5-6 Quadratic Formula and Discriminant

Alg. 2 std. 8.0

The solutions of a quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called the *quadratic formula.*

ex. 1 **Complete the square on $ax^2 + bx + c = 0$.**

$$\left(\frac{b}{2a}\right)^2$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c \cdot 4a}{a \cdot 4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex. 2 **Solve $2x^2 - 2x + 3 = 0$ by quadratic formula.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4(6)}}{4}$$

$$x = \frac{2 \pm \sqrt{-20}}{4}$$

$$x = \frac{(2 \pm 2i\sqrt{5}) \div 2}{(4) \div 2} = \frac{1 \pm i\sqrt{5}}{2}$$

$$\frac{1}{2} \pm \frac{\sqrt{5}}{2}i$$

OR

The ***discriminant*** **of a quadratic equation**
$ax^2 + bx + c = 0$ **is** $b^2 - 4ac$.

The discriminant allows us to describe the nature (how many and type) of the solutions without solving, if a, b, and c are real numbers.

$b^2 - 4ac > 0$ $\frac{-b \pm \sqrt{+}}{2a}$	**2 real solutions** **If a, b, c are integers and $b^2 - 4ac$ = perfect square, then there are 2 (real) rational solutions.** $\frac{-b \pm \sqrt{36}}{2a}$ * the trinomial is factorable
$b^2 - 4ac < 0$	**2 conjugate imaginary solutions** $\frac{-b \pm i\sqrt{\ }}{2a}$
$b^2 - 4ac = 0$	**1 real solution** $\frac{-b \pm \sqrt{0}}{2a}$

ex. 3 **Find the discriminant and describe the solutions.**

a) $-3x^2 - 5x - 9 = 0$

$b^2 - 4ac = 25 - 4(27) = -83$

2 conjugate imag.

b) $6x^2 + 31x + 40 = 0$

$b^2 - 4ac = 961 - 4(240) = 1$

2 real rat'l.

$x = \frac{-31 \pm \sqrt{1}}{12} = \frac{-31 \pm 1}{12} = \frac{-30}{12}, \frac{-32}{12} =$

ex. 4 **For what values of *k* will $x^2 - 8x + k = 0$ have 2 real solutions?**

$b^2 - 4ac > 0$

$64 - 4k > 0$

$k < 16$