14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

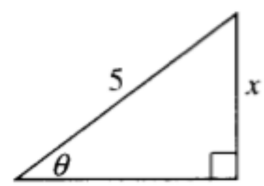
14. C $v(t) = 2t - 6$; $v(t) = 0$ for $t = 3$

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

20. E $\left.\frac{d\left(\sqrt[3]{x}\right)}{dt}\right|_{x=8} = \left.\frac{1}{3}x^{-\frac{2}{3}} \cdot \frac{dx}{dt}\right|_{x=8} = \frac{1}{3}(8)^{-\frac{2}{3}} \cdot \frac{dx}{dt} = \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{dx}{dt} = \frac{1}{12} \cdot \frac{dx}{dt} \Rightarrow k = 12$

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40



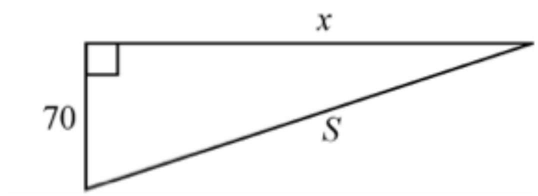
23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

79. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

81. A Let x be the distance of the train from the crossing. Then $\frac{dx}{dt} = 60$.



$S^2 = x^2 + 70^2 \Rightarrow 2S\frac{dS}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dS}{dt} = \frac{x}{S}\frac{dx}{dt}$.

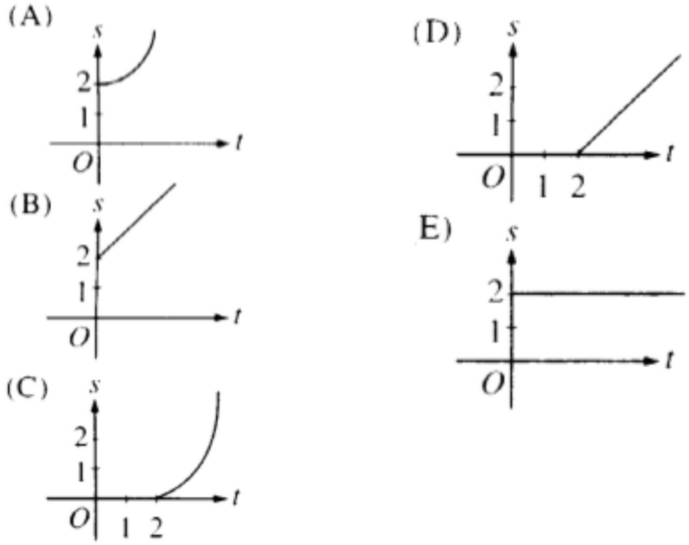
After 4 seconds, $x = 240$ and so $S = 250$.

Therefore $\frac{dS}{dt} = \frac{240}{250}(60) = 57.6$

23. E $x = 5\sin\theta$; $\frac{dx}{dt} = 5\cos\theta \cdot \frac{d\theta}{dt}$; When $x = 3, \cos\theta = \frac{4}{5}$; $\frac{dx}{dt} = 5\left(\frac{4}{5}\right)(3) = 12$

79. D Count the number of places where the graph of $y(t)$ has a horizontal tangent line. Six places.

90. A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t ?



t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

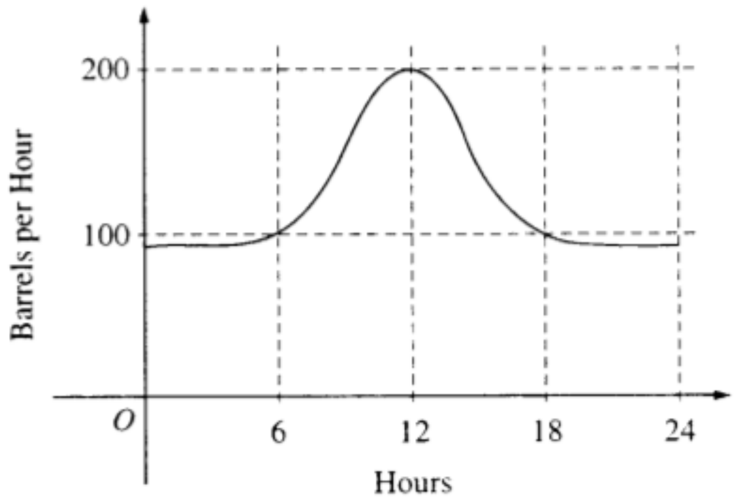
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

90. A Constant acceleration means linear velocity which in turn leads to quadratic position. Only the graph in (A) is quadratic with initial $s = 2$.

91. E $v(t) = 11 + \int_0^t a(x)\,dx \approx 11 + \left[2 \cdot 5 + 2 \cdot 2 + 2 \cdot 8\right] = 41$ ft/sec.

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

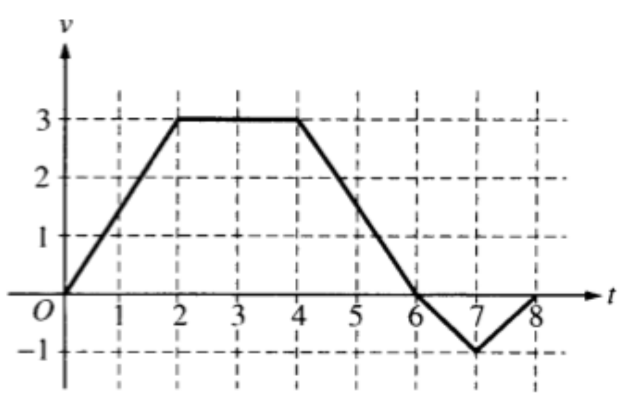


9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

39. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2



A bug begins to crawl up a vertical wire at time $t=0$. The velocity v of the bug at time t, $0\le t\le 8$, is given by the function whose graph is shown above.

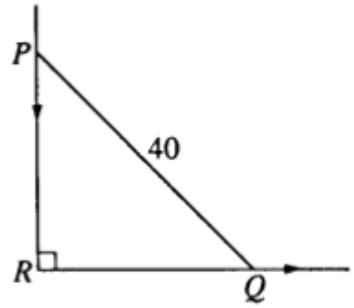
9. What is the total distance the bug traveled from $t=0$ to $t=8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about $3{,}000$ barrels.

39. C Want $\dfrac{y(4)-y(1)}{4-1}$ where $y(x)=\ln|x|+C$. This gives $\dfrac{\ln 4-\ln 1}{3}=\dfrac{1}{3}\ln 4=\dfrac{1}{3}\ln 2^2=\dfrac{2}{3}\ln 2$.

9. B Let A_1 be the area between the graph and t-axis for $0\le t\le 6$, and let A_2 be the area between the graph and the t-axis for $6\le t\le 8$ Then $A_1=12$ and $A_2=1$. The total distance is $A_1+A_2=13$.



34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

(A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

12. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) -9

34. E Let $y = PR$ and x=RQ.

$x^2 + y^2 = 40^2$, $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, $x \cdot \frac{3}{4}\left(-\frac{dy}{dt}\right) + y\frac{dy}{dt} = 0 \Rightarrow y = \frac{3}{4}x$.

Substitute into $x^2 + y^2 = 40^2$. $x^2 + \frac{9}{16}x^2 = 40^2$, $\frac{25}{16}x^2 = 40^2$, $x = 32$

4. B If $x = 2$ then $y = 5$. $x\frac{dy}{dt} + y\frac{dx}{dt} = 0$; $2(3) + 5 \cdot \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{6}{5}$

12. E $v(t) = 2\cos 2t + 3\sin 3t$, $a(t) = -4\sin 2t + 9\cos 3t$, $a(\pi) = -9$.

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

37. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$

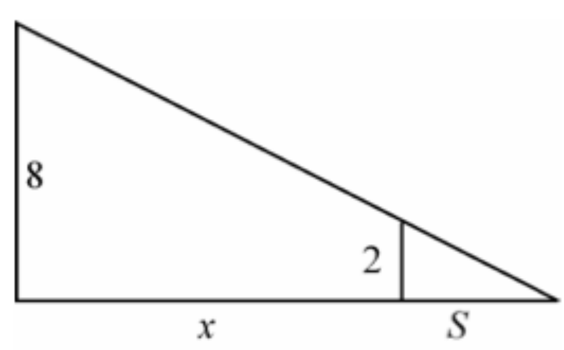
34. D Let x and y represent the horizontal and vertical sides of the triangle formed by the ladder, the wall, and the ground.

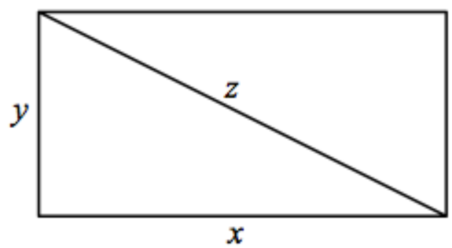
$x^2 + y^2 = 25$; $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$; $2(24)\frac{dx}{dt} + 2(7)(-3) = 0$; $\frac{dx}{dt} = \frac{7}{8}$.

39. D $A = \pi r^2$ and $C = 2\pi r$; $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ and $\frac{dC}{dt} = 2\pi\frac{dr}{dt}$. For $\frac{dA}{dt} = \frac{dC}{dt}$, $r = 1$.

37. D $\frac{x+S}{8} = \frac{S}{2} \Rightarrow x = 3S$

$\frac{dx}{dt} = 3\frac{dS}{dt} = 3\cdot\frac{4}{9} = \frac{4}{3}$





40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

40. B $x^2 + y^2 = z^2$, take the derivative of both sides with respect to t. $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$

Divide by 2 and substitute: $4 \cdot \frac{dx}{dt} + 3 \cdot \frac{1}{3}\frac{dx}{dt} = 5 \cdot 1 \Rightarrow \frac{dx}{dt} = 1$

26. D $v(t) = 4\sin t - t$; $a(t) = 4\cos t - 1 = 0$ at $t = \cos^{-1}(1/4) = 1.31812$; $v(1.31812) = 2.55487$

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

20. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

11. C $a(t)=6t-2; v(t)=3t^2-2t+C$ and $v(3)=25 \Rightarrow 25=27-6+C$; $v(t)=3t^2-2t+4$
$x(t)=t^3-t^2+4t+K$; Since $x(1)=10$, $K=6$; $x(t)=t^3-t^2+4t+6$.

20. E $v(t)=-\frac{1}{2}e^{-2t}+3$ and $x(t)=\frac{1}{4}e^{-2t}+3t+4$

22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

22. A $A=\pi r^2$, $A=64\pi$ when $r=8$. Take the derivative with respect to *t*.

$$\frac{dA}{dt}=2\pi r\cdot\frac{dr}{dt};\ 96\pi=2\pi(8)\cdot\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=6$$

25. C At rest when $v(t)=0$. $v(t)=e^{-2t}-2te^{-2t}=e^{-2t}(1-2t)$, $v(t)=0$ at $t=\frac{1}{2}$ only.

16. C At rest when $0=v(t)=x'(t)=3t^2-6t-9=3\left(t^2-2t-3\right)=3(t-3)(t+1)$

$t=-1, 3$ and $t\geq 0\Rightarrow t=3$

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

28. C Let $x(t) = -5t^2$ be the position at time t. Average velocity $= \dfrac{x(3) - x(0)}{3 - 0} = \dfrac{-45 - 0}{3} = -15$

31. C $V = \dfrac{1}{3}\pi r^2 h,\ \dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2rh\dfrac{dr}{dt} + r^2\dfrac{dh}{dt}\right) = \dfrac{1}{3}\pi\left(2(6)(9)\left(\dfrac{1}{2}\right) + 6^2\left(\dfrac{1}{2}\right)\right) = 24\pi$

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

26. E $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = S \cdot \frac{dr}{dt} = 100\pi(0.3) = 30\pi$

11. B $v(t) = 2t + 4 \Rightarrow a(t) = 2 \therefore\ a(4) = 2$

9. C $A = \pi r^2$, $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ and from the given information in the problem $\frac{dA}{dt} = 2\frac{dr}{dt}$.

So, $2\frac{dr}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow r = \frac{1}{\pi}$

12. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

(A) 20 m (B) 14 m (C) 7 m (D) 6 m (E) 3 m

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

15. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

(A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

12. B $v(t) = 3t + C$ and $v(2) = 10 \Rightarrow C = 4$ and $v(t) = 3t + 4$.

$$\text{Distance} = \int_0^2 (3t+4)\,dt = \frac{3}{2}x^2 + 4t\Big|_0^2 = 14$$

35. A $a(t) = 24t^2$, $v(t) = 8t^3 + C$ and $v(0) = 0 \Rightarrow C = 0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4\Big|_0^2 = 32$.

14. D Since $v(t) \geq 0$, distance = $\int_0^4 |v(t)|\,dt = \int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}\right) dt = \left(2t^{\frac{3}{2}} + 2t^{\frac{5}{2}}\right)\Big|_0^4 = 80$

15. C $x(t) = 4 + \int_0^t (2w-4)\,dw = 4 + (w^2 - 4w)\Big|_0^t = 4 + t^2 - 4t = t^2 - 4t + 4$

or, $x(t) = t^2 - 4t + C$, $x(0) = 4 \Rightarrow C = 4$ so, $x(t) = t^2 - 4t + 4$

3. A $\text{Distance} = \int_0^2 |v(t)|\,dt = \int_0^2 e^t\,dt = e^t\Big|_0^2 = e^2 - e^0 = e^2 - 1$

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

8. B Distance $= \int_1^2 \left| t^2 \right| dx = \int_1^2 t^2\, dt = \frac{1}{3}t^3 \Big|_1^2 = \frac{1}{3}(2^3 - 1^3) = \frac{7}{3}$

13. D $v(t) = 8t - 3t^2 + C$ and $v(1) = 25 \Rightarrow C = 20$ so $v(t) = 8t - 3t^2 + 20$.
$s(4) - s(2) = \int_2^4 v(t)\, dt = (4t^2 - t^3 + 20t)\Big|_2^4 = 32$

28. C $v(t) = 8 - 6t$ changes sign at $t = \frac{4}{3}$. Distance $= \left| x(1) - x\left(\frac{4}{3}\right) \right| + \left| x(2) - x\left(\frac{4}{3}\right) \right| = \frac{5}{3}$.

Alternative Solution: Distance $= \int_1^2 \left| v(t) \right| dt = \int_1^2 \left| 8 - 6t \right| dt = \frac{5}{3}$

6. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)^3(t-6)$. The particle is moving to the right for

(A) $0 < t < 5$

(B) $2 < t < 6$

(C) $t > 5$

(D) $t \geq 0$

(E) never

No Calculators

10. A particle starts at $(5, 0)$ when $t = 0$ and moves along the x-axis in such a way that at time $t > 0$ its velocity is given by $v(t) = \frac{1}{1+t}$. Determine the position of the particle at $t = 3$.

(A) $\frac{97}{16}$

(B) $\frac{95}{16}$

(C) $\frac{79}{16}$

(D) $1 + \ln 4$

(E) $5 + \ln 4$

Ans

6. C p. 25

$$x(t) = (t-2)^3 (t-6)$$
$$x'(t) = 3(t-2)^2 (t-6) + (t-2)^3$$
$$= (t-2)^2 [\,3(t-6) + (t-2)\,]$$
$$= (t-2)^2 (4t-20)$$
$$= 4(t-2)^2 (t-5)$$

This is positive-valued when $t > 5$.

10. E p. 48

$$v(t) = \frac{1}{1+t}$$
$$s(t) = \int v(t)\,dt = \ln|1+t| + C$$
$$s(0) = 5 \quad \Rightarrow \quad C = 5$$
$$s(t) = \ln|1+t| + 5 \quad \Rightarrow \quad s(3) = \ln(4) + 5$$

13. Two particles move along the x-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos 2t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0

(B) 1

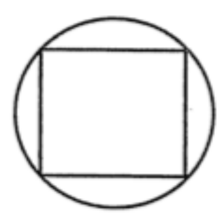
(C) 2

(D) 3

(E) 4

Calculator Active

4. A square is inscribed in a circle as shown in the figure at the right. As the square expands, the circle expands to maintain the four points of intersection. The perimeter of the square is increasing at the rate of 8 inches per second.
(For the circle: $A = \pi r^2$ and $C = 2\pi r$.)



(a) Find the rate at which the circumference of the circle is increasing.
(b) At the instant when the area of the square is 16 square inches, find the rate at which the area enclosed between the square and the circle is increasing.

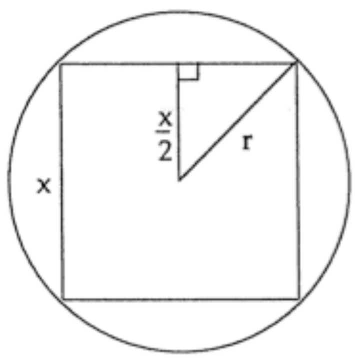
13. E p. 59

$x_1'(t) = -2\sin(2t)$

$x_2'(t) = \frac{1}{2}e^{(t-3)/2}$

Graph these two velocity functions. There are four intersection points.

4. p. 42



(a) Let the side of the square be denoted by x.

Since the perimeter $P = 4x$, we have $\frac{dP}{dt} = 4\frac{dx}{dt}$

Hence $\frac{dp}{dt} = 8 \quad \Rightarrow \quad \frac{dx}{dt} = 2.$

In addition, in the isosceles right triangle in the figure above,

$r = \frac{x}{2}\sqrt{2} = \frac{x}{\sqrt{2}}.$

$r = \frac{x}{\sqrt{2}} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{1}{\sqrt{2}}\frac{dx}{dt} = \sqrt{2}.$

$C = 2\pi r \quad \Rightarrow \quad \frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi\sqrt{2}.$

(b) $A = \pi r^2 - (r\sqrt{2})^2$

$= (\pi - 2)\, r^2$

$\frac{dA}{dt} = (\pi - 2)\, 2r\frac{dr}{dt}$

$= (\pi - 2)\,(2 \cdot 2\sqrt{2})\,\sqrt{2} = 8(\pi - 2)$

When the square has side 4, the circle has radius $2\sqrt{2}$.