

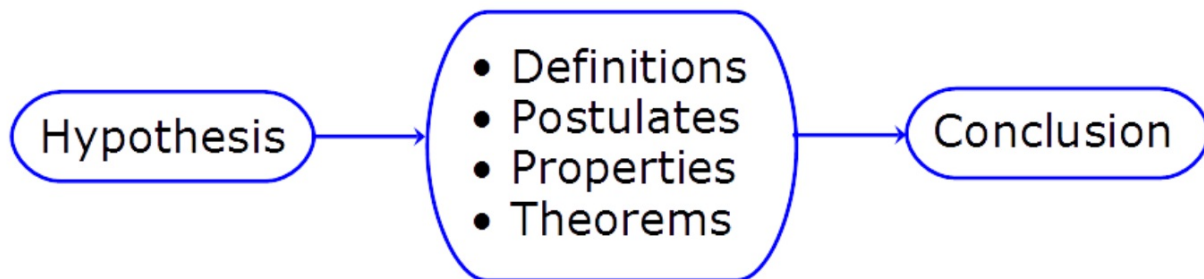
Lesson 2 - 6

Geometric Proof Going Deeper

Essential question: *How can you organize the deductive reasoning of a geometric proof?*



When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.



Definitions

Examples: A right angle measures 90°

Supplementary angles add together to be 180°

Postulates: A statement accepted without proof

Examples: Angle Addition Postulate

Segment Addition Postulate

Properties

Examples: The algebraic properties from yesterday

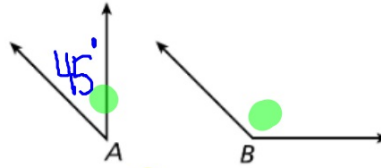
Theorems: A statement that can be proven

Once you prove a theorem, you can use it to prove other theorems

A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

Write a justification for each step, given that $\angle A$ and $\angle B$ are supplementary and $m\angle A = 45^\circ$.

Prove that $m\angle B = 135^\circ$.



Statements

Reasons

1. $\angle A$ and $\angle B$ are supplementary.

$$m\angle A = 45^\circ$$

2. $m\angle A + m\angle B = 180^\circ$

3. $45^\circ + m\angle B = 180^\circ$

4. $m\angle B = 135^\circ$

Given information

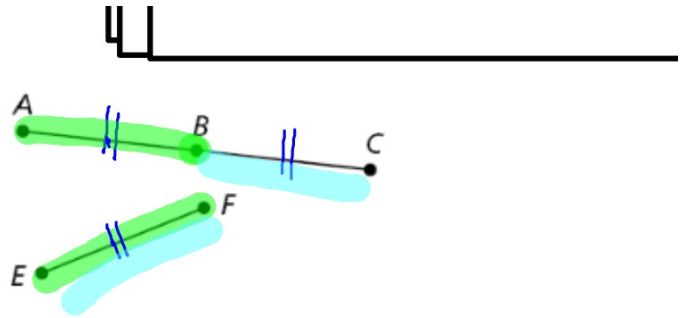
Def. of supp \angle s

Subst. Prop of =
Steps 1, 2

Subtr. Prop of =

Write a justification for each step, given that B is the midpoint of \overline{AC} and $\overline{AB} \cong \overline{EF}$.

Prove $\overline{BC} \cong \overline{EF}$



Statements	Reasons
1. B is the midpoint of \overline{AC} .	Given information
2. $\overline{AB} \cong \overline{BC}$	Def. of <u>mdpt.</u>
3. $\overline{AB} \cong \overline{EF}$	Given information
4. $\overline{BC} \cong \overline{EF}$	Trans. Prop. of \cong SUBST.

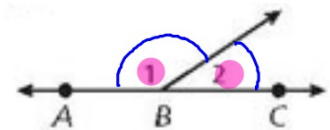
$1=2$
 $1=3$

 $2=3$

We are going to prove the theorem below

Theorem

THEOREM	HYPOTHESIS	CONCLUSION
2-6-1 Linear Pair Theorem If two angles form a linear pair, then they are supplementary.	$\angle A$ and $\angle B$ form a linear pair.	$\angle A$ and $\angle B$ are supplementary.



Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair.	1. Given
2. \overrightarrow{BA} and \overrightarrow{BC} form a line.	2. Def. of lin. pair
3. $m\angle ABC = 180^\circ$	3. Def. of straight \angle
4. $m\angle 1 + m\angle 2 = m\angle ABC$	4. \angle Add. Post.
5. $m\angle 1 + m\angle 2 = 180^\circ$	5. Subst. Steps 3, 4
6. $\angle 1$ and $\angle 2$ are supplementary.	6. Def. of supp. \angle

We can now use the theorem in other proofs

Theorem

$\angle 1 + \angle 2$ sup
 $\angle 2 + \angle 3$ sup



THEOREM	HYPOTHESIS	CONCLUSION
<p>2-6-2 Congruent Supplements Theorem If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.</p>	<p>$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.</p>	<p>$\angle 1 \cong \angle 3$</p>

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supp., and $\angle 2$ and $\angle 3$ are supp.	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	2. Def. of supp. \triangle
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. Subst.
4. $m\angle 2 = m\angle 2$	4. Reflex. Prop. of =
5. $m\angle 1 = m\angle 3$	5. <u>Subtr.</u> Prop. of =
6. $\angle 1 \cong \angle 3$	6. Def. of $\cong \triangle$

Theorems

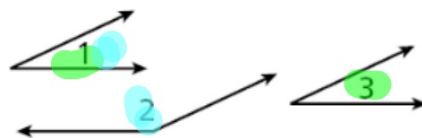
THEOREM	HYPOTHESIS	CONCLUSION
2-6-3 Right Angle Congruence Theorem All right angles are congruent.	$\angle A$ and $\angle B$ are right angles.	$\angle A \cong \angle B$
2-6-4 Congruent Complements Theorem If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	$\angle 1 \cong \angle 3$

Use the given plan to write a two-column proof.

Given: $\angle 1$ and $\angle 2$ are supplementary, and

$$\angle 1 \cong \angle 3$$

Prove: $\angle 3$ and $\angle 2$ are supplementary.

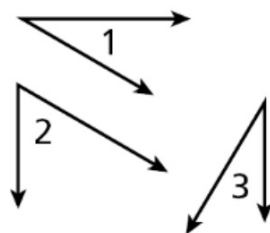


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 3$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$	2. Def. of supp. \angle s
3. $m\angle 1 = m\angle 3$	3. Def. of $\cong \angle$ s
4. $m\angle 3 + m\angle 2 = 180^\circ$	4. <u>Subst.</u>
5. $\angle 3$ and $\angle 2$ are supplementary	5. Def. of supp. \angle s



Given: $\angle 1$ and $\angle 2$ are complementary, and
 $\angle 2$ and $\angle 3$ are complementary.

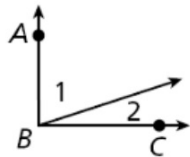
Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	1. <input type="text"/>
2. <input type="text"/>	2. Def. of comp. \angle s
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. <input type="text"/>
4. $m\angle 2 = m\angle 2$	4. <input type="text"/>
5. <input type="text"/>	5. <u>Subtr.</u> Prop. of =
6. $\angle 1 \cong \angle 3$	6. <input type="text"/>

Lesson Quiz: Part I

Write a justification for each step, given that $m\angle ABC = 90^\circ$ and $m\angle 1 = 4m\angle 2$. Prove $m\angle 2 = 18^\circ$



Statements

Reasons

1. $m\angle ABC = 90^\circ$ and $m\angle 1 = 4m\angle 2$

2. $m\angle 1 + m\angle 2 = m\angle ABC$

3. $4m\angle 2 + m\angle 2 = 90^\circ$

4. $5m\angle 2 = 90^\circ$

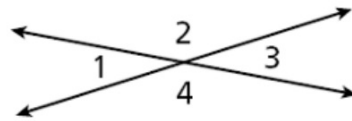
5. $m\angle 2 = 18^\circ$

Lesson Quiz: Part II

2. Use the given plan to write a two-column proof.

Given: $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$

Prove: $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$



1. $\angle 1$ and $\angle 2$ are supp.
 $\angle 1$ and $\angle 4$ are supp.

2. $m\angle 1 + m\angle 2 = 180^\circ$,
 $m\angle 1 + m\angle 4 = 180^\circ$

3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$

Pages 113-115 #1, 2, 4, 7, 8, 11-13, 20-22

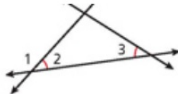
#4, 7, 8: copy ENTIRE proof on paper including given, prove, picture, and 2 columns (try your best and we will go over them)

#11-13, 20-22: draw pictures on paper and show work

Vocabulary Apply the vocabulary from this lesson to answer each question.

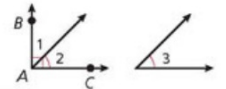
- In a *two-column proof*, you list the in the left column and the in the right column. (*statements* or *reasons*)
 - A is a statement you can prove. (*postulate* or *theorem*)
4. Fill in the blanks to complete the two-column proof.

Given: $\angle 2 \cong \angle 3$
 Prove: $\angle 1$ and $\angle 3$ are supplementary.
 Proof:



Statements	Reasons
1. $\angle 2 \cong \angle 3$	1. Given
2. $m\angle 2 = m\angle 3$	2. a. <u> </u> ?
3. b. <u> </u> ?	3. Lin. Pair Thm.
4. $m\angle 1 + m\angle 2 = 180^\circ$	4. Def. of supp. Δ
5. $m\angle 1 + m\angle 3 = 180^\circ$	5. c. <u> </u> ? Steps 2, 4
6. d. <u> </u> ?	6. Def. of supp. Δ

8. Given: $\angle BAC$ is a right angle. $\angle 2 \cong \angle 3$
 Prove: $\angle 1$ and $\angle 3$ are complementary.

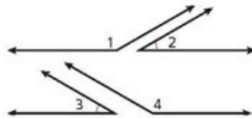


Proof:

Statements	Reasons
1. $\angle BAC$ is a right angle.	1. Given
2. $m\angle BAC = 90^\circ$	2. a. <u> </u> ?
3. b. <u> </u> ?	3. \angle Add. Post.
4. $m\angle 1 + m\angle 2 = 90^\circ$	4. Subst. Steps 2, 3
5. $\angle 2 \cong \angle 3$	5. Given
6. c. <u> </u> ?	6. Def. of $\cong \Delta$
7. $m\angle 1 + m\angle 3 = 90^\circ$	7. d. <u> </u> ? Steps 4, 6
8. e. <u> </u> ?	8. Def. of comp. Δ

Fill in the blanks to complete each two-column proof.

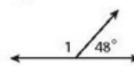
7. Given: $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 2 \cong \angle 3$
 Prove: $\angle 1 \cong \angle 4$
 Proof:



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	1. Given
2. a. <u> </u> ?	2. Def. of supp. Δ
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. b. <u> </u> ?
4. $\angle 2 \cong \angle 3$	4. Given
5. $m\angle 2 = m\angle 3$	5. Def. of $\cong \Delta$
6. c. <u> </u> ?	6. Subtr. Prop. of = Steps 3, 5
7. $\angle 1 \cong \angle 4$	7. d. <u> </u> ?

Find each angle measure.

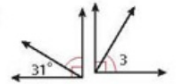
11. $m\angle 1$



12. $m\angle 2$

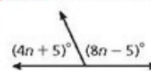


13. $m\angle 3$

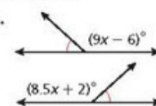


Algebra Find the value of each variable.

- 20.



- 21.



- 22.

