11.4 Infinite Geometric Series

std. 22.0

March 7

compare sums for these series:

$1 + 2 + 4 + 8 + 16 + \ldots$

$S_1 = 1$

$S_2 = 3$

$S_3 = 7$

$S_4 = 15$

$S_5 = 31$

$S_n = 2^n - 1$

$r = 2$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$S_1 = 1$

$S_2 = 1\frac{1}{2}$

$S_3 = 1\frac{3}{4}$

$r = \frac{1}{2}$

$S_4 = 1\frac{7}{8}$

$S_5 = 1\frac{15}{16}$

$S_n \approx 2$

infinite geometric series sum: if $|r|<1$ $S=\frac{a_1}{1-r}$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

as $n \to \infty$, $r^n \to 0$ if $|r|<1$, $1-r^n = 1$

$-1 < r < 1$ $r \neq 0$

ex. 1 **Find the sum:**

a) $-30 + 15 - \frac{15}{2} + \frac{15}{4} - \ldots$

$a_1 = -30$

$r = -\frac{1}{2}$

$S = \frac{a_1}{1-r}$

$= \frac{-30}{1-\left(\frac{-1}{2}\right)} = \frac{-30}{\frac{3}{2}} = -20$

b) $\sum_{i=1}^{\infty} 2(0.1)^{i-1}$

$a_1 = 2$ $r = .1$

$S = \frac{2}{1-.1}$

$\frac{20}{.9} = \frac{20}{9}$

ex. 2 **Use a series to write $\overline{.63}$ as a fraction.**

$r = .01$

$a_1 = .63$

$= .63 + .0063 + .000063 + \cdots$

$\times .01 \quad \times .01$

$S = \frac{.63}{1 - .01} = \frac{7}{11}$

ex. 3 **An infinite geometric series with $a_1 = 5$ has sum $S_n = \frac{27}{5}$. Find the common ratio.**

$S = \frac{a_1}{1 - r}$

$\frac{27}{5} = \frac{5}{1 - r}$

$27 - 27r = 25$

$2 = 27r$

$\frac{2}{27} = r$