Section 9.4
Composition of Functions

Warm Up

Determine the coordinates of the image of *P*(4, –7) under each transformation.

1. a translation 3 units left and 1 unit up

(1, –6)

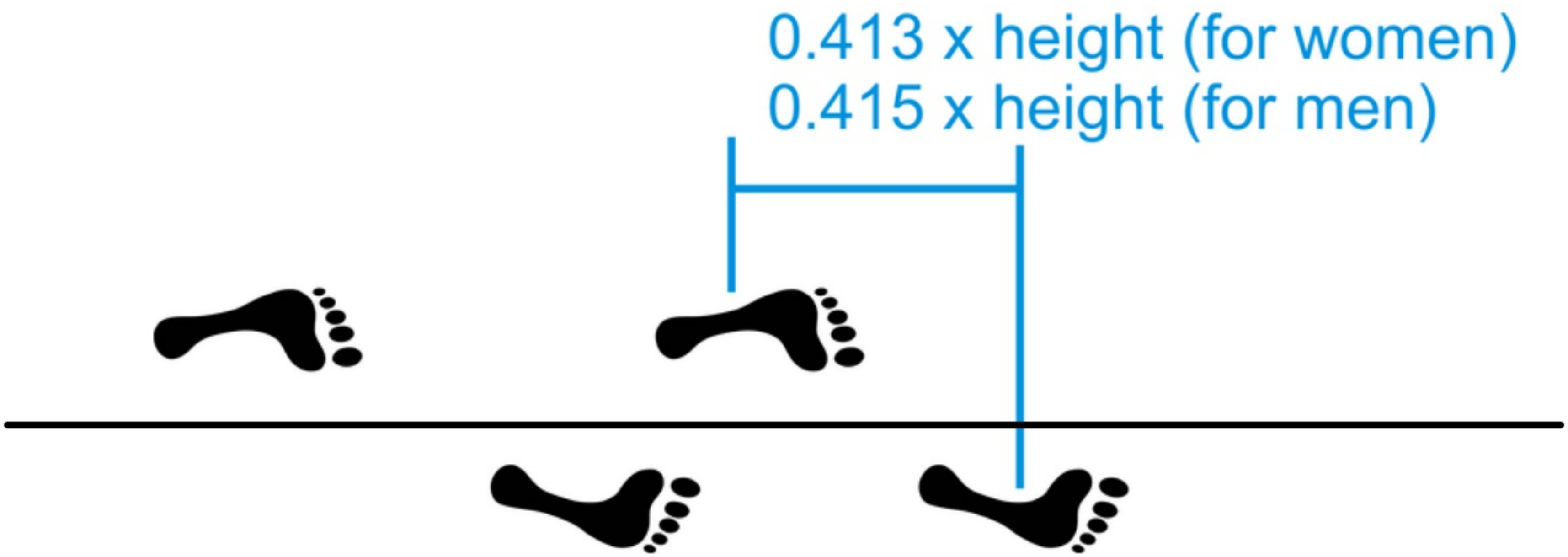
2. a rotation of 90° about the origin

(7, 4)

3. a reflection across the *y*-axis

(–4, –7)

A **composition of transformations** is one transformation followed by another. For example, a **glide reflection** is the composition of a translation and a reflection across a line parallel to the translation vector.

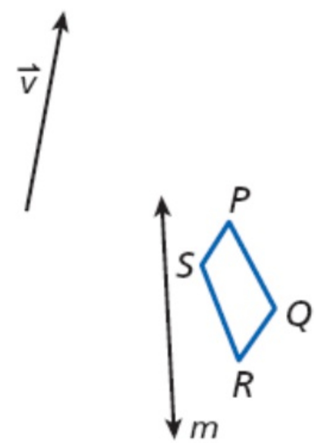


Theorem 12-4-1

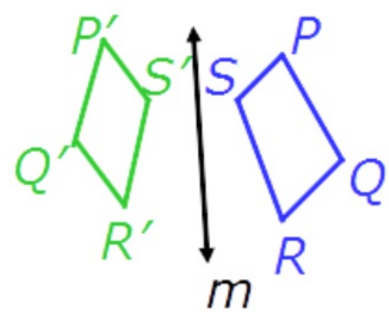
A composition of two isometries is an isometry.

Draw the result of the composition of isometries.

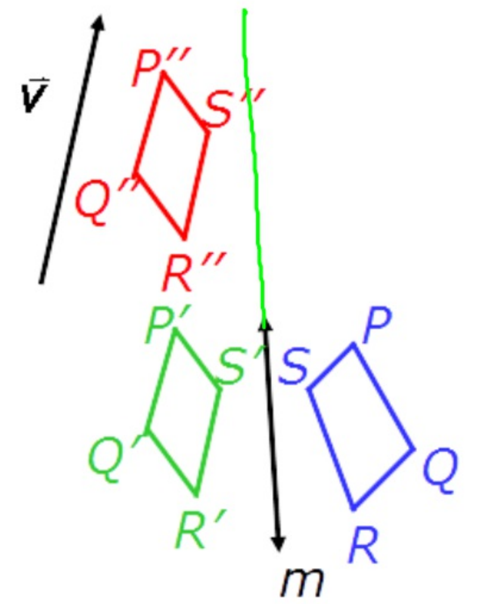
Reflect *PQRS* across line *m* and then translate it along $\vec{v}$.



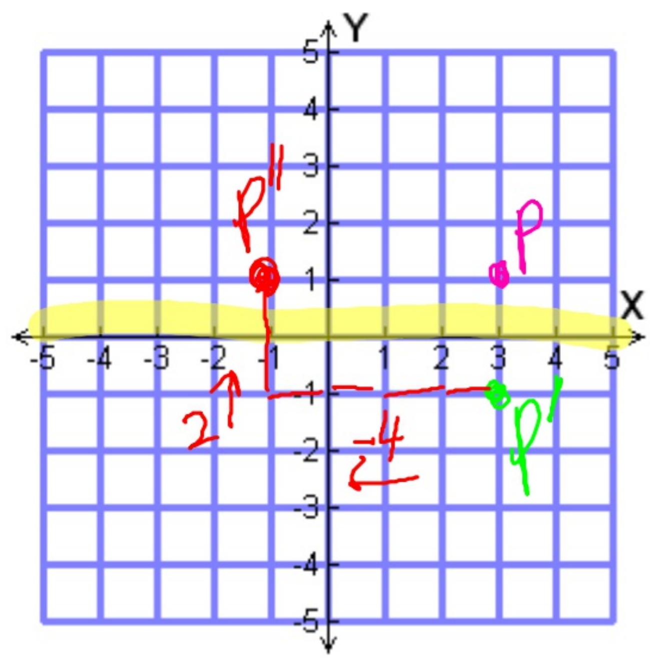
Step 1 Draw *P'Q'R'S'*, the reflection image of *PQRS*.

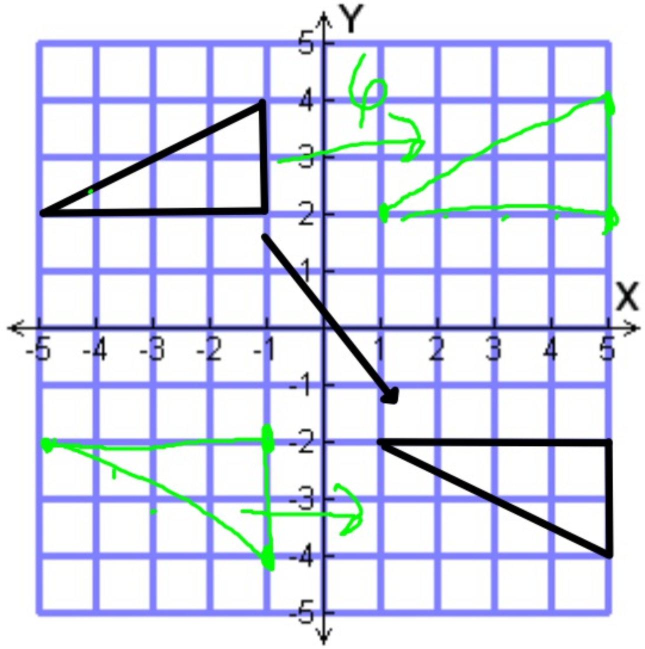


Step 2 Translate *P′Q′R′S′* along $\vec{v}$ to find the final image, *P″Q″R″S″*.



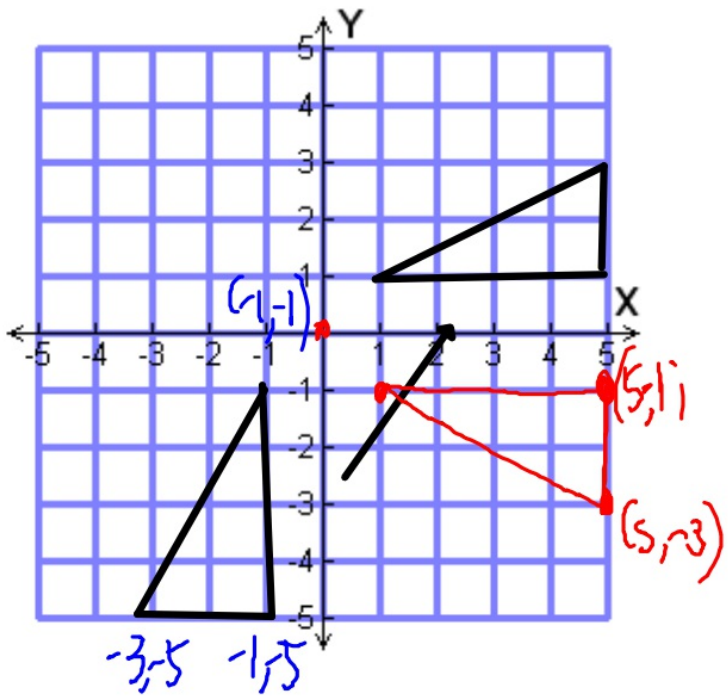
The point (3, 1) is reflected about the x-axis
and then translated according to the vector <-4, 2>. What is the coordinate of the image? (-1, 1)





What combinations of transformations gives us the image?

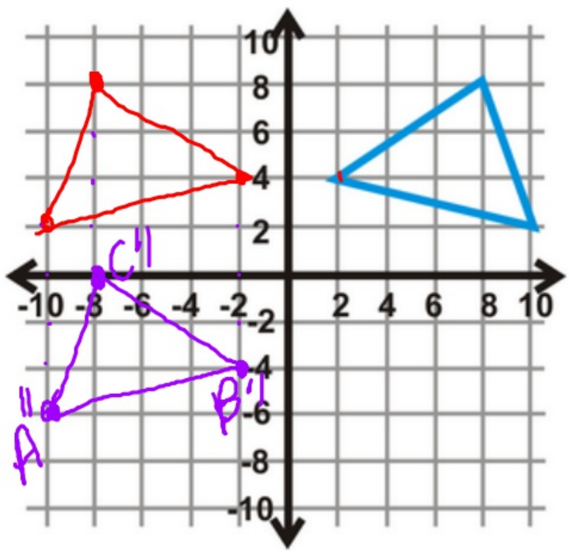
Translation <6, 0>
Reflection about x-axis



What combinations of transformations gives us the image?

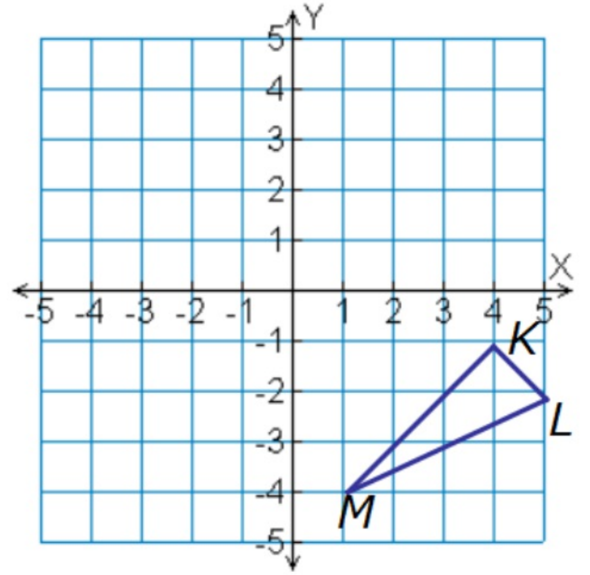
90 deg rotation about origin
Reflection about x-axis

Example 1: Reflect triangle ABC over the y-axis and then translate the image 8 units down.



Draw the result of the composition of isometries.

Δ*KLM* has vertices *K*(4, –1), *L*(5, –2), and *M*(1, –4). Rotate Δ*KLM* 180° about the origin and then reflect it across the *y*-axis.

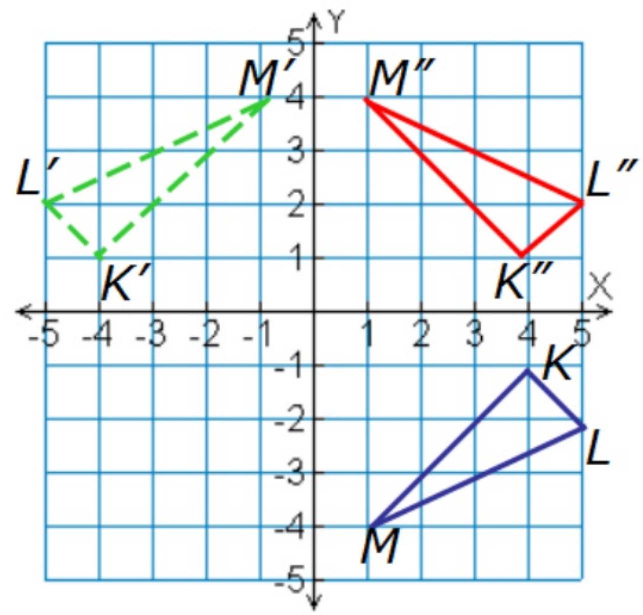


Step 1 The rotational image of (*x*, *y*) is (–*x*, –*y*).

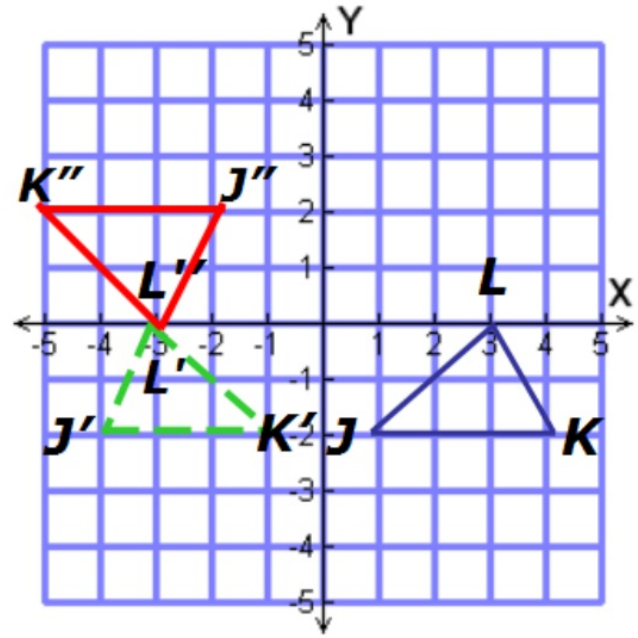
K(4, –1) → *K*′(–4, 1),
L(5, –2) → *L*′(–5, 2), and
M(1, –4) → *M*′(–1, 4).

Step 2 The reflection image of (*x*, *y*) is (–*x*, *y*). change x

K′(–4, 1) → *K*″(4, 1),
L′(–5, 2) → *L*″(5, 2), and
M′(–1, 4) → *M*″(1, 4).



Δ*JKL* has vertices *J*(1,–2), *K*(4, –2), and *L*(3, 0). Reflect Δ*JKL* across the *x*-axis and then rotate it 180° about the origin.



Step 1 The reflection image of (x, y) is $(-x, y)$.

$J(1, -2) \rightarrow J'(-1, -2)$,
$K(4, -2) \rightarrow K'(-4, -2)$, and
$L(3, 0) \rightarrow L'(-3, 0)$.

Step 2 The rotational image of (x, y) is $(-x, -y)$.

$J'(-1, -2) \rightarrow J''(1, 2)$,
$K'(-4, -2) \rightarrow K''(4, 2)$, and
$L'(-3, 0) \rightarrow L''(3, 0)$.

$\triangle RST$ **has vertices** $R(1, 2)$, $S(1, 4)$, **and** $T(-3, 4)$. **Rotate** $\triangle RST$ **90° about the origin and then reflect it across the** y**-axis.**

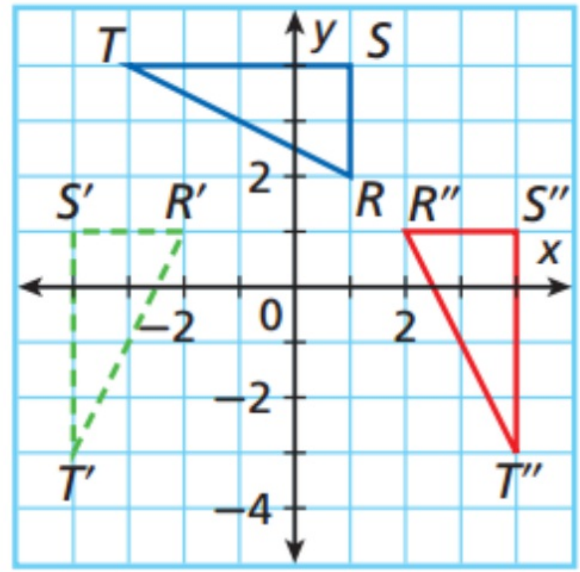
Step 1 The rotation image of (x, y) is $(-y, x)$.

$R(1, 2) \rightarrow R'(-2, 1)$, $S(1, 4) \rightarrow S'(-4, 1)$, and $T(-3, 4) \rightarrow T'(-4, -3)$.

Step 2 The reflection image of (x, y) is $(-x, y)$.

$R'(-2, 1) \rightarrow R''(2, 1)$, $S'(-4, 1) \rightarrow S''(4, 1)$, and $T'(-4, -3) \rightarrow T''(4, -3)$.

Step 3 Graph the preimage and images.

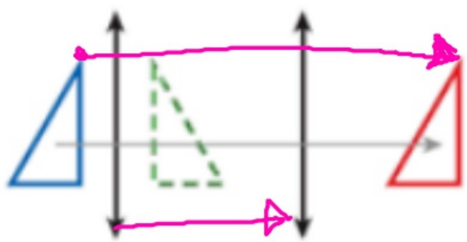


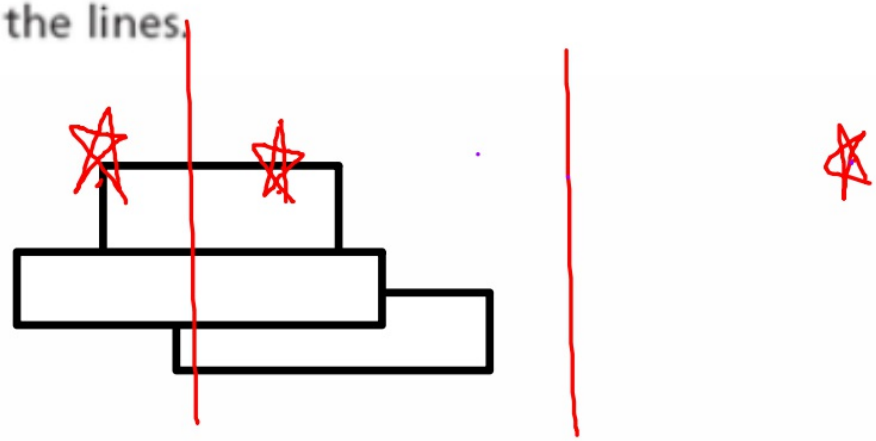
9-4 Compositions of Transformations

Theorem 12-4-2

The composition of two reflections across two parallel lines is equivalent to a translation.

- The translation vector is perpendicular to the lines.
- The length of the translation vector is twice the distance between the lines.

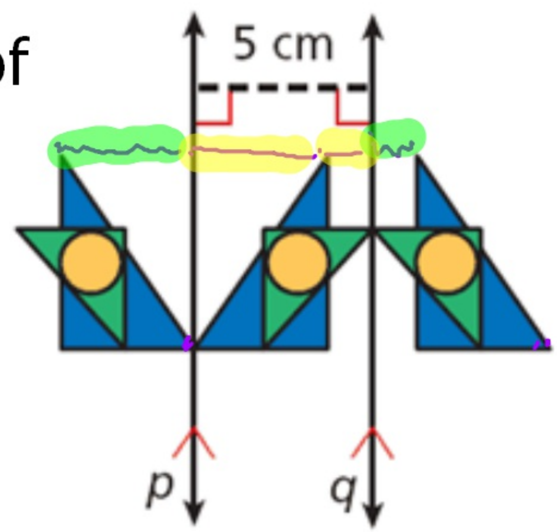




Example 2: Art Application

Sean reflects a design across line *p* and then reflects the image across line *q*. Describe a single transformation that moves the design from the original position to the final position.

By Theorem 12-4-2, the composition of two reflections across parallel lines is equivalent to a translation perpendicular to the lines.



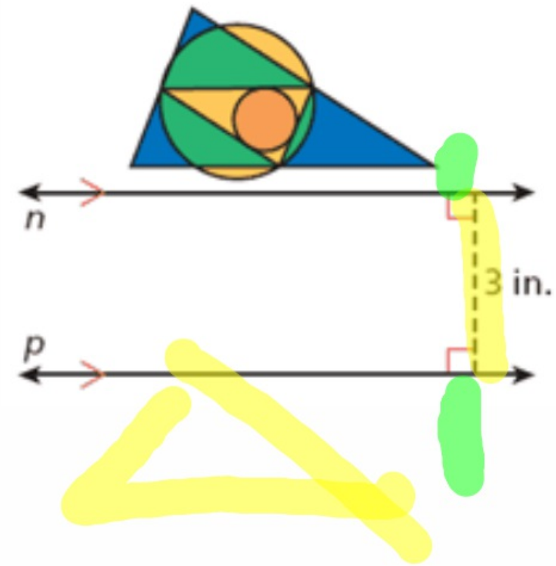
By Theorem 12-4-2, the translation vector is

2(5 cm) = 10 cm to the right.

Check It Out! Example 2

What if…? Suppose Tabitha reflects the figure across line *n* and then the image across line *p*. Describe a single transformation that is equivalent to the two reflections.

A translation in direction ⊥ to *n* and *p*, by distance of 6 in.



Theorem 12-4-3

Any translation or rotation is equivalent to a composition of two reflections.

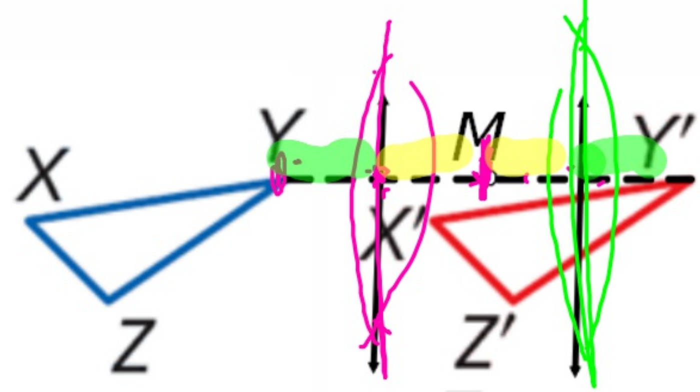
Example 3A: Describing Transformations in Terms of Reflections

Copy each figure and draw two lines of reflection that produce an equivalent transformation.

translation: ΔXYZ□ ΔX′Y′Z′.

Step 1 Draw $\overline{YY'}$ and locate the midpoint *M* of $\overline{YY'}$

Step 2 Draw the perpendicular bisectors of *YM* and *Y′M.*



Lesson Quiz: Part I

***PQR* has vertices *P*(5, –2), *Q*(1, –4), and *P*(–3, 3).**

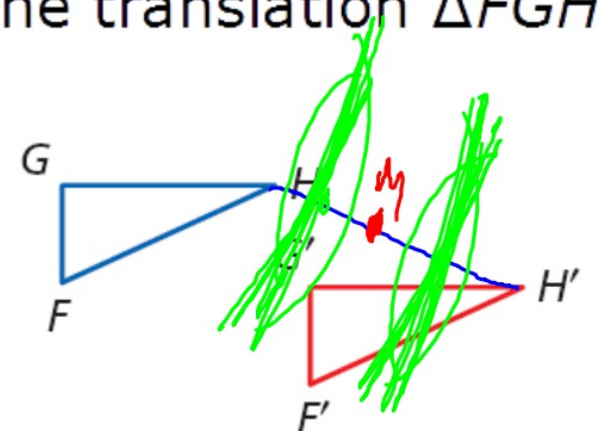
1. Translate Δ*PQR* along the vector <–2, 1> and then reflect it across the *x*-axis.

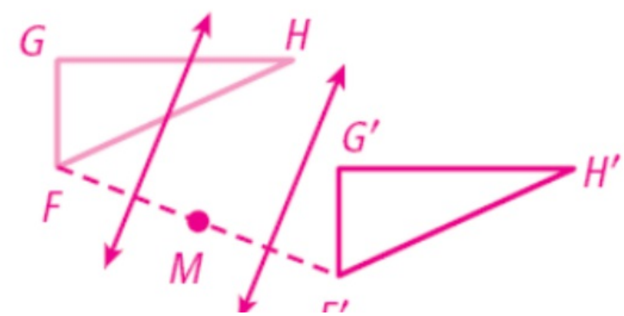
P″(3, 1), *Q*″(–1, –3), *R*″(–5, –4)

2. Reflect Δ*PQR* across the line *y* = *x* and then rotate it 90° about the origin.

P″(–5, –2), *Q*″(–1, 4), *R*″(3, 3)

3. Copy the figure and draw two lines of reflection that produce an equivalent transformation of the translation Δ*FGH* → Δ*F*′*G*′*H*′.





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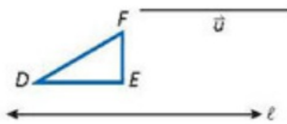
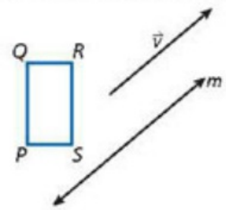
#2-10, 12-13, 15-19, 23-25

Draw the result of each composition of isometries.

2. Translate $\triangle DEF$ along $\vec{u}$ and then reflect it across line ℓ.

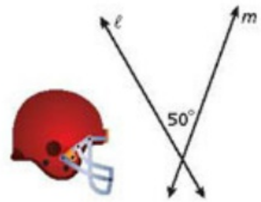


3. Reflect rectangle $PQRS$ across line m and then translate it along $\vec{v}$.



4. $\triangle ABC$ has vertices $A(1, -1)$, $B(4, -1)$, and $C(3, 2)$. Reflect $\triangle ABC$ across the y-axis and then translate it along the vector $\langle 0, -2 \rangle$.

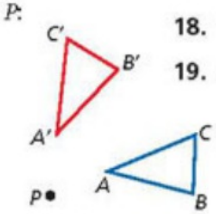
5. **Sports** To create the opening graphics for a televised football game, an animator reflects a picture of a football helmet across line ℓ. She then reflects its image across line m, which intersects line ℓ at a 50° angle. Describe a single transformation that moves the helmet from its starting position to its final position.



Copy each figure and draw two lines of reflection that produce an equivalent transformation.

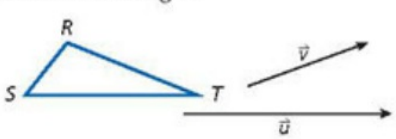
6. translation: $\triangle EFG \rightarrow \triangle E'F'G'$

7. rotation with center P: $\triangle ABC \rightarrow \triangle A'B'C'$

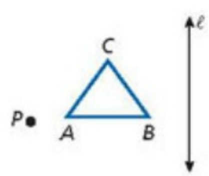


Draw the result of each composition of isometries.

8. Translate $\triangle RST$ along $\vec{u}$ and then translate it along $\vec{v}$.



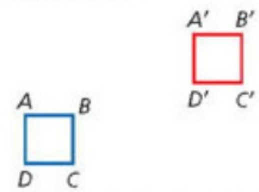
9. Rotate $\triangle ABC$ 90° about point P and then reflect it across line ℓ.



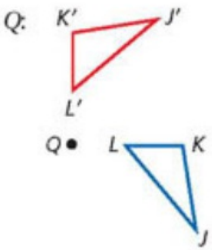
10. $\triangle GHJ$ has vertices $G(1, -1)$, $H(3, 1)$, and $J(3, -2)$. Reflect $\triangle GHJ$ across the line $y = x$ and then reflect it across the x-axis.

Copy each figure and draw two lines of reflection that produce an equivalent transformation.

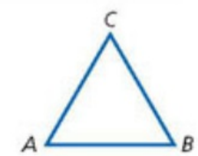
12. translation: $ABCD \rightarrow A'B'C'D'$



13. rotation with center Q: $\triangle JKL \rightarrow \triangle J'K'L'$



15. Equilateral $\triangle ABC$ is reflected across $\overline{AB}$. Then its image is translated along $\overrightarrow{BC}$. Copy $\triangle ABC$ and draw its final image.



Tell whether each statement is sometimes, always, or never true.

16. The composition of two reflections is equivalent to a rotation.

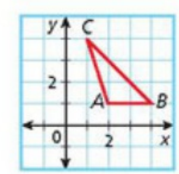
17. An isometry changes the size of a figure.

18. The composition of two isometries is an isometry.

19. A rotation is equivalent to a composition of two reflections.

23. $\triangle ABC$ is reflected across the y-axis. Then its image is rotated 90° about the origin. What are the coordinates of the final image of point A under this composition of transformations?

   (A) $(-1, -2)$ (B) $(-2, 1)$ (C) $(1, 2)$ (D) $(-2, -1)$



24. Which composition of transformations maps $\triangle ABC$ into the fourth quadrant?

   (F) Reflect across the x-axis and then reflect across the y-axis.
   (G) Rotate about the origin by 180° and then reflect across the y-axis.
   (H) Translate along the vector $\langle -5, 0 \rangle$ and then rotate about the origin by 90°.
   (J) Rotate about the origin by 90° and then translate along the vector $\langle 1, -2 \rangle$.

25. Which is equivalent to the composition of two translations?

   (A) Reflection (B) Rotation (C) Translation (D) Glide reflection