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

$f'(x) = \dfrac{10}{(1-3x)^2}$ D: $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

26. $g(t) = \frac{1}{\sqrt{t}}$

$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}} \right) \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{t} - \sqrt{t+h}}{\sqrt{t+h} \cdot \sqrt{t}} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t+h} \cdot \sqrt{t}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}}$$

$$= \lim_{h \to 0} \frac{t - (t+h)}{h\sqrt{t+h} \cdot \sqrt{t} \, (\sqrt{t} + \sqrt{t+h})}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{t+h} \cdot \sqrt{t} \, (\sqrt{t} + \sqrt{t+h})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{t+h} \cdot \sqrt{t} \, (\sqrt{t} + \sqrt{t+h})}$$

$$= \frac{-1}{\sqrt{t} \cdot \sqrt{t} \, (\sqrt{t} + \sqrt{t})}$$

$$= \frac{-1}{t(2\sqrt{t})}$$

$$= \frac{-1}{2t^{3/2}}$$

$g(t) = \frac{1}{\sqrt{t}}$ D: $(0, \infty)$

$g'(t) = \frac{-1}{2t^{\frac{3}{2}}}$ D: $(0, \infty)$

30. SKIP

31 (a) $U'(t)$ is the rate at which the unemployment rate is changing with respect to time. Its units are percent per year.

32 (a) $P'(t)$ is the rate at which the percentage of Americans under the age of 18 is changing with respect to time. Its units are percent per year (%/yr).

33. f is not diff at $x=-4$ since f has a corner at $x=4$

f is not diff at $x=0$ since f is discontinuous at $x=0$. f is discontinuous at $x=0$ since $\lim_{x\to 0} f(x)$ DNE.

$\lim_{x\to 0} f(x)$ DNE since $\underbrace{\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)}$

Note: no specific values to list since not on graph

34. f is not diff at $x=0$ since f is discontinuous at $x=0$. f is discontinuous at $x=0$ since $-\infty = \lim_{x\to 0} f(x) \neq f(0) = 0$

f is not diff at $x=3$ since f has a vertical tangent at $x=3$

35. f is not diff at $x=-1$ since f has a vertical tangent at $x=-1$

f is not diff at $x=4$ since f has a corner at $x=4$

36. f is not diff at $x=-1$ since f is discontinuous at $x=-1$. f is discontinuous at $x=-1$ since $\lim_{x \to -1} f(x)$ DNE.

$\lim_{x\to -1} f(x)$ DNE since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$

f is not diff at $x=2$ since f has a corner at $x=2$

43. $f(x) = 1+4x-x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[1+4(x+h)-(x+h)^2]-(1+4x-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{1+4x+4h-(x^2+2hx+h^2)-1-4x+x^2}{h}$$

$$= \lim_{h\to 0} \frac{4h-2xh-h^2}{h}$$

$$= \lim_{h\to 0} (4-2x-h)$$

$$= 4-2x$$

$f'(x) = 4-2x$

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h} = \lim_{h\to 0} \frac{[4-2(x+h)]-(4-2x)}{h}$$

$$= \lim_{h\to 0} \frac{4-2x-2h-4+2x}{h}$$

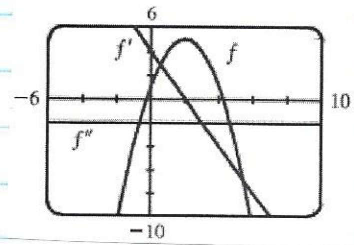
$$= \lim_{h\to 0} \frac{-2h}{h}$$

$$= \lim_{h\to 0} -2$$

$$= -2$$

$f''(x) = -2$

43. Cont'd



We see from the graph that our answers are reasonable because the graph of f' is that of a linear function and the graph of f'' is that of a constant function.