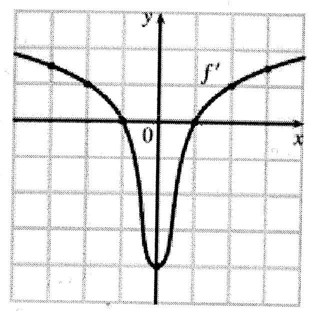
3.2a

1. ⓐ $f'(-3) \approx 1.5$
ⓑ $f'(-2) \approx 1$
ⓒ $f'(-1) \approx 0$
ⓓ $f'(0) \approx -4$
ⓔ $f'(1) \approx 0$
ⓕ $f'(2) \approx 1$
ⓖ $f'(3) \approx 1.5$



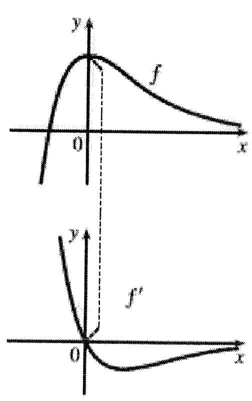
3. (a)′ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)′ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)′ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)′ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

5.



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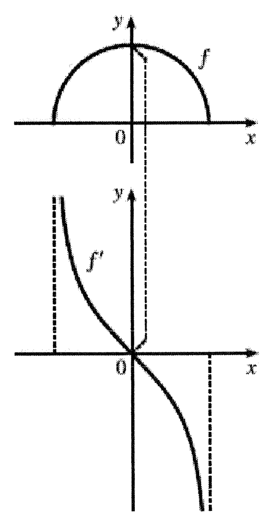

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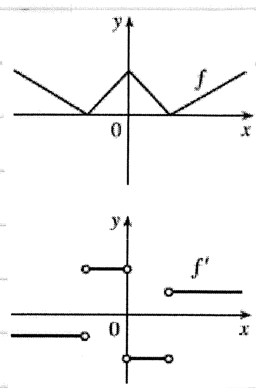
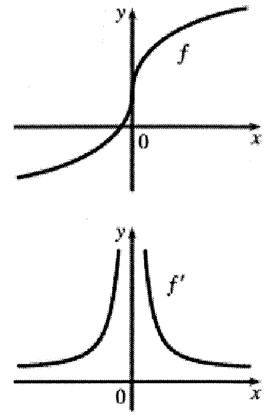


11.



17. $f(x) = \frac{1}{2}x - \frac{1}{3}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h) - \frac{1}{3}\right] - \left(\frac{1}{2}x - \frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}h}{h}$$

$$= \lim_{h\to 0} \frac{1}{2}$$

$$= \frac{1}{2}$$

$f(x) = \frac{1}{2}x - \frac{1}{3}$ D: $\mathbb{R}$

$f'(x) = \frac{1}{2}$ D: $\mathbb{R}$

19. $f(t) = 5t - 9t^2$

$$f'(t) = \lim_{h \to 0} \frac{f(h+t) - f(t)}{h}$$
$$= \lim_{h \to 0} \frac{[5(h+t) - 9(h+t)^2] - (5t - 9t^2)}{h}$$
$$= \lim_{h \to 0} \frac{5h + 5t - 9(h^2 + 2ht + t^2) - 5t + 9t^2}{h}$$
$$= \lim_{h \to 0} \frac{5h - 18th - 9h^2}{h}$$
$$= \lim_{h \to 0} (5 - 18t - 9h)$$
$$= 5 - 18t$$

$f(t) = 5t - 9t^2$ D: $\mathbb{R}$
$f'(t) = 5 - 18t$ D: $\mathbb{R}$

21. $f(x) = x^3 - 3x + 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{[(x+h)^3 - 3(x+h) + 5] - (x^3 - 3x + 5)}{h}$$
$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 3x - 3h + 5 - x^3 + 3x - 5}{h}$$
$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 3h}{h}$$
$$= \lim_{h \to 0} (3x^2 + 3xh + h^2 - 3)$$
$$= 3x^2 - 3$$

$f(x) = x^3 - 3x + 5$ D: $\mathbb{R}$
$f'(x) = 3x^2 - 3$ D: $\mathbb{R}$

23. $g(x) = \sqrt{1+2x}$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h \to 0} \frac{(1+2x+2h) - (1+2x)}{h(\sqrt{1+2x+2h} + \sqrt{1+2x})}$$

$$= \lim_{h \to 0} \frac{2h}{h(\sqrt{1+2x+2h} + \sqrt{1+2x})}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \frac{2}{2\sqrt{1+2x}}$$

$$= \frac{1}{\sqrt{1+2x}}$$

$g(x) = \sqrt{1+2x}$ D: $[-\frac{1}{2}, \infty)$

$g'(x) = \frac{1}{\sqrt{1+2x}}$ D: $(-\frac{1}{2}, \infty)$

25. $G(t) = \frac{4t}{t+1}$

$$G'(t) = \lim_{h\to 0} \frac{G(t+h) - G(t)}{h} = \lim_{h\to 0} \frac{\frac{4(t+h)}{(t+h)+1} - \frac{4t}{t+1}}{h}$$

$$= \lim_{h\to 0} \left(\frac{4t+4h}{t+h+1} - \frac{4t}{t+1}\right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(4t+4h)(t+1) - 4t(t+h+1)}{(t+h+1)(t+1)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4t^2} \cancel{+4t} \cancel{+4ht} + 4h \cancel{-4t^2} \cancel{-4ht} \cancel{-4t}}{h(t+h+1)(t+1)}$$

$$= \lim_{h\to 0} \frac{4h}{h(t+h+1)(t+1)}$$

$$= \lim_{h\to 0} \frac{4}{(t+h+1)(t+1)}$$

$$= \frac{4}{(t+1)^2}$$

$G(t) = \frac{4t}{t+1}$ D: $(-\infty, -1) \cup (-1, \infty)$

$G'(t) = \frac{4}{(t+1)^2}$ D: $(-\infty, -1) \cup (-1, \infty)$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

27. $f(x) = x^4$

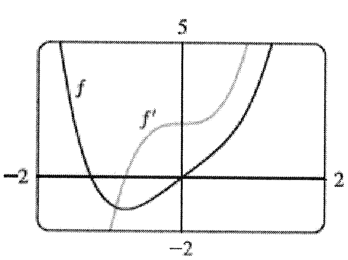
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$$
$$= \lim_{h\to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$
$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$
$$= \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3)$$
$$= 4x^3$$

$f(x) = x^4$ D: $\mathbb{R}$
$f'(x) = 4x^3$ D: $\mathbb{R}$

29. (a) $f(x) = x^4 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^4 + 2(x+h)] - (x^4+2x)}{h}$$
$$= \lim_{h\to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 2x + 2h - x^4 - 2x}{h}$$
$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 2h}{h}$$
$$= \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3 + 2)$$
$$= 4x^3 + 2$$

(b)



Notice that $f'(x) = 0$ when f has a horizontal tangent, $f'(x)$ is positive when the tangents have positive slope, and $f'(x)$ is negative when the tangents have negative slope.