Area of a Circle

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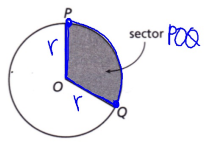
The area A of a circle with radius r is given by $A = \pi r^2$.

A **sector** of a circle is a region bounded by two radii and their intercepted arc. A sector is named by the endpoints of the arc and the center of the circle. For example, the figure shows sector POQ.

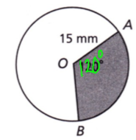
In the same way that you used proportional reasoning to find the length of an arc, you can use proportional reasoning to find the area of a sector.



CC.9-12.G.C.5

2 EXAMPLE Finding the Area of a Sector

Find the area of sector AOB. Express your answer in terms of π and rounded to the nearest tenth.



A First find the area of the circle.

$A = \pi r^2 = \pi(\underline{15})^2$ Substitute

$= \underline{225\pi}$ Simplify.

B The entire circle is 360°, but $\angle AOB$ measures 120°. Therefore, the sector's area is $\boxed{\frac{120}{360}}$ or $\frac{1}{3}$ of the circle's area.

Area of sector $AOB = \frac{1}{3} \cdot \underline{225\pi}$ The area is $\frac{1}{3}$ of the circle's area.

$= \underline{75\pi}$ Simplify.

$= \underline{235.6}$ Use a calculator to evaluate. Then round.

So, the area of sector AOB is $\underline{75\pi \text{ mm}^2}$ or $\underline{235.6 \text{ mm}^2}$

REFLECT

2a. How could you use the above process to find the area of a sector of the circle whose central angle measures $m°$?

$\frac{m°}{360°} \cdot 225\pi$

2b. Make a conjecture: What do you think is the formula for the area of a sector with a central angle of $m°$ and radius r?

$\frac{m°}{360°} \cdot \pi r^2$

The proportional reasoning process you used in the example can be generalized. Given a sector with a central angle of $m°$ and radius r, the area of the entire circle is πr^2 and the area of the sector is $\frac{m}{360}$ times the circle's area. This gives the following formula.

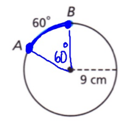
Area of a Sector

The area A of a sector of a circle with a central angle of $m°$ and radius r is given by $A = \frac{m}{360} \cdot \pi r^2$.

Arc length is understood to be the distance along a circular arc measured in linear units (such as feet or centimeters). You can use proportional reasoning to find arc lengths.

CC.9–12.G.CO.1

3 EXAMPLE Finding Arc Length



Find the arc length of $\overset{\frown}{AB}$. Express your answer in terms of π and rounded to the nearest tenth.

A First find the circumference of the circle.

$C = 2\pi r =$ 18π   Substitute 9 for r.

B The entire circle is 360°, but $\overset{\frown}{AB}$ measures 60°. Therefore, the arc's length is $\boxed{\frac{60}{360} \text{ or } \frac{1}{6}}$ of the circumference.

Arc length of $\overset{\frown}{AB} = \frac{1}{6} \cdot$ 18π   Arc length is $\frac{1}{6}$ of the circumference.

= 3π   Multiply.

= 9.4   Use a calculator to evaluate. Then round.

So, the arc length of $\overset{\frown}{AB}$ is 3π cm or 9.4 cm.

REFLECT

3a. How could you use the above process to find the length of an arc of the circle that measures $m°$?

$\frac{m°}{360°} \cdot 18\pi$

The proportional reasoning process you used above can be generalized. Given a circle with radius r, its circumference is $2\pi r$ and the arc length s of an arc with measure $m°$ is $\frac{m}{360}$ times the circumference. This gives the following formula.

Arc Length

The arc length s of an arc with measure $m°$ and radius r is given by the formula $s = \frac{m}{360} \cdot 2\pi r$.

circumference