36. The approximate value of $y=\sqrt{4+\sin x}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$, is

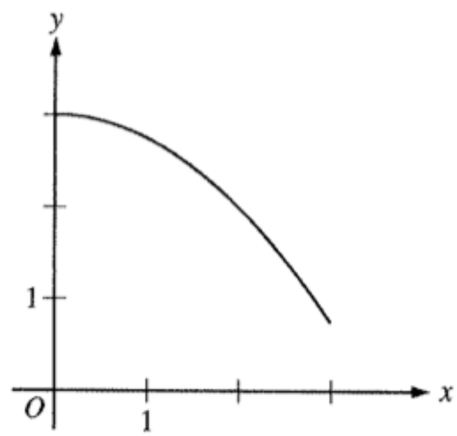
(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

92. Let f be the function given by $f(x)=x^2-2x+3$. The tangent line to the graph of f at $x=2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

36. B $y=\sqrt{4+\sin x}$, $y(0)=2$, $y'(0)=\dfrac{\cos 0}{2\sqrt{4+\sin 0}}=\dfrac{1}{4}$. The linear approximation to y is $L(x)=2+\dfrac{1}{4}x$. $L(1.2)=2+\dfrac{1}{4}(1.2)=2.03$

92. D $f'(x)=2x-2$, $f'(2)=2$, and $f(2)=3$, so an equation for the tangent line is $y=2x-1$. The difference between the function and the tangent line is represented by $(x-2)^2$. Solve $(x-2)^2<0.5$. This inequality is satisfied for all x such that $2-\sqrt{0.5}<x<2+\sqrt{0.5}$. This is the same as $1.293<x<2.707$. Thus the largest value in the list that satisfies the inequality is 2.7.



Graph of f

10. The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

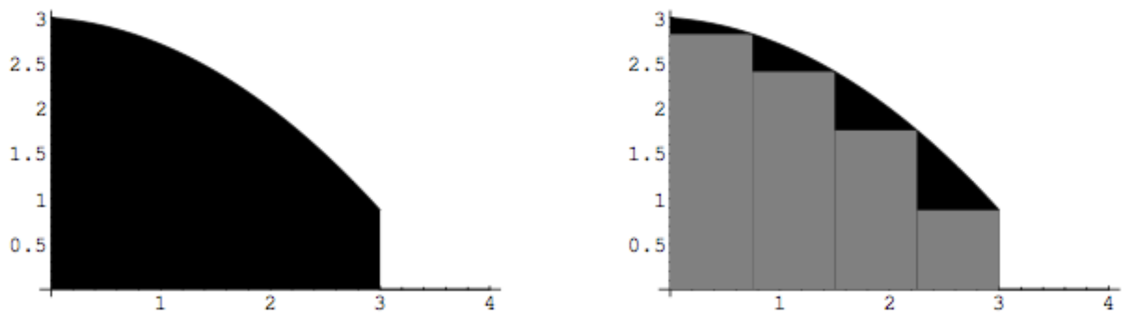
(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

C 10. There are five areas to compare. Three of the five areas are obtained by summing the areas of four rectangles, one of the areas is the sum of the areas of four trapezoids and the last is the area under the curve shown in the problem.

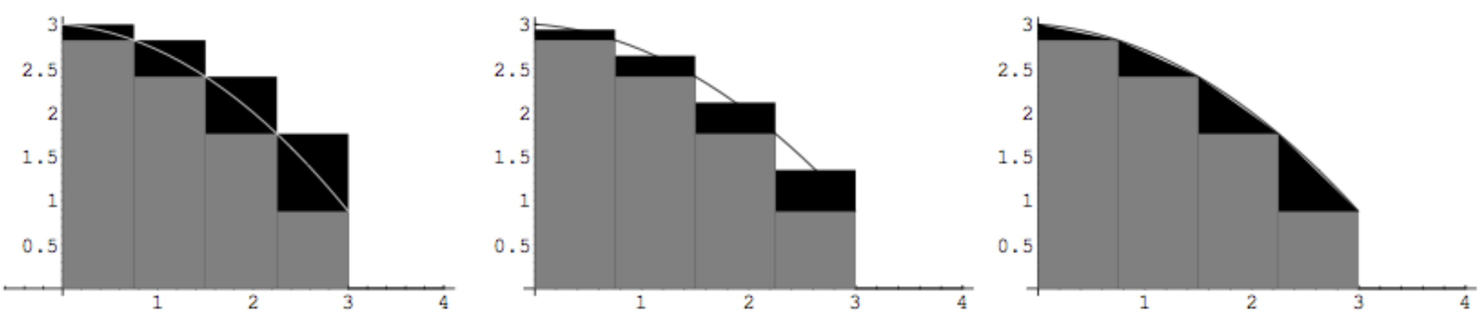
Since the function is decreasing, on a given interval the low point on the graph occurs at the right endpoint. So the area of a given rectangle is smaller than the area under the curve. This is illustrated in the figure below where in the figure to the left the dark area is $\int_0^3 f(x)dx$ and the figure to the right shows the area under the curve with the four rectangles whose are is given by the right Riemann sum is shown overlayed on the area under the curve.



Since there is still some black area showing after the gray area is overlayed, the gray area must be smaller. Therefore,

$$\text{Right Riemann Sum Approximation} < \int_0^3 f(x)dx.$$

Similarly, if we overlay the area for the right Riemann sum approximation on the areas for the other approximations we have the figures shown below and it is clear that the right Riemann sum approximation is the smallest of the values.

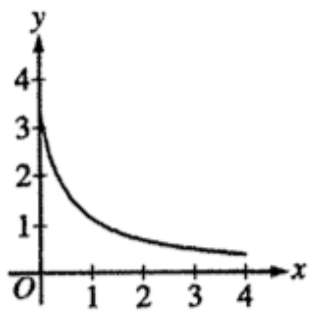


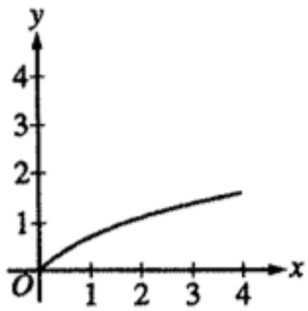
x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

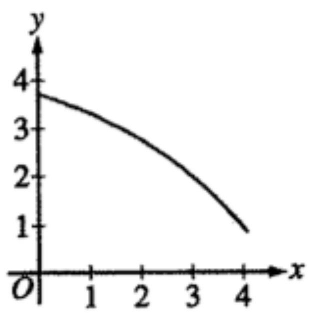
8. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

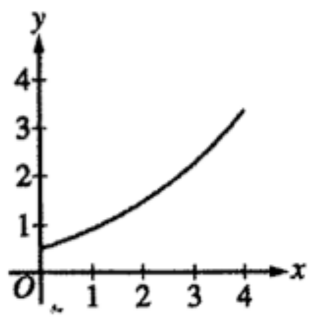
(A) 6 (B) 14 (C) 28 (D) 32 (E) 50

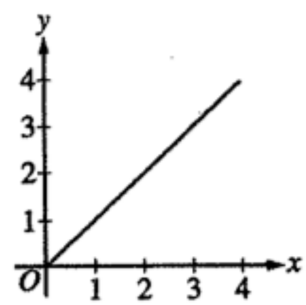
85. **If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?**

(A)


(B)


(C)


(D)


(E)


x	2	5	10	14
$f(x)$	12	28	34	30

25. **The function f is continuous on the closed interval $[2, 14]$ and has values as shown in the table above. Using the subintervals $[2, 5]$, $[5, 10]$, and $[10, 14]$, what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?**

(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

B 8. Recall that for a Riemann sum the left endpoint of the i-th subinterval is x_{i-1} and the length of the i-th subinterval is $\Delta x_i = x_i - x_{i-1}$. Thus, the left Riemann sum for the function given is

$$\int_2^{13} f(x)dx \approx \sum_{i=1}^{4} f(x_{i-1})\Delta x_i = f(x_0)\Delta x_1 + f(x_1)\Delta x_2 + f(x_2)\Delta x_3 + f(x_3)\Delta x_4$$
$$= f(2)(3-2) + f(3)(5-3) + f(5)(8-5) + f(8)(13-8)$$
$$= (6)(1) + (-2)(2) + (-1)(3) + (3)(5) = 14.$$

A 85. Since the trapezoidal rule over-approximates $\int_0^4 f(x)dx$, on each subinterval the curve lies below the line segment joining the points on the curve at the endpoints of the interval. This means that the curve is concave upward over the subintervals. This eliminates curves (B), (C), and (E). Since the right Riemann sum under-approximates $\int_0^4 f(x)dx$, over each subinterval the point on the curve at the right endpoint is lower than the point on the curve at the left endpoint. This means that the curve should be decreasing. Therefore, curve (D) is eliminated and the graph of $y = f(x)$ is (A).

D 25. Recall that for a Riemann sum the right endpoint of the i-th subinterval is x_i and the length of the i-th subinterval is $\Delta x_i = x_i - x_{i-1}$. Thus, the right Riemann sum for the function given is

$$\int_2^{14} f(x)dx \approx \sum_{i=1}^{3} f(x_i)\Delta x_i$$
$$= f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3$$
$$= f(5)(5-2) + f(10)(10-5) + f(14)(14-10)$$
$$= (28)(3) + (34)(5) + (30)(4)$$
$$= 374.$$

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

85. C There are 3 trapezoids. $\frac{1}{2} \cdot 3\left(f(2) + f(5)\right) + \frac{1}{2} \cdot 2\left(f(5) + f(7)\right) + \frac{1}{2} \cdot 1\left(f(7) + f(8)\right)$

18. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$?

(A) $e^2 + e^0 + e^{-2}$

(B) $e^4 + e^2 + e^0$

(C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$

(E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}} + \sqrt{\frac{2}{50}} + \sqrt{\frac{3}{50}} + \cdots + \sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

18. E $\Delta x = \dfrac{4-(-2)}{3} = 2,\ T = \dfrac{1}{2}(2)\left(\dfrac{e^4}{2} + 2\cdot\dfrac{e^2}{2} + 2\cdot\dfrac{e^0}{2} + \dfrac{e^{-2}}{2}\right) = \dfrac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

36. D Rectangle approximation = $e^0 + e^1 = 1 + e$
Trapezoid approximation. = $\left(1 + 2e + e^4\right)/2$.
Difference = $(e^4 - 1)/2 = 26.799$.

24. B Let $[0,1]$ be divided into 50 subintervals. $\Delta x = \dfrac{1}{50};\ x_1 = \dfrac{1}{50}, x_2 = \dfrac{2}{50}, x_3 = \dfrac{3}{50}, \cdots, x_{50} = 1$
Using $f(x) = \sqrt{x}$, the right Riemann sum $\sum_{i=1}^{50} f(x_i)\Delta x$ is an approximation for $\int_0^1 \sqrt{x}\, dx$.