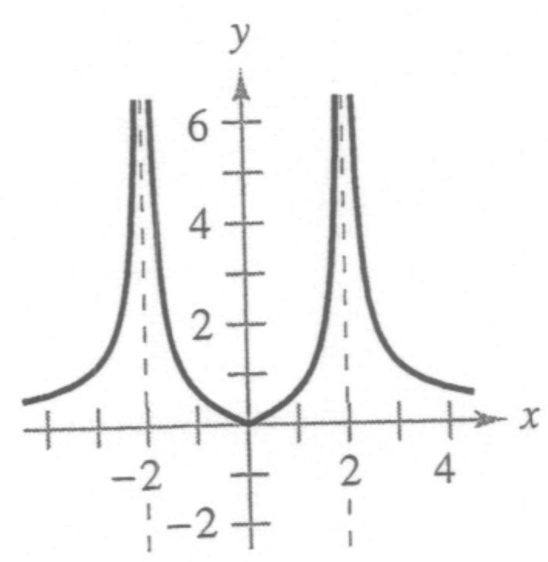
Exercises for Section 1.5

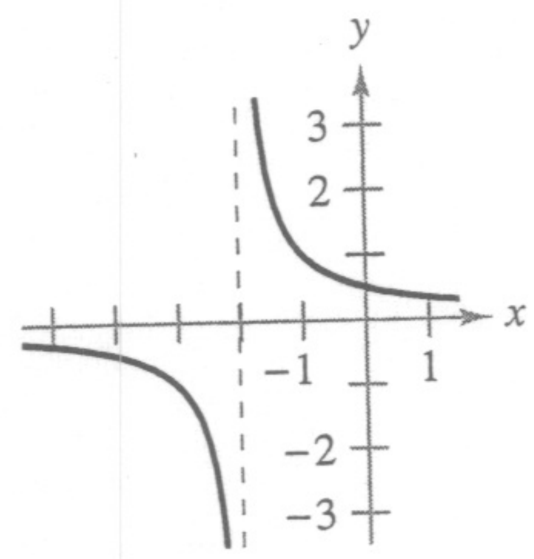
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

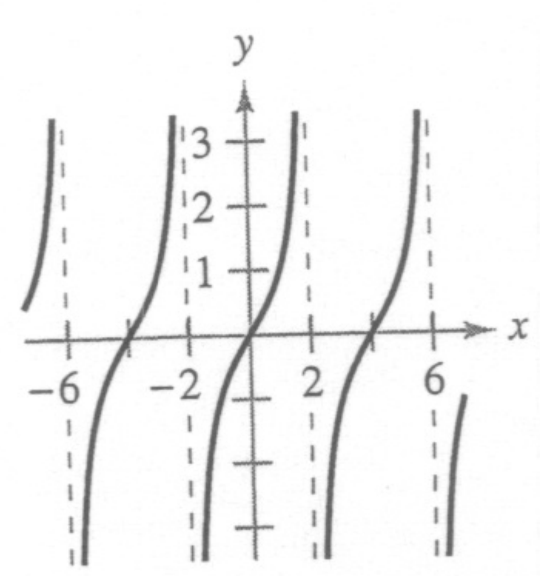
1. $f(x) = 2\left|\dfrac{x}{x^2 - 4}\right|$



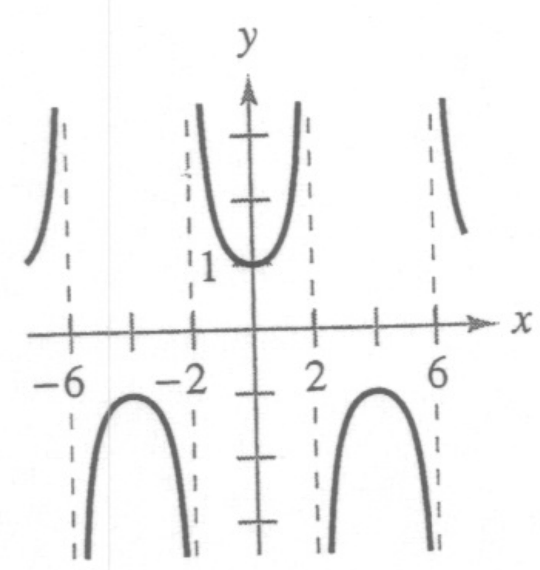
2. $f(x) = \dfrac{1}{x + 2}$



3. $f(x) = \tan \dfrac{\pi x}{4}$



4. $f(x) = \sec \dfrac{\pi x}{4}$



Numerical and Graphical Analysis **In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.**

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

5. $f(x) = \dfrac{1}{x^2 - 9}$

6. $f(x) = \dfrac{x}{x^2 - 9}$

7. $f(x) = \dfrac{x^2}{x^2 - 9}$

8. $f(x) = \sec \dfrac{\pi x}{6}$

In Exercises 9–28, find the vertical asymptotes (if any) of the graph of the function.

9. $f(x) = \dfrac{1}{x^2}$

10. $f(x) = \dfrac{4}{(x - 2)^3}$

11. $h(x) = \dfrac{x^2 - 2}{x^2 - x - 2}$

12. $g(x) = \dfrac{2 + x}{x^2(1 - x)}$

13. $f(x) = \dfrac{x^2}{x^2 - 4}$

14. $f(x) = \dfrac{-4x}{x^2 + 4}$

15. $g(t) = \dfrac{t - 1}{t^2 + 1}$

16. $h(s) = \dfrac{2s - 3}{s^2 - 25}$

17. $f(x) = \tan 2x$

18. $f(x) = \sec \pi x$

19. $T(t) = 1 - \dfrac{4}{t^2}$

20. $g(x) = \dfrac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$

21. $f(x) = \dfrac{x}{x^2 + x - 2}$

22. $f(x) = \dfrac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

23. $g(x) = \dfrac{x^3 + 1}{x + 1}$

24. $h(x) = \dfrac{x^2 - 4}{x^3 + 2x^2 + x + 2}$

25. $f(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

26. $h(t) = \dfrac{t^2 - 2t}{t^4 - 16}$

27. $s(t) = \dfrac{t}{\sin t}$

28. $g(\theta) = \dfrac{\tan \theta}{\theta}$

In Exercises 29–32, determine whether the graph of the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

29. $f(x) = \dfrac{x^2 - 1}{x + 1}$

30. $f(x) = \dfrac{x^2 - 6x - 7}{x + 1}$

31. $f(x) = \dfrac{x^2 + 1}{x + 1}$

32. $f(x) = \dfrac{\sin(x + 1)}{x + 1}$

In Exercises 33–48, find the limit.

33. $\displaystyle\lim_{x \to 2^+} \frac{x - 3}{x - 2}$

34. $\displaystyle\lim_{x \to 1^+} \frac{2 + x}{1 - x}$

35. $\displaystyle\lim_{x \to 3^+} \frac{x^2}{x^2 - 9}$

36. $\displaystyle\lim_{x \to 4^-} \frac{x^2}{x^2 + 16}$

37. $\displaystyle\lim_{x \to -3^-} \frac{x^2 + 2x - 3}{x^2 + x - 6}$

38. $\displaystyle\lim_{x \to (-1/2)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3}$

39. $\displaystyle\lim_{x \to 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)}$

40. $\displaystyle\lim_{x \to 3} \frac{x - 2}{x^2}$

41. $\displaystyle\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right)$

42. $\displaystyle\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right)$

43. $\displaystyle\lim_{x \to 0^+} \frac{2}{\sin x}$

44. $\displaystyle\lim_{x \to (\pi/2)^+} \frac{-2}{\cos x}$

45. $\displaystyle\lim_{x \to \pi} \frac{\sqrt{x}}{\csc x}$

46. $\displaystyle\lim_{x \to 0} \frac{x + 2}{\cot x}$

47. $\displaystyle\lim_{x \to 1/2} x \sec \pi x$

48. $\displaystyle\lim_{x \to 1/2} x^2 \tan \pi x$

In Exercises 49–52, use a graphing utility to graph the function and determine the one-sided limit.

49. $f(x) = \dfrac{x^2 + x + 1}{x^3 - 1}$

$\displaystyle\lim_{x \to 1^+} f(x)$

50. $f(x) = \dfrac{x^3 - 1}{x^2 + x + 1}$

$\displaystyle\lim_{x \to 1^-} f(x)$

51. $f(x) = \dfrac{1}{x^2 - 25}$

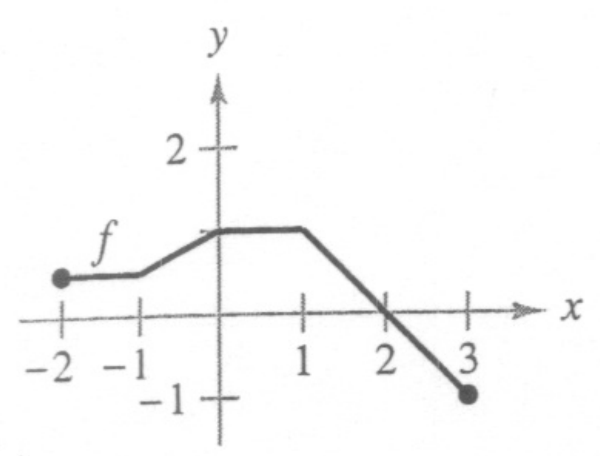
$\displaystyle\lim_{x \to 5^-} f(x)$

52. $f(x) = \sec \dfrac{\pi x}{6}$

$\displaystyle\lim_{x \to 3^+} f(x)$

Writing About Concepts

53. In your own words, describe the meaning of an infinite limit. Is ∞ a real number?
54. In your own words, describe what is meant by an asymptote of a graph.
55. Write a rational function with vertical asymptotes at $x = 6$ and $x = -2$, and with a zero at $x = 3$.
56. Does the graph of every rational function have a vertical asymptote? Explain.
57. Use the graph of the function f (see figure) to sketch the graph of $g(x) = 1/f(x)$ on the interval $[-2, 3]$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*



58. ***Boyle's Law*** For a quantity of gas at a constant temperature, the pressure P is inversely proportional to the volume V. Find the limit of P as $V \to 0^+$.

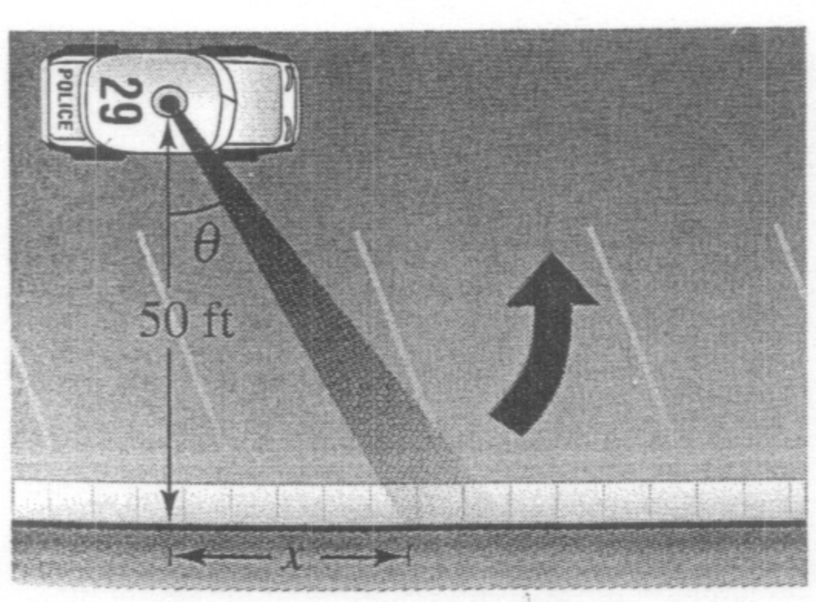
59. ***Rate of Change*** A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is

$r = 50\pi \sec^2 \theta$ ft/sec.

(a) Find the rate r when θ is $\pi/6$.

(b) Find the rate r when θ is $\pi/3$.

(c) Find the limit of r as $\theta \to (\pi/2)^-$.



60. ***Illegal Drugs*** The cost in millions of dollars for a governmental agency to seize $x\%$ of an illegal drug is

$$C = \frac{528x}{100 - x}, \quad 0 \le x < 100.$$

(a) Find the cost of seizing 25% of the drug.

(b) Find the cost of seizing 50% of the drug.

(c) Find the cost of seizing 75% of the drug.

(d) Find the limit of C as $x \to 100^-$ and interpret its meaning.

61. ***Relativity*** According to the theory of relativity, the mass m of a particle depends on its velocity v. That is,

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$$

where m_0 is the mass when the particle is at rest and c is the speed of light. Find the limit of the mass as v approaches c^-.

62. ***Rate of Change*** A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

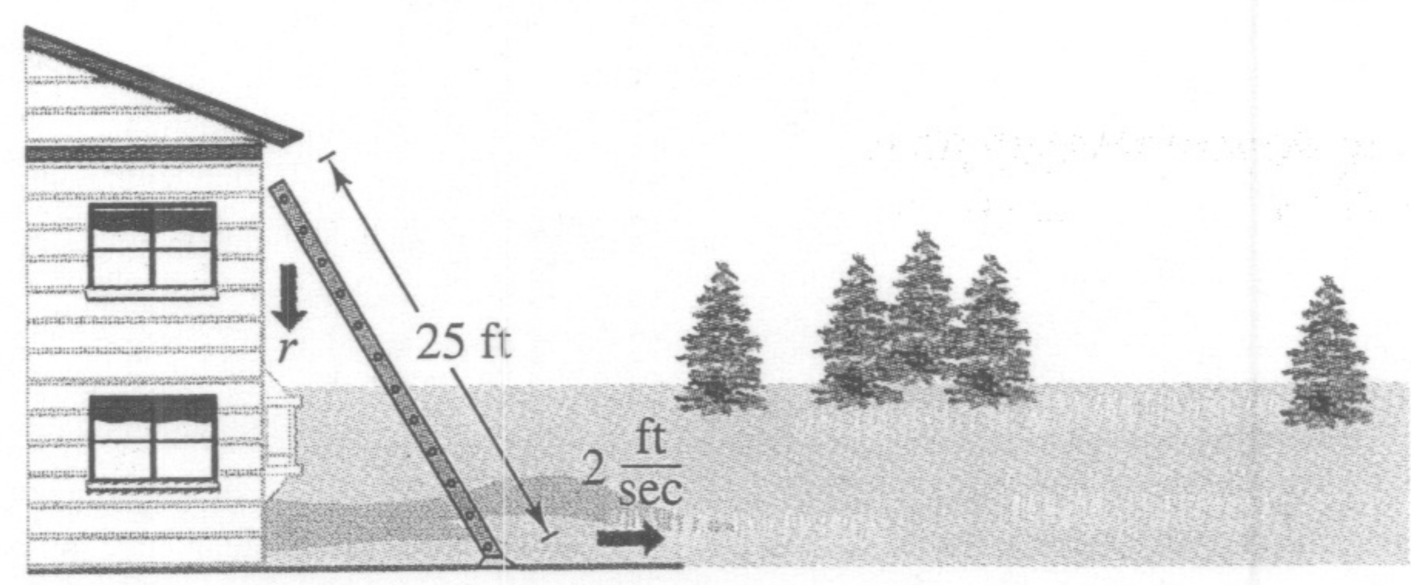
$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec}$$

where x is the distance between the base of the ladder and the house.

(a) Find the rate r when x is 7 feet.

(b) Find the rate r when x is 15 feet.

(c) Find the limit of r as $x \to 25^-$.



63. ***Average Speed*** On a trip of d miles to another city, a truck driver's average speed was x miles per hour. On the return trip the average speed was y miles per hour. The average speed for the round trip was 50 miles per hour.

(a) Verify that $y = \frac{25x}{x - 25}$. What is the domain?

(b) Complete the table.

x	30	40	50	60
y				

Are the values of y different than you expected? Explain.

(c) Find the limit of y as $x \to 25^+$ and interpret its meaning.

64. ***Numerical and Graphical Analysis*** Use a graphing utility to complete the table for each function and graph each function to estimate the limit. What is the value of the limit when the power on x in the denominator is greater than 3?

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$							

(a) $\lim\limits_{x \to 0^+} \frac{x - \sin x}{x}$ (b) $\lim\limits_{x \to 0^+} \frac{x - \sin x}{x^2}$

(c) $\lim\limits_{x \to 0^+} \frac{x - \sin x}{x^3}$ (d) $\lim\limits_{x \to 0^+} \frac{x - \sin x}{x^4}$