5. If $3x^2+2xy+y^2=2$, then the value of $\frac{dy}{dx}$ at $x=1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

40. If $\tan(xy)=x$, then $\frac{dy}{dx}=$

(A) $\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy)-y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy)-y}{x}$

7. If $y=\ln\left(x^2+y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

13. If $x^2+xy+y^3=0$, then, in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \dfrac{-2y - 6x}{2x + 2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\dfrac{dy}{dx}$ is not defined at $x = 1$.

40. E $\sec^2(xy) \cdot (xy' + y) = 1$, $xy' \sec^2(xy) + y\sec^2(xy) = 1$, $y' = \dfrac{1 - y\sec^2(xy)}{x\sec^2(xy)} = \dfrac{\cos^2(xy) - y}{x}$

7. D $\dfrac{dy}{dx} = \dfrac{2x + 2y \cdot \dfrac{dy}{dx}}{x^2 + y^2}$ at $(1,0) \Rightarrow y' = \dfrac{2}{1} = 2$

13. A $2x + x \cdot y' + y + 3y^2 \cdot y' = 0 \Rightarrow y' = -\dfrac{2x + y}{x + 3y^2}$

9. If $xy^2 + 2xy = 8$, then, at the point $(1, 2)$, y' is

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

6. If $y^2 - 2xy = 16$, then $\frac{dy}{dx} =$

(A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

9. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

9. B Take the derivative of each side of the equation with respect to x.

$2xyy' + y^2 + 2xy' + 2y = 0$, substitute the point $(1,2)$

$(1)(4)y' + 2^2 + (2)(1)y' + (2)(2) = 0 \Rightarrow y = -\frac{4}{3}$

6. C $2y \cdot y' - 2x \cdot y' - 2y = 0 \Rightarrow y' = \frac{y}{y-x}$

9. E $1 + (2x \cdot y' + 2y) - 2y \cdot y' = 0;\ y' = \frac{1+2y}{2y-2x}$. This cannot be evaluated at $(1,1)$ and so y' does not exist at $(1,1)$.

17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\dfrac{dy}{dx} =$

(A) $-\dfrac{x^2+y}{x+2y^2}$

(B) $-\dfrac{x^2+y}{x+y^2}$

(C) $-\dfrac{x^2+y}{x+2y}$

(D) $-\dfrac{x^2+y}{2y^2}$

(E) $\dfrac{-x^2}{1+2y^2}$

17. If $x^2 + y^2 = 25$, what is the value of $\dfrac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\dfrac{25}{27}$ (B) $-\dfrac{7}{27}$ (C) $\dfrac{7}{27}$ (D) $\dfrac{3}{4}$ (E) $\dfrac{25}{27}$

17. A Using implicit differentiation, $\frac{y+xy'}{xy}=1$. When $x=1$, $\frac{y+y'}{y}=1 \Rightarrow y'=0$.

Alternatively, $xy=e^x$, $y=\frac{e^x}{x}$, $y'=\frac{xe^x-e^x}{x^2}=\frac{e^x(x-1)}{x^2}$. $y'(1)=0$

4. A $3x^2+3(y+x\cdot y')+6y^2\cdot y'=0$; $y'(3x+6y^2)=-(3x^2+3y)$

$$y'=-\frac{3x^2+3y}{3x+6y^2}=-\frac{x^2+y}{x+2y^2}$$

17. A $x^2+y^2=25$; $2x+2y\cdot y'=0$; $x+y\cdot y'=0$; $y'(4,3)=-\frac{4}{3}$;

$x+y\cdot y'=0 \Rightarrow 1+y\cdot y''+y'\cdot y'=0$; $1+(3)y''+\left(-\frac{4}{3}\right)\cdot\left(-\frac{4}{3}\right)=0$; $y''=-\frac{25}{27}$

10. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

10. B $y = xy + x^2 + 1;\ y' = xy' + y + 2x;$ at $x = -1, y = 1;\ y' = -y' + 1 - 2 \Rightarrow y' = -\dfrac{1}{2}$

6. A Substitute $x = 2$ into the equation to find $y = 3$. Taking the derivative implicitly gives $\dfrac{d}{dx}\left(x^2 + xy\right) = 2x + xy' + y = 0$. Substitute for x and y and solve for y'.

$4 + 2y' + 3 = 0;\ y' = -\dfrac{7}{2}$

Calculator Active

4. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{-6}{7y^3}$ (B) $\frac{-3}{y^3}$ (C) 3 (D) $\frac{3}{2y}$ (E) $\frac{-9}{4y^3}$

15. If $\tan(x + y) = x$, then $\frac{dy}{dx} =$

(A) $\tan^2(x + y)$

(B) $\sec^2(x + y)$

(C) $\ln|\sec(x + y)|$

(D) $\sin^2(x + y) - 1$

(E) $\cos^2(x + y) - 1$

No Calculators

4. E p. 11

$$y^2 - 3x = 7 \quad \Rightarrow \quad 2y\frac{dy}{dx} - 3 = 0$$

$$\frac{dy}{dx} = \frac{3}{2y}$$

$$\frac{d^2y}{dx^2} = \frac{2y \cdot 0 - 3 \cdot 2\frac{dy}{dx}}{4y^2} = \frac{-6 \cdot \frac{3}{2y}}{4y^2} = -\frac{9}{4y^3}$$

15. E p. 28

$$\tan(x + y) = x$$

$$\sec^2(x + y)\left[1 + \frac{dy}{dx}\right] = 1$$

$$1 + \frac{dy}{dx} = \cos^2(x + y)$$

$$\frac{dy}{dx} = \cos^2(x + y) - 1$$

No Calculators

22. If $x^2 + 2xy - 3y = 3$, then the value of $\frac{dy}{dx}$ at $x = 2$ is

(A) 1

(B) 2

(C) -2

(D) $\frac{10}{3}$

(E) $-\frac{1}{2}$

22. C p. 52

We use implicit differentiation to obtain $\frac{dy}{dx}$.

$x^2 + 2xy - 3y = 3$

$2x + 2y + 2x\frac{dy}{dx} - 3\frac{dy}{dx} = 0$

$(2x - 3)\frac{dy}{dx} = -(2x + 2y)$

$\frac{dy}{dx} = -\frac{2x + 2y}{2x - 3}$

We also need to find the y-coordinate that pairs with $x = 2$.

$x = 2 \quad \Rightarrow \quad 4 + 4y - 3y = 3$

$\Rightarrow \quad y = -1$

$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{4 - 2}{3 - 4} = -2$