2.5a

1. $\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$

ⓐ $\lim_{x\to 2}[f(x)+5g(x)] = 4+5(-2) = -6$

ⓑ $\lim_{x\to 2}[g(x)]^3 = \left[\lim_{x\to 2} g(x)\right]^3 = (-2)^3 = -8$

ⓒ $\lim_{x\to 2}\sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)} = \sqrt{4} = 2$

ⓓ $\lim_{x\to 2}\frac{3f(x)}{g(x)} = \frac{3(4)}{-2} = -6$

ⓔ $\lim_{x\to 2}\frac{g(x)}{h(x)}$ DNE since $\lim_{x\to 2} h(x) = 0$

ⓕ $\lim_{x\to 2}\frac{g(x)h(x)}{f(x)} = \frac{(-2)(0)}{4} = 0$

2. ⓐ $\lim_{x\to 2}[f(x)+g(x)] = 2+0 = 2$

ⓑ $\lim_{x\to 1}[f(x)+g(x)]$ DNE since $\lim_{x\to 1} g(x)$ DNE

$\lim_{x\to 1} g(x)$ DNE since $2 = \lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x) = 1$

ⓒ $\lim_{x\to 0}[f(x)g(x)] \approx (0)(1.3) = 0$

ⓓ $\lim_{x\to -1}\frac{f(x)}{g(x)}$ DNE since $\lim_{x\to -1} g(x) = 0$

ⓔ $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = (2^3)(2) = 16$

ⓕ $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{\lim_{x\to 1}(3) + \lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

3. $\lim_{x \to -2} (3x^4 + 2x^2 - x + 1)$

$= 3\lim_{x \to -2} x^4 + 2\lim_{x \to -2} x^2 - \lim_{x \to -2} x + \lim_{x \to -2} 1$

$= 3(-2)^4 + 2(-2)^2 - (-2) + 1$

$= 59$

4. $\lim_{x \to 2} \dfrac{2x^2 + 1}{x^2 + 6x - 4}$

$= \dfrac{\lim_{x \to 2} (2x^2 + 1)}{\lim_{x \to 2} (x^2 + 6x - 4)} = \dfrac{2(2)^2 + 1}{(2)^2 + 6(2) - 4} = \dfrac{3}{4}$

5. $\lim_{x \to 8} (1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$

$= \lim_{x \to 8} (1 + \sqrt[3]{x}) \cdot \lim_{x \to 8} (2 - 6x^2 + x^3)$

$= (1 + \sqrt[3]{8}) \cdot (2 - 6(8)^2 + (8)^3)$

$= 390$

6. $\lim_{t \to -1} (t^2 + 1)^3 (t + 3)^5$

$= \left[\lim_{t \to -1} (t^2 + 1) \right]^3 \cdot \left[\lim_{t \to -1} (t + 3) \right]^5$

$= [(-1)^2 + 1]^3 \cdot (-1 + 3)^5$

$= 256$

7. $\lim_{x \to 1} \left(\dfrac{1 + 3x}{1 + 4x^2 + 3x^4} \right)^3 = \dfrac{\left[\lim_{x \to 1} (1 + 3x) \right]^3}{\left[\lim_{x \to 1} (1 + 4x^2 + 3x^4) \right]^3}$

$= \dfrac{[1 + 3(1)]^3}{[1 + 4(1)^2 + 3(1)^4]^3} = \dfrac{1}{8}$

8. $\lim\limits_{u\to -2} \sqrt{u^4+3u+6}$

$= \sqrt{\lim\limits_{u\to -2}(u^4+3u+6)}$

$= \sqrt{(-2)^4+3(-2)+6}$

$= 4$

9. $\lim\limits_{x\to 4^-} \sqrt{16-x^2}$

$= \sqrt{\lim\limits_{x\to 4^-}(16-x^2)}$

$= \sqrt{16-(4)^2}$

$= 0$

11. $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x-2} = \lim\limits_{x\to 2} \dfrac{(x+3)(x-2)}{(x-2)}$

$= \lim\limits_{x\to 2}(x+3)$

$= 2+3$

$= 5$

13 $\lim\limits_{x\to 2} \dfrac{x^2-x+6}{x-2}$

$\lim\limits_{x\to 2^-} \dfrac{x^2-x+6}{x-2} = -\infty$ $\left[\begin{array}{l} \text{as } x\to 2^-, (x^2-x+6)\to 8 \\ \text{as } x\to 2^-, (x-2)\to 0^- \end{array}\right]$

$\lim\limits_{x\to 2^+} \dfrac{x^2-x+6}{x-2} = \infty$ $\left[\begin{array}{l} \text{as } x\to 2^+, (x^2-x+6)\to 8 \\ \text{as } x\to 2^+, (x-2)\to 0^+ \end{array}\right]$

$\lim\limits_{x\to 2} \dfrac{x^2-x+6}{x-2}$ DNE since $\lim\limits_{x\to 2^-} \dfrac{x^2-x+6}{x-2} \neq \lim\limits_{x\to 2^+} \dfrac{x^2-x+6}{x-2}$

15. $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3} = \lim_{t\to -3} \frac{(t+3)(t-3)}{(2t+1)(t+3)}$

$= \lim_{t\to -3} \frac{t-3}{2t+1}$

$= \frac{-3-3}{2(-3)+1}$

$= \frac{6}{5}$

17. $\lim_{h\to 0} \frac{(4+h)^2-16}{h} = \lim_{h\to 0} \frac{16+8h+h^2-16}{h}$

$= \lim_{h\to 0} \frac{h(8+h)}{h}$

$= \lim_{h\to 0} (8+h)$

$= 8$

19. $\lim_{x\to -2} \frac{x+2}{x^3+8} = \lim_{x\to -2} \frac{(x+2)}{(x+2)(x^2-2x+4)}$

$= \lim_{x\to -2} \frac{1}{x^2-2x+4}$

$= \frac{1}{(-2)^2-2(-2)+4}$

$= \frac{1}{12}$

21. $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}} = \lim_{t\to 9} \frac{9-t}{3-\sqrt{t}} \cdot \frac{3+\sqrt{t}}{3+\sqrt{t}}$

$= \lim_{t\to 9} \frac{(9-t)(3+\sqrt{t})}{(9-t)}$

$= \lim_{t\to 9} (3+\sqrt{t})$

$= 3+\sqrt{9} = 6$

23. $$\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7} = \lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}\cdot\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$$

$$= \lim_{x\to 7}\frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$= \lim_{x\to 7}\frac{x-7}{(x-7)(\sqrt{x+2}+3)}$$

$$= \lim_{x\to 7}\frac{1}{\sqrt{x+2}+3}$$

$$= \frac{1}{\sqrt{7+2}+3}$$

$$= \frac{1}{6}$$

25. $$\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4}\frac{x+4}{4x}\cdot\frac{1}{4+x}$$

$$= \lim_{x\to -4}\frac{1}{4x}$$

$$= \frac{1}{4(-4)}$$

$$= \frac{-1}{16}$$

27. $$\lim_{x\to 16}\frac{4-\sqrt{x}}{16x-x^2} = \lim_{x\to 16}\frac{4-\sqrt{x}}{x(16-x)}\cdot\frac{4+\sqrt{x}}{4+\sqrt{x}}$$

$$= \lim_{x\to 16}\frac{16-x}{x(16-x)(4+\sqrt{x})}$$

$$= \lim_{x\to 16}\frac{1}{x(4+\sqrt{x})}$$

$$= \frac{1}{16(4+\sqrt{16})}$$

$$= \frac{1}{128}$$

29. $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

$= \lim_{t \to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}}$

$= \lim_{t \to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}} \cdot \frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}$

$= \lim_{t \to 0} \frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \lim_{t \to 0} \frac{-t}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \lim_{t \to 0} \frac{-1}{\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \frac{-1}{\sqrt{1+0}\,(1+\sqrt{1+0})}$

$= \frac{-1}{2}$