17 Archimedes' Claw

At last the Romans were reduced to such a state of alarm that if they saw so much as a length of rope or a piece of timber appear over the top of the wall, it was enough to make them cry out, "Look, Archimedes is aiming one of his machines at us!" and they would turn their backs and run.
—Plutarch (ca. 46–120 C.E.), Greek biographer, *Parallel Lives*

The position of Archimedes as the most creative and original mathematician of Antiquity has never been in question—indeed he is usually ranked with Newton and Gauss as one of the supreme mathematical geniuses of all time.
—Stuart Hollingdale (1910–), British author, *Makers of Mathematics*

It sounded as if he were bragging, although everyone knew that Archimedes (ar-kuh-MEE-deez) wasn't a braggart. But he did tell King Hiero II (ruler of the Greek city-state of Syracuse), "Give me somewhere to stand and I will move the earth."

Now that would be quite a feat! The Greek god Atlas was supposed to be holding the heavens on his shoulders—but hardly anyone took that tale seriously. They did pay attention to Archimedes (287–212 B.C.E.), who had discovered so many mathematical theorems, written so many scientific books and papers, and invented so many things, that hardly anyone could keep up with his accomplishments.

This portrait of Archimedes was painted long after his lifetime. The Italian artist Giuseppe Nogari (1699–ca.1763) depicts Archimedes with a compass, a tool that wasn't invented until the first century C.E.

Archimedes especially loved geometry, which is all about shapes—flat shapes and solid shapes. He studied triangles, circles, ellipses, squares, rectangles, and other polygons (to name a few flat shapes). And he studied pyramids, cubes, cones, cylinders, spheres, and other polyhedrons (to name a few solid shapes). Then he figured out how to measure them all. (Euclid defined, compiled, and organized. Archimedes innovated. Both kinds of minds are needed.)

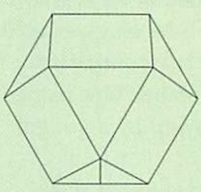
A POLYGON (meaning "many-angled" in Greek) has two dimensions (length and width) and three or more sides and angles. A POLYHEDRON is a solid, a three-dimensional (length, width, depth) figure with four or more faces.

Why would you want to measure the area of a circle? Or the volume of a sphere? Or of a cylinder? For starters, it would let you measure the water in a pipeline, the wood in a log, or the size of the universe. Those first two kinds of measurements had important practical uses in Archimedes' time, as they do now. But the universe? Could someone, even then, have been thinking of space travel? For Archimedes, with one of the most creative minds the world has known, nothing was beyond consideration.

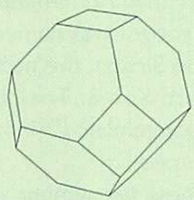
In a book called *The Sand Reckoner*, Archimedes estimated

ARCHIMEDES' SOLIDS

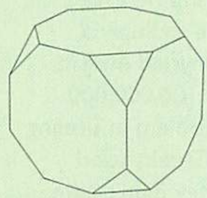
Archimedes found 13 ways to make a polyhedron in which all the faces are regular polygons. For some solids, he just cut off the corners of one of Plato's five perfect solids (see page 99). The third shape, for example, is a truncated cube.



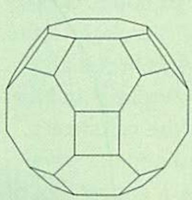
cuboctahedron:* *8 triangles, 6 squares



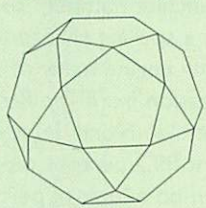
truncated octahedron:* *6 squares, 8 hexagons



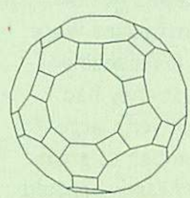
truncated cube:* *8 triangles, 6 octagons



truncated cuboctahedron:* *12 squares, 8 hexagons, 6 octagons

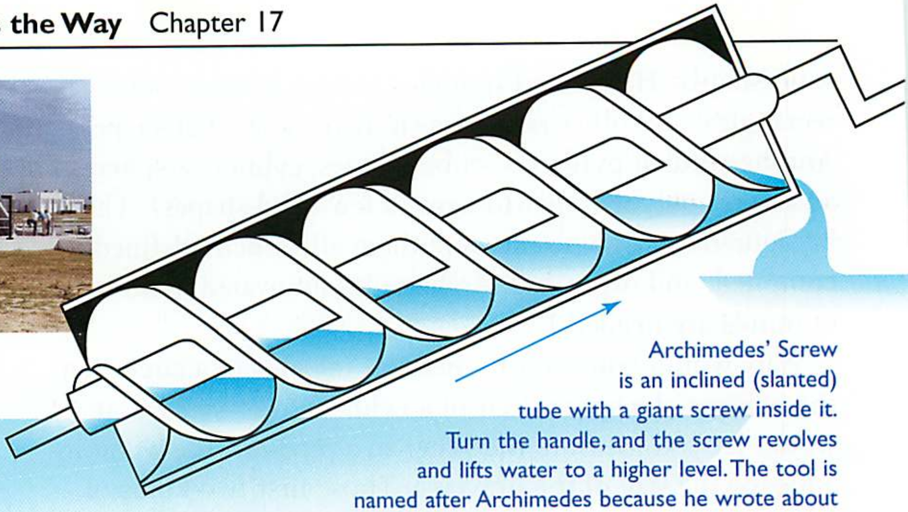


icosidodecahedron:* *20 triangles, 12 pentagons



truncated icosidodecahedron:* *30 squares, 20 hexagons, 12 decagons

Archimedes' Screws are still in use today. Seven of them pump wastewater in a treatment plant in Memphis, Tennessee. Each screw is 2.5 meters (8.2 feet) in diameter and can lift 75,000 liters (almost 20,000 gallons) per minute. (That's about 600 bathtubs full of water.)



Archimedes' Screw is an inclined (slanted) tube with a giant screw inside it. Turn the handle, and the screw revolves and lifts water to a higher level. The tool is named after Archimedes because he wrote about it in 236 B.C.E., but it's doubtful that he invented it.

the number of grains of sand needed to fill the universe. He was trying to prove two things: that numbers are infinite—you can never have a last number—and that it is possible to deal with really big numbers.

THINK BIG—AND BIGGER

Archimedes wanted to prove that no number is too big to measure. He began by creating his own big number. It was 10,000, and he called it a myriad. Then he said you could multiply a myriad by a myriad and have a myriad myriad, which is written today as 10^8 or 100,000,000.

Archimedes had a serious problem in dealing with these large numbers. The Greeks used alphabet letters for numbers. They didn't have digit symbols, as we do, and zero was missing. A myriad was an *M* with a little alpha (α) on top: $\overset{\alpha}{\mathrm{M}}$. To say that Greek mathematicians were hampered is an understatement. It's amazing what they accomplished with what they had.

Archimedes used his system of very large numbers to estimate that the number of grains of sand in the visible world is 10^{63} (as we would write it). That's far too many, but his point was that nothing is too big to measure.

He may be right. We're still finding ever-bigger numbers. A favorite of mine is a googol: 10^{100} (1 with 100 zeroes after it), which is bigger than Archimedes' number of sand grains. The googol was named in 1955 by nine-year-old Milton Sirotta, the nephew of mathematician Edward Kasner. The biggest number unit named is a googolplex: 10^{googol} (1 followed by a googol of zeroes).

These big number words have found their way into everyday language. A *myriad* now means "an infinite number," too many to count. *Googol* with a spelling change (Google) is an Internet search engine many of us use. And here's a quotation from *The Record*, a newspaper in New Jersey: "In North Jersey, where the reservoirs are at 99.2 percent capacity and puddles have taken on a look of permanence, a googolplex of baby mosquitoes is being incubated." That means: If you go to North Jersey in a rainy year, you'd better bring mosquito repellent.

Archimedes sent off a letter to King Hiero II announcing his book. This time he said, "I will try to show you by means of geometrical proofs, which you will be able to follow, that, of the numbers named by me . . . some exceed . . . the number of the mass of sand equal in magnitude to the . . . universe."

Grains of sand to fill the whole universe? That would be some number, even though the universe to the Greeks was only what you can see with your eyes.

Archimedes was the son of an astronomer and, like his dad and Aristotle and most others of his time, he thought the stars and planets were attached to solid but transparent spheres. He built small planetariums in order to make the heavens understandable. Those models told him there was a problem with the concept of an Earth-centered universe. So, like Aristarchus of Samos (who was about 23 years older), Archimedes believed in the less-accepted idea that the Earth and planets revolve around the Sun. (We wouldn't know about Aristarchus if Archimedes hadn't written about him.)

Archimedes thought ideas were more important than things. As to practical science, the Greek historian Plutarch said that Archimedes believed engineering was "sordid and ignoble [as is] every sort of art that lends itself to mere use and profit." (That was Plato's idea. It was a tough influence to shake.)

Archimedes was fooling himself because every time he put his mind to it, he seemed to invent something that was useful—and usually profitable. You could call him an engineering genius—and not be wrong. The truth is, when pure thought and practicality are balanced, civilizations work at their best. And those two extremes were combined in this one amazing man. It doesn't happen often, in people or civilizations.

Archimedes lived in Syracuse, a city-state on the island of Sicily, in the middle of the Mediterranean Sea. (See the map on page 150.) When he was ready for what we'd call college, he sailed off to Alexandria. His teacher was Conon, whose teacher was Euclid. For the rest of his life, Archimedes stayed in touch with the scholars in Alexandria.

Here is what Plutarch says about Archimedes the writer. "It is not possible to find in all geometry more difficult and intricate questions, or more simple and lucid explanations. Some ascribe this to his natural genius; while others think that incredible effort and toil produced these, to all appearances, easy and unlaboured results."

Did Archimedes work hard at writing? I don't know, but it takes hard work for most people—like me—to produce clear and simple sentences. I have to write and rewrite.

ENGINEERING is about using the laws of science to build things—bridges, tunnels, temples, monuments, machines, and so on. If building useful things is SORDID ("dirty, vile") and IGNOBLE ("unworthy"), where does that leave Hero, the Alexandrian engineer who built mostly frivolous things?

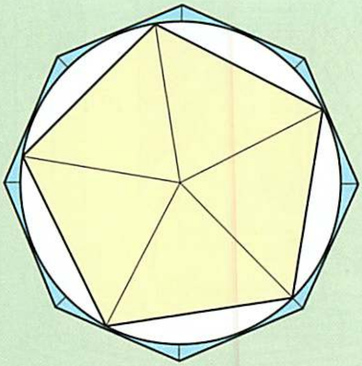
A copper coin shows the head of Hiero II, who ruled Syracuse for 60 years. Plutarch describes Archimedes as a near relative of Hiero, a king who often sought Archimedes' advice on military and other matters.

That wasn't difficult. The Mediterranean world was linked by the sea. It made travel and communication easy—for traders, travelers, and armies on the move. And in Archimedes' time, two big armies were moving. The Romans (from Italy) and the Carthaginians (from Africa) were fighting for control of the sea-lapped lands.

Most of Sicily was ruled by Carthage. But prosperous Syracuse, at one end of the island, managed to stay independent. That wasn't easy. Carthage wanted Syracuse. So did Rome. Syracuse's location and wealth made it especially attractive to power seekers.

Politics didn't interest Archimedes. It was math and science that he cared about. But King Hiero II, who may have been his cousin, kept pestering him for help. So Archimedes invented some war machines, in case Syracuse needed to defend itself.

Then the king came to him with a personal problem. He had given a big hunk of gold to a jeweler to make a gold crown. But when he got the crown, King Hiero believed he



When Close Counts

How do you measure the curve of a circle? Archimedes worked out something called the method of exhaustion. It was a first step toward calculus, which he might have discovered if he had had decent number symbols to use. He came up with a value for π (pi) that was closer than any yet achieved (see page 71). He did that by drawing polygons inside and outside a circle. As the polygons were given more and more sides (this *was* exhausting), they came close to becoming a circle. The circumference of the circle—a measurement needed to calculate π—was in between the circumferences of the inner and outer polygons.

had been cheated. He thought the jeweler had mixed cheaper silver with the gold and kept some of the gold for himself. But no one knew how to tell a mixture of metals—an alloy—from a pure metal. And that included Archimedes.

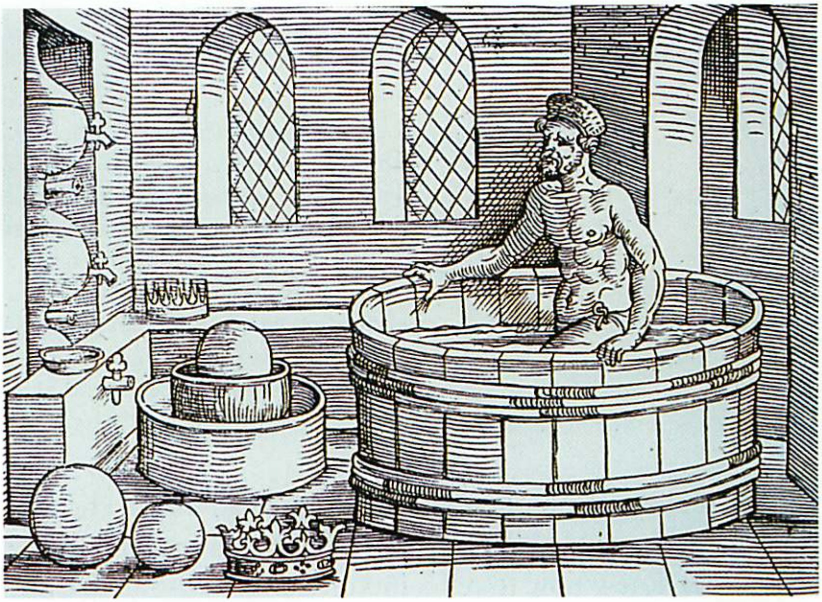
He started to think about the problem. Soon he couldn't get it out of his mind. How can you tell a pure metal from an alloy? You can compare color or shine, and you can test hardness. The crown passed all those tests. A different approach was needed. King Hiero told Archimedes he was not to harm the crown. Archimedes thought about the problem when he was eating, when he was walking, and he must have dreamed about it when he was sleeping.

Density seemed a possible way to go. Gold is more dense—heavier for its size—than silver. So a pure gold crown would weigh more than a crown *of the same size* with both gold and silver in it. But this crown weighed the same as the chunk of gold the king had given the goldsmith. Another dead end.

If you're thinking scientifically, it helps to restate a problem. Archimedes may have reworded it this way: A gold-silver crown that weighs the same as a pure-gold crown *would have to be a bit bigger*; its volume would be greater. Was this crown bigger than the chunk of gold? Had the jeweler added extra silver to make its weight right? Measuring the exact volume of an irregular shape seemed impossible. How could it be done?

Archimedes was in the public bath when the answer came

To understand density, it helps to define mass and volume. MASS is the measure of the matter in an object. On the Moon, an object's mass (its quantity of matter) is the same as on Earth, but its weight is less because the Moon's gravity is weaker. On Earth, mass generally equals weight. (The gravitational pull on a mountaintop is slightly less than in a valley, so even objects on Earth can vary in weight depending on their location.) VOLUME is the amount of space an object takes up. It's not exactly the same thing as size. Even if a Wiffle™ ball and a baseball are the same size, the hollow Wiffle™ ball has less volume. DENSITY is a ratio of mass to volume. It's expressed as mass (m) divided by volume (v), or $d = m/v$.



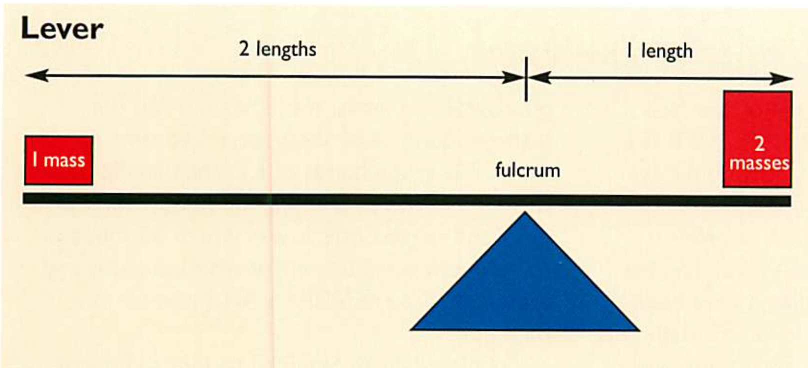
The woodcut at right (probably from the nineteenth century) imaginatively depicts Archimedes' "Eureka!" moment in the bathtub. In Archimedes' time, crowns were wreath shaped, like the one above.

Did Archimedes really run naked through the streets? One source is a Roman named Vitruvius, a military engineer in Africa for Julius Caesar. He tells of Archimedes' "Eureka!" moment in a 10-volume book on architecture and engineering. His work has a base in science that is unusual for books of any era.

to him. He was so excited that he jumped out of the bath and ran home through the streets of Syracuse—stark naked. He was too intent on his thoughts to take time to put on his clothes. (The Greeks didn't worry about nakedness as much as we do.) He kept shouting, "Eureka (yoo-REE-kuh)! Eureka!"—which means "I have it! I have it!"

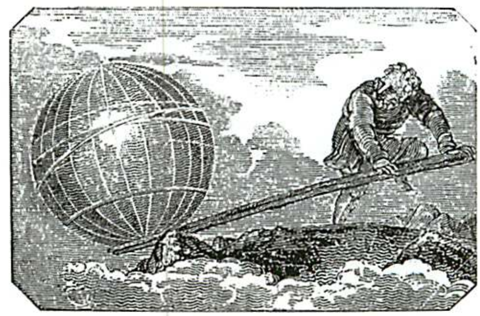
When Archimedes had stepped into the full bath, his body displaced water. It splashed above the rim and onto the floor. That's what gave him his big insight. He realized that the water he displaced was equal in volume to the part of his body that was underwater.

Scientific breakthroughs are often made by seeing connections, and that's exactly what happened. Archimedes suddenly realized that his body and a chunk of gold must react the same way when put in water. **Any object—a man, a crown, or a piece of gold—when immersed in a liquid will displace a volume of liquid equal to the object's own volume.** Now all Archimedes had to do was to lower the crown into a bowl filled with water to the rim. Which he did. Then he measured the water that spilled over. He did the same thing with a piece of pure gold the same size as the one the king had given the jeweler.



Archimedes' Law of the Lever led to the notion of center of gravity: For every object, there is a single point at which the force of gravity appears to act on that object. In this drawing, that point is where the lever rests on the fulcrum. The mass that is twice as heavy is half the distance to the fulcrum.

Did the king's crown displace the same amount of water as the pure gold? No. It displaced more water. The crown had a greater volume than the gold. It was padded with silver. The king had been cheated.



What about moving the Earth? Remember, Archimedes said that if he had somewhere to stand, he could move the Earth. To prove his point, Archimedes told the king to find something very heavy, and he would move it. The king picked his biggest ship and loaded it with cargo and sailors. The ship was in dry dock, so it couldn't glide through water, which would have made things easier. Archimedes had to lift it all by himself. He did. With one hand, or so the story goes.

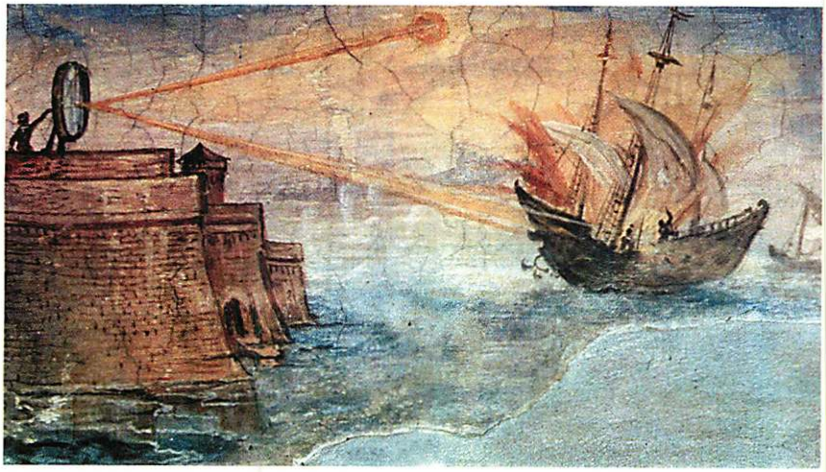
In case you were wondering, the Earth's mass is just under 6 sextillion metric tons, or 6 followed by 21 zeroes. (A metric ton is equal to 1.1 standard U.S. tons.)

He worked out a combination of levers and pulleys and proved that with a lever that's long enough and balanced properly, you can lift anything, even the Earth. Archimedes was one of the world's first experts in mechanics.

In simple terms, his Law of the Lever, still a basic law in physics, says this: If you have a heavy person on one side of a seesaw and a light one on the other, you need to move the heavy person closer to the fulcrum and give the lightweight a lot of board. To balance, the product of weight times distance must be the same on both sides of the seesaw (see illustration above).

A LEVER is a rigid bar or beam that rests on a FULCRUM (fixed support). A PULLEY is a wheel-and-rope combination used to lift or pull objects. MECHANICS is the science of motion and machines (like levers and pulleys) that move things.

How about those war machines that Archimedes invented for King Hiero II? He hoped they would never have to be

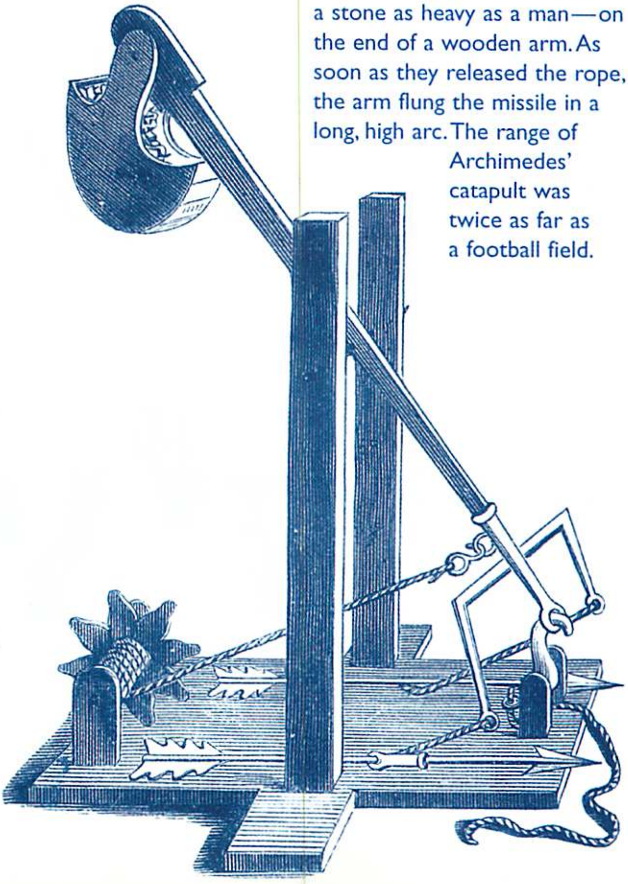


Did sunlight and shields really set those Roman sails on fire? We have the word of Plutarch, a Greek historian writing more than 200 years later, but modern physicists are still debating the science and technology of Archimedes' mirrors.

used, but in 215 B.C.E. the Romans were on the march—and they wanted Syracuse. By this time Hiero was dead and his grandson, Hieronymus, made a big mistake in picking Carthage as the potential winner in the Second Punic War.

The Romans were confident. The powerful Roman legions (armies) had been squashing everyone who got in their way. Syracuse was a treasure to put in their chest. Its strategic location made it vital to any power that wanted control of the Mediterranean. Besides, the Romans hated the Carthaginians—and they worried that some people in Syracuse, like the new king, were getting too friendly with Carthage.

So Roman infantrymen—15,000 of them with shields and armor and fancy weaponry—were loaded onto a fleet of 60 ships and sent to capture Syracuse. The soldiers would have to climb the city's walls, but they were used to doing that. Theirs was the best army anywhere. They were prepared for almost anything—except the mind of a scientist.



Ancient catapults were powered by the tension of a wound-up rope. Soldiers used a winch and ratchet to pull the rope as taut as they could and to secure it in place. They placed a missile—a stone as heavy as a man—on the end of a wooden arm. As soon as they released the rope, the arm flung the missile in a long, high arc. The range of Archimedes' catapult was twice as far as a football field.

Archimedes is said to have arranged a large number of flat mirrors, or polished shields, that could be focused on ships approaching the island. The mirrors reflected the sun's rays and blinded the Roman sailors. The sum of all those small spots of sunlight was so intense that it set their cotton sails on fire. The ships turned back.

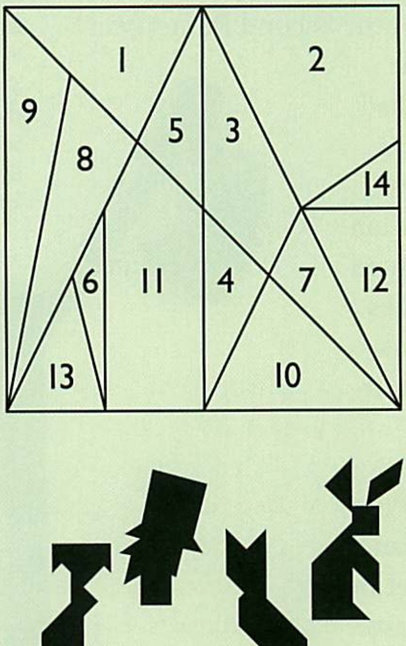
Meanwhile, the Romans weren't finished. They rebuilt their ships. This time they set sail on a cloudy day.

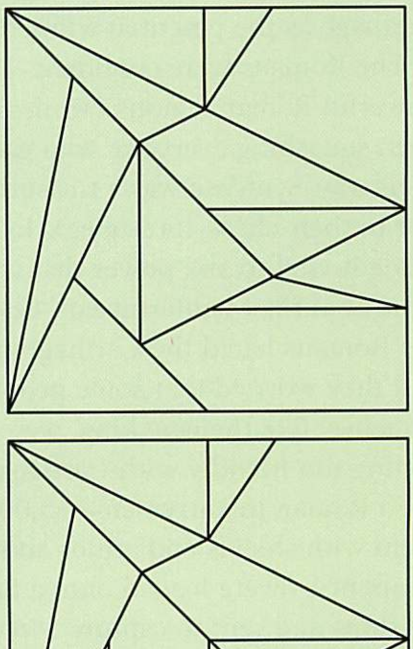
DIABOLICAL means "fiendish" or "wicked." It comes from the Greek word for devil.

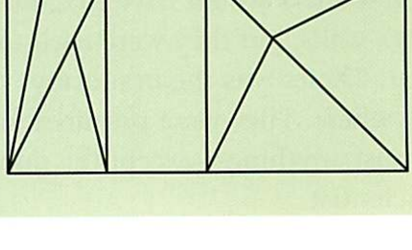
Archimedes was ready for them with some diabolical machines. Giant catapults, like slingshots, hurled ballistic missiles (lethal stones) at the ships. That wasn't all. Cranes swung out from the city's walls, dropping huge boulders on those who got too near. At the same time, archers shot deadly arrows. Still, that wasn't the worst of it.

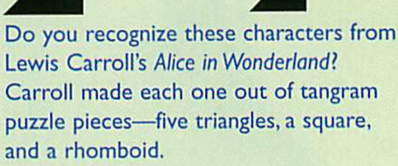
SQUARE DEAL

Combinatorics asks: How many possible ways can the 14 pieces on page 154 be put together to make a square? That's what Archimedes was trying to figure out. We still don't know his answer. (There are tears and gaps in his manuscript.) We do know what modern mathematicians have found: There are 17,151 ways to make a square from Archimedes' 14 shapes. Here are three of the solutions.









Do you recognize these characters from Lewis Carroll's *Alice in Wonderland*? Carroll made each one out of tangram puzzle pieces—five triangles, a square, and a rhomboid.

DEATH BY MATH

Despite Archimedes' war machines, the Romans finally besieged and captured Syracuse.

According to the story, Archimedes was concentrating on a mathematical problem, scratching numbers in the dirt at his feet, when a Roman soldier called out to him. Archimedes was so immersed in his work, he didn't respond. (He was famous for both his ability to concentrate and his absentmindedness.) The soldier ran a sword through his body.

A copy of a Roman mosaic shows Archimedes reaching for his chalkboard while an impatient soldier commands him to follow.

The Roman fleet was in for a horror beyond anything anyone could have imagined. It was about to meet Archimedes' Claw! Here is a description of what happened, written by the Greek historian Plutarch:

> *The ships, drawn by engines within, and whirled about, were dashed against steep rocks that stood jutting out under the walls, with great destruction of the soldiers that were aboard them. A ship was frequently lifted up to a great height in the air (a dreadful thing to behold) and was rolled to and fro, and kept swinging, until mariners [sailors] were all thrown out, when at length it was dashed against the rocks, or let fall.*

A 75-year-old scientist had stopped the world's greatest army with the power of his brain.