Name: ______________________ Date: ______________ Row: _____ Period: ____ Assignment #: ____

SECTION 2.1 WORKSHEET

YOU MUST SHOW ALL WORK ON A SEPARATE SHEET OF PAPER

Section 2.1 The Tangent and Velocity Problems

This section illustrates two types of problems to which calculus may be applied:

- finding the slope of a line tangent to a curve, and
- finding the velocity of a moving object.

These are two examples of instantaneous rate of change found as a "limit" of average rates of change.

Concepts to Master

A. Slope of secant line to the graph of a function; Slope of tangent line

B. Interpretation of slope as instantaneous velocity

Summary and Focus Questions

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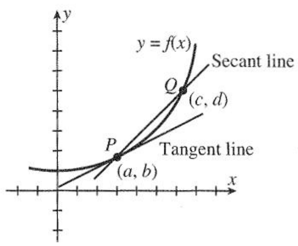
A. A **secant line** at $P(a, b)$ for the graph of $y = f(x)$ is a line joining P and another point Q also on the graph. If Q has coordinates (c, d), then the slope of the secant line is

$$\frac{d-b}{c-a}.$$



A **tangent line** to a graph of a function touches the graph at a point much like a tangent line to a circle.

To find the slope of the tangent line at P, we repeatedly select points Q closer and closer to P; the slopes of the secant lines become better and better estimates of the slope of the tangent line at P. Finally, our guess for the slope of the tangent line is the number that the slopes of the secant lines seem to be approaching as points Q get closer and closer to P.

Once we have determined the slope m of the tangent line to the curve, then using the point P as a point on the line, the equation of the tangent line is

$$y - b = m(x - a).$$

1) Let P be the point (3, 10) on the graph of $y = f(x)$. Each point Q in the table below is also on the graph of f. Find the slopes of each secant line PQ.

Q	Slope of PQ
(6, 18)	
(5, 15)	
(4, 12.3)	
(3.5, 11.1)	
(3.1, 10.21)	

2) What is your guess for the slope of the tangent line to $y = f(x)$ at $x = 3$ in question 1?

3) Let $P(1, 5)$ be a point on the graph of $f(x) = 6x - x^2$. Let $Q(x, f(x))$ be on the graph. Find the slope of the secant line PQ for each given x value for Q.

x	
3	
2	
1.5	
1.01	

4) Use your answer to question 3 to guess the slope of the tangent to $f(x)$ at P.

5) What is the equation of the tangent line to $f(x)$ at P in question 3?

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B. If $f(x)$ represents the distance an object is located from an initial starting point along an axis at time t, then

the slope of the secant line between points P and Q is the **average velocity** the object has traveled from P to Q.

the slope of the tangent line at the point P is the **instantaneous velocity** the object has at P.

Thus, given a distance function:

To find an average velocity, calculate the slope of a secant line.
To find an instantaneous velocity, calculate the slope of a tangent line.

6) Let $f(x) = x^2 + 3x$ be the distance in feet a race car has traveled from its starting point after x seconds.

a) How far has the race car traveled after 2 seconds? After 4 seconds?

b) what is the average velocity over the time interval $x = 2$ to $x = 4$?

c) what is the instantaneous velocity when $x = 2$?

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8) The distance that a runner is from the starting line is given in this table:

t (seconds)	0	1	2	3	4
d (meters)	0	4	9	16	24

a) Find the average velocity over the time intervals [1, 4], [1, 3], [1, 2], and [0, 1].

b) Estimate the instantaneous velocity at $t = 1$.