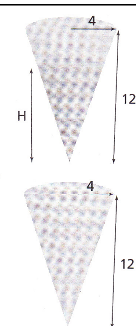
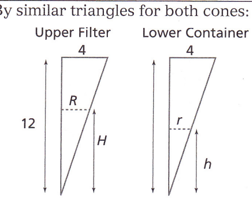
ANSWERS TO PRACTICE TEST #3 FREE RESPONSE QUESTIONS

1. The velocity of a particle moving along the x-axis is given by $v(t)=\dfrac{e^{t-1}}{2t^2+1}-t^2$ for $0\le t\le 12$. The position of the particle $x(t)$ is 3 when t is 2.
   a. During what interval is the particle moving to the left? Explain your reasoning.
   b. What is the position of the particle when it is farthest to the left?
   c. At what time in the interval $5\le t\le 10$ is the instantaneous velocity equal to the average velocity?
   d. How far did the particle travel on $0\le t\le 12$?

	Solution	Possible points
a.	The particle is moving to the left when $v(t)<0$. Therefore the particle is moving to the left in the interval $0.622<t<11.448$.	2: $\begin{cases}1: & v(t)<0\\ 1: & \text{answer}\end{cases}$
b.	$x(t)=x(0)+\int_0^{11.448} v(t)\,dt$ $x(t)\approx 2+(-335.690)=-333.690$ The particle is 336.69 units to the left of the starting point.	3: $\begin{cases}1: & \text{constant and limits}\\ 1: & \text{integral}\\ 1: & \text{answer}\end{cases}$
c.	$v_{avg}=\dfrac{1}{10-5}\int_5^{10} v(t)\,dt\approx\dfrac{1}{5}(-240.684)$ $v(t)=\dfrac{e^{t-1}}{2t^2+1}-t^2=-48.137$ $t\approx 7.292$	2: $\begin{cases}1: & \text{integral and constant}\\ 1: & \text{answer}\end{cases}$
d.	$x(t)=\int_0^{12}\lvert v(t)\rvert\,dt\approx 351.859$ Part d can also be answered by handling the particle's change in direction. $x(t)=\int_0^{0.622} v(t)\,dt-\int_{0.622}^{11.448} v(t)\,dt+\int_{11.448}^{12} v(t)\,dt$ $\Rightarrow x(t)\approx 351.859$	2: $\begin{cases}1: & \text{integral on } \lvert v(t)\rvert\\ 1: & \text{answer}\end{cases}$

2. A solution is draining through a conical filter into an identical conical container as shown in the diagram to the right. The solution drips from the upper filter into the lower container at a rate of π cm³/sec $\left(V_{cone}=\dfrac{\pi}{3}r^2h\right)$.
   a. How fast is the level in the upper filter dropping when the solution level in the upper filter is at 6 cm?
   b. If the conical filter is initially full, what is the level of the solution in the lower level when the solution in the level in the upper filter is at 6 cm and how fast is the level in the lower filter rising?
   c. How fast is the surface area of the solution in the lower filter increasing when the volume in the upper filter equals the volume in the lower container?

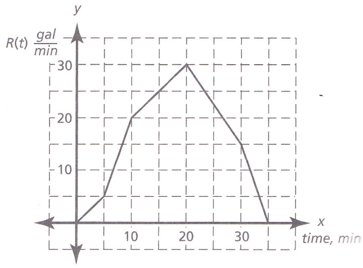


	Solution	Possible points
a.	By similar triangles for both cones:  $\dfrac{4}{12}=\dfrac{1}{3}=\dfrac{R}{H}\Rightarrow R=\dfrac{1}{3}H$ and $V_{cone}=\dfrac{\pi}{3}R^2H\Rightarrow\dfrac{\pi}{27}H^3$. Then $\dfrac{dV}{dt}=\dfrac{\pi}{9}H^2\left(\dfrac{dH}{dt}\right)\Rightarrow-\pi=\dfrac{\pi}{9}6^2\left(\dfrac{dH}{dt}\right)$ $\Rightarrow\dfrac{dH}{dt}=-\dfrac{1}{4}$ cm/sec.	3: $\begin{cases}1: & R=\dfrac{1}{3}H\\ 1: & \dfrac{dV}{dt}\\ 1: & \text{answer}\end{cases}$
b.	When $H=6$, the volume left in the upper filter is $V_{upper}=\dfrac{\pi}{27}6^3=8\pi$ cm³ and the volume in the lower container is $V_{lower}=\dfrac{\pi}{3}4^2(12)-8\pi=64\pi-8\pi=56\pi$ cm³. Solving for h in the lower container, $56\pi=\dfrac{\pi}{27}h^3\Rightarrow h=6\sqrt[3]{7}\approx 11.4776$. Since $\dfrac{dV}{dt}=\dfrac{\pi}{9}h^2\left(\dfrac{dh}{dt}\right)\Rightarrow\pi=\dfrac{\pi}{9}(11.4776)^2\left(\dfrac{dh}{dt}\right)$ and $\dfrac{dh}{dt}\approx 0.068$ cm/sec.	3: $\begin{cases}1: & V_{lower}\\ 1: & h_{lower}\\ 1: & \text{answer}\end{cases}$

	Solution	Possible points
c.	$V=\dfrac{\pi}{3}(4)^2(12)=64\pi$ Half full: $V=32\pi=\dfrac{\pi}{27}h^3\Rightarrow h=6\sqrt[3]{4}\approx 9.5244$ $\dfrac{dV}{dt}=\dfrac{\pi}{9}h^2\left(\dfrac{dh}{dt}\right)\Rightarrow\pi=\dfrac{\pi}{9}(9.5244)^2\left(\dfrac{dh}{dt}\right)$ and $\dfrac{dh}{dt}\approx 0.099$ cm/sec and $r=\dfrac{1}{3}h$, then $r\approx 3.175$ and $\dfrac{dr}{dt}=\dfrac{1}{3}\dfrac{dh}{dt}\Rightarrow 0.033$ cm/sec. $A=\pi r^2\Rightarrow\dfrac{dA}{dt}=2\pi r\left(\dfrac{dr}{dt}\right)$ and $\dfrac{dA}{dt}\approx 2\pi(3.175)^2(0.033)\approx 0.658$ cm²/sec.	3: $\begin{cases}1: & h\\ 1: & \dfrac{dh}{dt}\\ 1: & \dfrac{dA}{dt}\end{cases}$

3. Water is draining out of a tank at a variable rate as given by the chart and graph below.

t	$R(t)$ gal/min
0	0
5	5
10	20
20	30
30	15
35	0



a. Approximate the volume of water that has leaked from the tank for $0 \le t \le 35$ using a Riemann sum with a right-hand end point for the five unequal intervals indicated by the data in the chart.

b. Interpret the meaning of $\frac{1}{20}\int_{10}^{30} R(t)\,dt$ and find its value with the appropriate units using the data from part a.

c. Use the data from the table to find $R'(25)$. Show the computations that lead to your answer.

d. If the rate of the leak is modeled by $Q(t) = 16.78 \sin(0.15x - 1.25) + 14.6$, at what time is the rate of the leak increasing the fastest?

	Solution	Possible points
a.	5(5) + 5(20) + 10(30) + 10(15) + 5(0) = 575 gal	2: $\begin{cases} 1: \text{sum} \\ 1: \text{answer with units} \end{cases}$

	Solution	Possible points
b.	$\frac{1}{20}\int_{10}^{30} R(t)\,dt$ is the average rate of the leak over the 20-minute period, $10 \le t \le 30$. $\frac{1}{20}\int_{10}^{30} R(t)\,dt \approx \frac{1}{20}[10(30) + 10(15)] = 22.5$ gal/min	3: $\begin{cases} 1: \text{explanation} \\ 1: \text{use of Riemann sum} \\ 1: \text{answer} \end{cases}$
c.	$R'(25) \approx \frac{R(30) - R(20)}{30 - 20} = \frac{15 - 30}{10} = -1.5$ gal/min	2: $\begin{cases} 1: \text{difference quotient} \\ 1: \text{answer} \end{cases}$
d.	$Q'(t) = 16.78\ \cos(0.15t - 1.25)(0.15) = 0 \Rightarrow t \approx 18.805$ m $Q''(t) = -16.78\ \sin(0.15t - 1.25)(0.15)^2$ and $Q''(18.805) < 0$, thus the leak is at a maximum rate at $t = 18.805$ min.	2: $\begin{cases} 1: Q'(t) = 0 \\ 1: \text{answer with reason} \end{cases}$

4. Consider the curve defined by $x^2 + 4xy + y^2 = -12$.

a. Find $\frac{dy}{dx}$ in terms of x and y.

b. Find the equations of all horizontal tangents.

c. Find the equation of the tangent at the point (−4, 14).

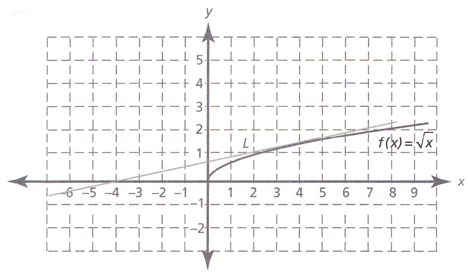
d. If $\frac{dy}{dt} = -\frac{1}{2}$ at the point (−4, 14), find $\frac{dx}{dt}$.

e. Use the tangent in part c to estimate the value of k for the point (−4.01, k) on the curve.

	Solution	Possible points
a.	$2x + 4y + 4x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$ $\frac{dy}{dx} = \frac{-(x + 2y)}{2x + y}$	2: $\begin{cases} 1: \text{implicit differentiation} \\ \quad < -1 > \text{each error} \\ 1: \text{answer} \end{cases}$

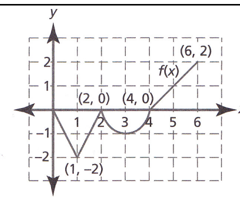
	Solution	Possible points
b.	$\frac{dy}{dx} = \frac{-(x + 2y)}{(2x + y)} = 0 \Rightarrow x = -2y$ $(-2y)^2 + 4(-2y)y + y^2 = -12$ and $-3y^2 = -12 \Rightarrow y \pm 2$. Verify that both values of y yield horizontal tangent lines by showing that $\frac{dy}{dx} = 0$ in both cases. When $y = 2$, $x = -4$ and $\frac{dy}{dx} = \frac{-(-4 + 4)}{-8 + 2} = 0$. When $y = -2$, $x = -4$ and $\frac{dy}{dx} = \frac{-(4 - 4)}{8 - 2} = 0$.	2: $\begin{cases} 1: \frac{dy}{dx} = 0 \\ 1: \text{solutions} \end{cases}$
c.	$\left.\frac{dy}{dx}\right\|_{(-4,14)} = \frac{-[-4 + 2(14)]}{2(-4) + 14} = \frac{-24}{6} = -4$ $y - 14 = -4(x + 4) \Rightarrow y = -4x - 2$	1: answer
d.	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy/dt}{dx/dt}$ $-4 = \frac{-1/2}{dx/dt} \Rightarrow \frac{dx}{dt} = \frac{1}{8}$	2: $\begin{cases} 1: \text{relationship} \\ 1: \text{answer} \end{cases}$
e.	$y = 14 - 4(-4.01 + 4) \Rightarrow y \approx 14.04$	1: answer

5. Let L be the tangent to $f(x)=\sqrt{x}$ at any point on the curve as shown in the diagram to the right.
   a. Show that the x-intercept of the tangent to the curve at the point $(h, \sqrt{h})$ is $-h$.
   b. Find the area of the region bounded by the tangent to the curve at $x = 4$, the curve, and the x-axis.
   c. What is the volume when the region found in part b is rotated about the line $x = 4$.



	Solution	Possible points
a.	$y'(x)=\dfrac{1}{2\sqrt{x}}$ and $y'(h)=\dfrac{1}{2\sqrt{h}}$ L: $y-\sqrt{h}=\dfrac{1}{2\sqrt{h}}(x-h)$ The x-intercept occurs when $y = 0$. Thus $\left(0-\sqrt{h}\right)2\sqrt{h}=(x-h)\Rightarrow x=-h$.	3: { 1: $y'(x)$ 1: L 1: $y=0\rightarrow$ answer
b.	The area is found by integrating with respect to y. Rewriting the functions, L: $y-2=\dfrac{1}{4}(x-4)\Rightarrow x=4y-4$ and $x=y^2$ $A=\int_0^2\left[y^2-(4y-4)\right]dy$ $=\left(\dfrac{y^3}{3}-2y^2+4y\right)\Big\vert_0^2$ $=\dfrac{8}{3}-8+8=\dfrac{8}{3}$ The area may also be determined by integrating with respect to x, but two integrals will be needed. L: $y-2=\dfrac{1}{4}(x-4)\Rightarrow y=\dfrac{1}{4}x+1$, with the x-intercept of -4. $A=\int_{-4}^{0}\left(\dfrac{1}{4}x+1\right)dx+\int_0^4\left(\dfrac{1}{4}x+1-x^{\frac{1}{2}}\right)dx$ $=\left(\dfrac{1}{8}x^2+x\right)\Big\vert_{-4}^{0}+\left(\dfrac{1}{8}x^2+x-\dfrac{2}{3}x^{\frac{3}{2}}\right)\Big\vert_0^4$ $=0-(2-4)+\left(2+4-\dfrac{16}{3}\right)=\dfrac{8}{3}$	3: { 1: integral with limits 1: antiderivative 1: answer
c.	$V=\pi\int_0^2\left[4-(4y-4)\right]^2-\left(4-y^2\right)^2dy$ $=\pi\int_0^2 16y^2-\left(16-8y^2+y^4\right)dy$ $=\pi\left(8y^3-16y-\dfrac{y^5}{5}\right)\Big\vert_0^2=\dfrac{128\pi}{5}$	3: { 1: integral with limits 1: antiderivative 1: answer

6. Let f be a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle. Let $g(x)=3+\int_2^x f(t)\,dt$.
   a. Find $g(1)$, $g'(1)$, and $g''(1)$.
   b. What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?
   c. What is the average value of $g(x)$ in the interval $2 \le x \le 6$?
   d. Identify the x-coordinate of any extrema of $g(x)$ on $0 < x < 6$. Explain your reasoning.
   e. Identify the x-coordinate of any points of inflection of $g(x)$ on $0 < x < 6$.



	Solution	Possible points
a.	$g(1)=3+\int_2^1 f(t)\,dt=3-\int_1^2 f(t)\,dt=3+\dfrac{1}{2}(1)(2)=4$ $g'(x)=f(x)\Rightarrow g'(1)=-2$ $g''(x)=f'(x)\Rightarrow g''(1)$ does not exist.	3: 1 for each answer
b.	$\dfrac{g(6)-g(2)}{6-2}=\dfrac{\left[3+\left(-\dfrac{\pi}{2}\right)+\dfrac{1}{2}(2)(2)\right]-(3)}{4}=\dfrac{4-\pi}{8}$	2: { 1: difference quotient 1: answer
c.	$g_{avg}=\dfrac{1}{6-2}\left(3+\int_2^6 f(t)\,dt\right)$ $=\dfrac{1}{4}\left(3-\pi+\dfrac{1}{2}(2)(2)\right)=\dfrac{5-\pi}{4}$	2: { 1: g_{avg} 1: value
d.	Extrema exist where $g'(x)=f(x)=0$ or $g'(x)$ does not exist. Extrema exist at $x = 2, 4$.	1: reason and answer
e.	Inflection points occur where $g''(x)=f'(x)$ change sign. The points of inflection exist at $x = 1, 2,$ and 3.	1: reason and answer

CALCULUS AB AND BC SCORING CHART

SECTION I: MULTIPLE CHOICE

________ × 1.2 = ________ = ________

correct (out of 45) — total (out of 54) — (round to nearest whole number)

SECTION II: FREE RESPONSE

Question 1 Score out of 9 points = ________

Question 2 Score out of 9 points = ________

Question 3 Score out of 9 points = ________

Question 4 Score out of 9 points = ________

Question 5 Score out of 9 points = ________

Question 6 Score out of 9 points = ________

Sum for Section II = ________

(out of 54)

Composite Score

Section I total	=
Section II total	=
Composite score	= ________ (out of 108)

Grade Conversion Chart*

Composite score range	AP Exam Grade
70–108	5
55–69	4
40–54	3
30–39	2
0–29	1

***Note:** The ranges listed above are only approximate. Each year the ranges for the actual AP Exam are somewhat different. The cutoffs are established after the exams are given to over 200,000 students, and are based on the difficulty level of the exam each year.